'Output-Only' Vibration based Condition Monitoring of Dynamical Systems

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CIVIL STRUCTURES

➤ MODAL IDENTIFICATION
  • Basic problem statement
  • Blind source separation

MECHANICAL SYSTEMS

➤ FAULT DETECTION
  • Basic problem statement
  • Synchrosqueezing transform
  • Time varying auto-regressive modelling
OUTPUT ONLY MODAL IDENTIFICATION

- Estimation of dynamic properties from response
  - Natural frequency
  - Damping
  - Mode-shapes (ψ)

INPUT data is a luxury

\[ \omega = \frac{1}{T} \]

\[ \xi_r \]

\[ T \]

\[ \omega = \frac{1}{T} \]
BLIND IDENTIFICATION

- Mathematical statement:
  \[ x = As \quad \Rightarrow \quad X = \Psi Q \]

- Estimate \( A \) and \( s \) using the information in \( x \)
- Infinite sets of \( A \) and \( s \) yield same \( x \)
- \( Q \) is the modal response
- Identify \( \omega_i, \xi_i, \psi \) using outputs only
- Synonymously used terms: Ambient modal identification, ambient vibration monitoring

*Traditional methods like SSI, NeXT-ERA, FDD etc. suffer from issues with model order, closely spaced modes, poor noise performance & poor damping estimates*
BLIND SOURCE SEPARATION

Sources

$\{s_1, s_2\}$

$\mathbf{s}$ sources ($m = 2$)

Mixing matrix $A$

$\mathbf{x} = \mathbf{A}\mathbf{s}$

Observations

$x_1$, $x_2$

$n$ observations ($n = 2$)

Second Order Blind Identification is a popular BSS method
SECOND ORDER BSS

• Uses 2\textsuperscript{nd} order temporal statistics of signals

STEPS:

• Orthogonalization: 
  \[ \hat{R}_x(0) = V_x \Lambda_x V_x^T \]
  \[ z = W x \]

• Unitary Transformation: 
  \[ \hat{R}_z(p) = V_z \Lambda_z V_z^T \]
  \[ \hat{A} = \hat{W}^{-1} V_z \]

*Second order blind identification (Belouchrani 1997)*
MODIFIED CROSS CORRELATION METHOD (MCC)

Formulation

Natural excitation technique allows forced responses to be cast into free responses (James & Farrar 1997)
• Orthogonalization: \( \tilde{r}(k) = Qr(k) \)

• Windowing

• Unitary Factorization: \( D(L, p) = V^T \hat{R}(p)V \)

\[
\langle \hat{R}(p) \rangle = \left\{ \begin{bmatrix} \vdots & \vdots & \vdots \\ 1 & \cdots & \cdots \\ \vdots & \vdots & \vdots \\ \end{bmatrix} \begin{bmatrix} \vdots & \vdots & \vdots \\ \cdots & \cdots & \cdots \\ \vdots & \vdots & \vdots \\ \end{bmatrix} \cdots \begin{bmatrix} \vdots & \vdots & \vdots \\ \cdots & \cdots & \cdots \\ \vdots & \vdots & \vdots \\ \end{bmatrix} \right\}
\]

• Joint Diagonalization: \( J(V, p) = \sum_{L} \left( \frac{1}{p} \sum_{p} \sum_{1 \leq i \neq j \leq m} |D_{ij}^p|^2 \right) \)

“Modified Cross-Correlation for blind Identification of Structures”
Matrix $V$ is a joint diagonalizer: $V^T R V = \text{diag} \left( d_1^2, d_2^2, \ldots \right)$

Transformation matrix: $V = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$
\[ \tan 2\theta = \frac{t_{off}}{t_{on} + (t_{on}^2 + t_{off}^2)^{1/2}} \]

\[ t_{on} = \mathbf{G}(1, 1) - \mathbf{G}(2, 2) \text{ and } t_{off} = \mathbf{G}(1, 2) + \mathbf{G}(2, 1) \]

\[
\mathbf{G} = \begin{bmatrix}
    a_k - d_k & e_k - g_k & m_k - p_k & \cdots & \cdots \\
    2b_k & 2f_k & 2n_k & \cdots & \cdots \\
\end{bmatrix}^T
\]

**Jacobi angles for JAD** *(Cardoso 1996)*
WMCC

• SWT of measurement ($r$) and source ($s_r$)

$$s_{r_m}^j (t) = \sum_k c_{km}^j \psi_{k}^j (t)$$

$$r_m^j (t) = \sum_k f_{km}^j \psi_{k}^j (t)$$

Shift invariance

$$d_k^j(x(t - \tau)) = d_k^j(x(t))$$

• Transformation: $r = A_r s_r \quad \rightarrow \quad f_1 = A_r c_1$

• Wavelet Covariance Matrix:

$$R_{s_r} (\tau) = R_{wsr} (\tau) = E \left[ c_l^j c_{l+\tau}^j \right]$$

$$R_r (\tau) = R_{wr} (\tau) = E \left[ f_l^j f_{l+\tau}^j \right]$$

*Shift invariance of SWT is the key trick utilized to formulate the wavelet covariance matrix. (Hazra and Narasimhan (2010))*
MCC & WMCC outperform conventional modal ID methods like SSI for high damping. Similar conclusions can be drawn for performance in presence of noise & damping estimates.
Applications: Separation of closely spaced modes, Underdetermined blind identification, Retuning of tuned mass dampers
**HYBRID BSS-1: EMD-MCC**

- Assumed mode shapes: Sinusoidal shape functions with stochastic shape parameters:

\[ f^i \left( \frac{x}{l} \right) = \sin \left[ \beta_i \frac{\pi}{2} \left( \frac{x}{l} - \delta_i \right) \right] \Rightarrow A_a \]

- Pseudo responses: \( \tilde{x}_e \approx A_a \Gamma \)

- Iterative update: \( \hat{A}_u = Q^{-1} V_{\tilde{r}_e} \Rightarrow \tilde{s}_e = \hat{A}^{-1}_u \tilde{x}_e \)

- Termination:

\[ \left\| \frac{\tilde{s}_e^j(\omega) - \tilde{s}_e^{j-1}(\omega)}{\tilde{s}_e^{j-1}(\omega)} \right\| = \| \gamma \| \leq \varepsilon \]

Pearson International Airport, Toronto

“Hybrid Time-Frequency Blind Source Separation Towards Ambient System Identification of Structures”. Hazra et al. (2011)
TABLE: Identified natural frequencies and damping for TMD restrained case using EMD-MCC (PARTIAL SENSOR)

<table>
<thead>
<tr>
<th>Mode #</th>
<th>Model ( f ) (Hz)</th>
<th>Natural frequencies (Hz)</th>
<th>Damping (% Critical)</th>
<th>MAC</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Mean (COV(%))</td>
<td>Mean (COV(%))</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>6-sensors 3-sensors</td>
<td>6-sensors 3-sensors</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.66</td>
<td>0.67(1.00) 0.67(1.30)</td>
<td>0.85(20.0) 0.85(22.0)</td>
<td>0.99</td>
</tr>
<tr>
<td>2</td>
<td>0.92</td>
<td>0.83(1.20) 0.83(1.20)</td>
<td>1.98(17.0) 1.92(20.0)</td>
<td>0.96</td>
</tr>
<tr>
<td>3</td>
<td>1.40</td>
<td>1.44(1.80) 1.48(2.90)</td>
<td>2.80(13.0) 2.70(16.0)</td>
<td>0.99</td>
</tr>
<tr>
<td>4</td>
<td>2.73</td>
<td>2.69(1.15) 2.67(1.40)</td>
<td>1.30(14.7) 1.50(15.8)</td>
<td>0.99</td>
</tr>
<tr>
<td>5</td>
<td>3.01</td>
<td>3.06(2.25) 3.13(3.00)</td>
<td>2.75(14.0) 2.65(15.3)</td>
<td>0.99</td>
</tr>
<tr>
<td>6</td>
<td>3.36</td>
<td>3.40(1.85) 3.42(2.75)</td>
<td>4.50(17.0) 4.35(18.5)</td>
<td>0.99</td>
</tr>
<tr>
<td>7</td>
<td>4.59</td>
<td>4.56(1.00) 4.64(1.35)</td>
<td>2.56(16.1) 2.68(17.5)</td>
<td>0.98</td>
</tr>
<tr>
<td>8</td>
<td>6.08</td>
<td>6.02(1.25) 6.15(2.15)</td>
<td>2.95(17.0) 2.90(19.2)</td>
<td>0.98</td>
</tr>
<tr>
<td>9</td>
<td>6.71</td>
<td>6.75(2.35) 6.62(3.00)</td>
<td>4.85(20.3) 4.70(21.0)</td>
<td>0.99</td>
</tr>
<tr>
<td>10</td>
<td>9.01</td>
<td>8.75(1.80) 8.71(2.25)</td>
<td>3.15(15.3) 3.19(19.0)</td>
<td>0.97</td>
</tr>
<tr>
<td>11</td>
<td>9.43</td>
<td>9.36(2.20) 9.52(3.20)</td>
<td>2.75(15.0) 2.72(18.0)</td>
<td>0.98</td>
</tr>
<tr>
<td>12</td>
<td>10.09</td>
<td>10.02(1.80) 10.19(2.30)</td>
<td>4.91(21.0) 4.88(23.0)</td>
<td>0.98</td>
</tr>
</tbody>
</table>

TABLE: Identified natural frequencies and damping for TMD unrestrained case using EMD-MCC

<table>
<thead>
<tr>
<th>Mode #</th>
<th>WMCC</th>
<th>Hybrid-EMD</th>
<th>SSI</th>
<th>WMCC</th>
<th>Hybrid-EMD</th>
<th>SSI</th>
</tr>
</thead>
<tbody>
<tr>
<td>1*</td>
<td>–</td>
<td>0.58(2.16)</td>
<td>–</td>
<td>–</td>
<td>5.81(22.0)</td>
<td>–</td>
</tr>
<tr>
<td>1</td>
<td>0.67(1.50)</td>
<td>0.68(1.30)</td>
<td>0.66(1.60)</td>
<td>0.95(26.0)</td>
<td>2.80(14.0)</td>
<td>1.36(33.0)</td>
</tr>
<tr>
<td>2</td>
<td>0.83(3.5)</td>
<td>0.80(2.60)</td>
<td>0.82(3.6)</td>
<td>4.90(33.0)</td>
<td>3.12(22.0)</td>
<td>7.42(42.0)</td>
</tr>
<tr>
<td>3</td>
<td>1.47(2.90)</td>
<td>1.44(2.10)</td>
<td>1.50(3.10)</td>
<td>2.85(25.0)</td>
<td>2.70(18.0)</td>
<td>3.80(31.0)</td>
</tr>
<tr>
<td>4</td>
<td>2.66(1.60)</td>
<td>2.68(2.30)</td>
<td>2.63(3.30)</td>
<td>1.39(18.0)</td>
<td>1.50(15.2)</td>
<td>2.50(29.3)</td>
</tr>
<tr>
<td>5</td>
<td>3.05(1.10)</td>
<td>3.07(2.00)</td>
<td>2.98(3.10)</td>
<td>2.82(17.2)</td>
<td>2.65(15.3)</td>
<td>3.85(29.3)</td>
</tr>
<tr>
<td>6</td>
<td>3.40(1.65)</td>
<td>3.40(1.75)</td>
<td>3.56(5.75)</td>
<td>4.63(22.3)</td>
<td>4.35(18.2)</td>
<td>6.25(39.2)</td>
</tr>
<tr>
<td>7</td>
<td>4.63(1.00)</td>
<td>4.63(0.98)</td>
<td>4.78(4.00)</td>
<td>2.71(16.8)</td>
<td>2.62(14.0)</td>
<td>4.12(34.0)</td>
</tr>
<tr>
<td>8</td>
<td>6.00(1.41)</td>
<td>6.13(1.35)</td>
<td>6.26(4.35)</td>
<td>3.06(21.5)</td>
<td>2.90(19.0)</td>
<td>4.80(42.0)</td>
</tr>
<tr>
<td>9</td>
<td>6.62(2.50)</td>
<td>6.66(1.85)</td>
<td>–</td>
<td>4.69(25.2)</td>
<td>4.70(21.0)</td>
<td>–</td>
</tr>
<tr>
<td>10</td>
<td>8.79(2.20)</td>
<td>8.80(1.90)</td>
<td>–</td>
<td>3.39(26.0)</td>
<td>3.18(19.0)</td>
<td>–</td>
</tr>
<tr>
<td>11</td>
<td>–</td>
<td>9.35(2.38)</td>
<td>–</td>
<td>–</td>
<td>2.68(16.2)</td>
<td>–</td>
</tr>
<tr>
<td>12</td>
<td>–</td>
<td>9.98(1.00)</td>
<td>–</td>
<td>–</td>
<td>4.82(23.0)</td>
<td>–</td>
</tr>
</tbody>
</table>
Modes corresponding to $0.58 \text{ Hz}$ & $0.68 \text{ Hz}$ are closely spaced
Another way of solving the underdetermined problem

\[ f_{p,k}^{2,v} = A_{pl} e_{i,k}^{2,v} \]

\[ p = [i, j, \ldots]^{T} \times n_{m} \]
\[ l = [i, j, \ldots]^{T} \times n_{s} \]

\( n_{m} \) = no of measurements
\( n_{s} \) = no of sources
\( k \) = shift index
\( v \) = packet number

Line of orientation = mode-shape
SPARSE BSS

Broadband Excitation

\[ |X_5(\omega)| \]

\[ |X_4(\omega)| \]

\[ \left| F_5^{1,0}(\omega) \right| \]
\[ \left| F_5^{2,0}(\omega) \right| \]
\[ \left| F_5^{3,0}(\omega) \right| \]

\[ \left| F_5^{4,0}(\omega) \right| \]
\[ \left| F_5^{5,0}(\omega) \right| \]
\[ \left| F_5^{6,0}(\omega) \right| \]
SPARSE BSS

\[ f_{4,0}^{6,0}, v(t) \]

\[ F_5^{6,0}(\omega), F_4^{6,0}(\omega) \]

\[ \sigma_2^2 = 0.0001, \sigma_1^2 = 0.16 \]

\[ \text{RCN} = 0.0006, m = 0.82 \]

\[ m_1 = 0.82 \]

\[ \omega_1 = 0.91 \text{ Hz} \]

\[ m_2 = -0.092 \]

\[ \omega_2 = 3.39 \text{ Hz} \]

\[ m_3 = -1.29 \]

\[ \omega_3 = 7.10 \text{ Hz} \]

\[ m_4 = -2.5 \]

\[ \omega_4 = 10.65 \text{ Hz} \]

\[ m_5 = -3.31 \]

\[ \omega_5 = 12.75 \text{ Hz} \]
Closure on BSS based SHM

- BSS is a robust tool for SHM/ambient vibration monitoring
- BSS methods allow integrating time-frequency decompositions directly into one formulation
- The notion of under-determined BI using hybrid BSS is introduced in the field of ambient vibration monitoring
- EMD-MCC identifies full scale structures more accurately with fewer measurements compared to SSI
- Sparse-BSS can be used as a tool to solve underdetermined problems with applications to wireless SHM
Mechanical systems: Gearbox fault diagnosis

- Gearbox dynamics is difficult: rotor-dynamics, unbalance, misalignment, shaft vibration, gear meshing dynamics, bearing dynamics – convolved effects

- Convolutive mixing problem: Limited success using BSS

- Gearbox fault diagnosis:
  
  Signal processing to generate features → Pattern classification

- Operating conditions can also change: fluctuating rpms, loads
HISTORY & BACKGROUND

Where it all came from?

Gear-motors of Toronto airport baggage handling system
June – August 2013
Processing of gearbox signals

- **Traditional approaches**: Fourier spectra, cepstrum, envelop spectrum

- **Time frequency techniques**: STFT, Wavelets, Wigner Ville distribution

- **Parametric approaches**: AR, ARMA, VARMA, KF, EKF based approaches

- **Modern techniques**: Empirical mode decomposition (EMD), Hilbert-Huang transform, blind source separation (BSS)
Synchro-squeezing transform

- **Motivation:** Fuzzy frequency resolution of CWT around an instantaneous frequency
- **Nonlinear squeezing** to evaluate instantaneous frequencies and reduce smearing
Basic steps in SST

- Calculate the CWT of the signal \( s(t) \)

\[
W_s(a, b) = \frac{1}{a} \int_R s(t) \psi \left( \frac{t - b}{a} \right) dt
\]

- Calculate the instantaneous frequencies

\[
\omega(a, b) = \frac{-i}{W_s(a, b)} \frac{\partial}{\partial b} W_s(a, b)
\]

- Apply Synchro-squeezing at frequency centers of successive bins \([\omega_c - \frac{1}{2} \Delta \omega, \omega_c + \frac{1}{2} \Delta \omega]\)

\[
T_s(\omega_c, b) = \frac{1}{\Delta \omega} \sum_{a_k : |\omega(a_k, b) - \omega_c| \leq \frac{\Delta \omega}{2}} W_s(a, b) a_k^{-\frac{3}{2}} \Delta a_k
\]
Signal decomposition

• IMF extraction around a curve \( Y_t = \{a : \omega \approx c_t \} \)

\[
h(t) = \int_{\omega \approx c(t)} T_s(\omega, t) d\omega
\]

• The curve satisfies the inequality:

\[
Y_t = \left\{ a : \frac{\omega \psi - \varepsilon \psi}{c_t} \leq a \leq \frac{\omega \psi + \varepsilon \psi}{c_t} \right\}
\]

• Daubechies, Lu & Wu (2011)
• Oberlin & Meignen (2012)
NUMERICAL EXAMPLE

Results of decomposition
Fault detection: TVAR modelling

\[ x(n) = [a_1(n), a_2(n), \ldots, a_p(n)]^T \]

\[ x(n) = \Gamma(n - 1)x(n - 1) + w(n) \]

\[ y(n) = C(n)x(n) + v(n) \]

Kalman gain

\[ K(n) = P_x(n|n - 1)C(n)^T \sigma_y^2(n|n - 1)^{-1} \]

\[ x(n|n) = x(n|n - 1) + K(n)(y(n) - y(n|n - 1)) \]

- TVAR coeff
- States

State space model
Fault detection: example

Signal: 3-sinusoids with baseband freq 320 Hz, 340 Hz & 360 Hz and with FM frequencies of 10 Hz, 30 Hz and 65 Hz concatenated end to end
Case study: DDS

http://spectraquest.com/prognostics/details/dds/
DDS experiments

<table>
<thead>
<tr>
<th>STATE</th>
<th>Data range</th>
<th>Fault state</th>
<th>Braking state</th>
<th>RPM</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1-50K</td>
<td>NO</td>
<td>3V</td>
<td>10</td>
</tr>
<tr>
<td>2</td>
<td>50K-100K</td>
<td>NO</td>
<td>NO</td>
<td>20</td>
</tr>
<tr>
<td>3</td>
<td>100K-150K</td>
<td>NO</td>
<td>3V</td>
<td>20</td>
</tr>
<tr>
<td>4</td>
<td>150K-200K</td>
<td>Chipped</td>
<td>NO</td>
<td>20</td>
</tr>
<tr>
<td>5</td>
<td>200K-250K</td>
<td>Chipped</td>
<td>3V</td>
<td>20</td>
</tr>
<tr>
<td>6</td>
<td>250K-300K</td>
<td>Chipped</td>
<td>3V</td>
<td>40</td>
</tr>
</tbody>
</table>

AIM: Extraction of 6 states from the data & 2 fault conditions

RAW ACCELERATION DATA
Case study: DDS

Hazra et al. (2015) (Journal of Vibration and Control)
Case study: DDS

Hazra et al. (2015) (Journal of Vibration and Control)
$C'(k) = (1 - \frac{1}{k})C(k - 1) + \frac{1}{k}x(k)x(k)x(k)x(k)$

$\kappa(k) = \frac{C(k)}{\sigma^2(k)^2} - 3$
Automated widow selection using Sequential Karhunen Loeve Transform applied on the recursive CI’s

Hazra et.al (2015) (Journal of vibration and Control)
Case study: DDS

Hazra et.al (2015) (Journal of vibration and Control)
Case study: DDS

Hazra et al. (2015) (Journal of vibration and Control)
Closure

• Synchro-squeezing transform is introduced in the context of rotating machinery diagnosis
• SST provides sharper frequency TF-representation than CWT
• SST is also able to decompose non-stationary AM-FM like signals accurately
• Sequential karhunen Loeve transform can be utilized for automatic window selection
• Time varying auto-regressive model applied on the extracted IMFs generates diagnostic feature space
• The framework facilitated separation of operating condition fluctuations and faulted states successfully
Acknowledgements

- Erasmus Mundus Euphrates
- IIT-Guwahati
- Prof. Paul Fanning, UCD
- Prof. Vikram Pakrashi, UCD
Thank you for patient hearing
EXTRA SLIDES
Automated window selection

- \( \alpha = \{X_1, \ldots, X_q\} \)
- \( \alpha = U\Sigma V \)
- \( p \times (q + r) \) matrix \( \beta \)

\[
\begin{bmatrix}
\alpha \\
\beta
\end{bmatrix} =
\begin{bmatrix}
U & \bar{\beta}
\end{bmatrix}
\begin{bmatrix}
\sum_0 & \bar{\beta} \\
0 & \bar{\beta}^T \bar{\beta}
\end{bmatrix}
\begin{bmatrix}
V^T & 0 \\
0 & I
\end{bmatrix}
= [U \beta] [R]
\begin{bmatrix}
V^T & 0 \\
0 & I
\end{bmatrix}
= U*\Sigma*V*
\]

Ross (2008)

- In the proposed approach we don’t use raw data, we use condition indicators obtained recursively

For example:

\[
\begin{align*}
C(k) & = (1 - \frac{1}{k})C(k - 1) + \frac{1}{k}x(k)x(k)x(k)x(k) \\
\kappa(k) & = \frac{C(k)}{\sigma^2(k)^2} - 3
\end{align*}
\]
“Retuning tuned mass dampers based on ambient vibration measurements”. *Smart materials and structures, Hazra et al.* (2010)
“Retuning tuned mass dampers based on ambient vibration measurements”. *Smart materials and tructures*, Hazra et al. (2010)
“Retuning tuned mass dampers based on ambient vibration measurements”. *Smart materials and structures, Hazra et al.* (2010)
RETUNING ALGORITHM

- Primary structure estimation (frequency): \( \hat{\Omega} = \frac{\omega_1 + \omega_2}{2} \)

- Modeshape: \( \Phi_{N+1,N+1} \rightarrow \hat{\Phi}_{N,N} \)

- Iterative retuning:
  \[
  \left\| \frac{\tilde{s}_e^j(\omega) - \tilde{s}_e^{j-1}(\omega)}{\tilde{s}_e^{j-1}(\omega)} \right\| = \| \gamma \| \leq \varepsilon
  \]

- \( \mu \) level retuning:
  \[
  \tilde{\Omega} = E[\Omega], \quad \tilde{f}_{opt} = E[f_{opt}]
  \]
  \[
  \tilde{\xi}_{opt} = E[\xi_{opt}]
  \]

- \( \mu + \sigma \) level retuning:
  \[
  \tilde{\Omega} = E[\Omega] + \sigma_{\Omega}, \quad \tilde{f}_{opt} = E[f_{opt}] + \sigma_{f_{opt}}
  \]
  \[
  \tilde{\xi}_{opt} = E[\xi_{opt}] + \sigma_{\xi_{opt}}
  \]

“Retuning tuned mass dampers based on ambient vibration measurements”. *Smart materials and structures*, Hazra et al. (2010)
EXAMPLE WITH SIMULATED DATA

- 6DOF spring-mass-dashpot system (5+TMD)
- Excitation: Elcentro ground motion (0.3g)
- $\omega$ (Hz) = 0.78, 0.99, 3.38, 7.10, 10.66, 12.73
- $\omega$ (Hz) identified = 0.80, 1.01, 3.42, 7.03, 10.74, 12.47

“Retuning tuned mass dampers based on ambient vibration measurements”. *Smart materials and structures, Hazra et al.* (2010)
EMPIRICAL MODE DECOMPOSITION

\[ \ddot{x} \approx \sum_i \Gamma_i(t) + \hat{e}(t) \]
TABLE-1: Identified natural frequencies and damping for TMD restrained case (FULL SENSOR DATA)

<table>
<thead>
<tr>
<th>Mode No</th>
<th>Model $f$ (Hz)</th>
<th>Natural frequencies (Hz)</th>
<th>Damping (% Critical)</th>
<th>MAC</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Mean (COV(%))</td>
<td>MCC</td>
<td>WMCC</td>
</tr>
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<td>MCC</td>
<td>WMCC</td>
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<tr>
<td>1</td>
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<td>0.67(1.50)</td>
<td>0.67(1.30)</td>
<td>0.85(23.0)</td>
</tr>
<tr>
<td>2</td>
<td>0.92</td>
<td>0.83(1.30)</td>
<td>0.83(1.20)</td>
<td>1.98(20.0)</td>
</tr>
<tr>
<td>3</td>
<td>1.40</td>
<td>1.50(3.90)</td>
<td>1.44(2.90)</td>
<td>2.80(23.0)</td>
</tr>
<tr>
<td>4</td>
<td>2.73</td>
<td>2.64(2.15)</td>
<td>2.68(1.40)</td>
<td>1.30(15.0)</td>
</tr>
<tr>
<td>5</td>
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<td>2.94(2.25)</td>
<td>3.00(1.00)</td>
<td>2.75(16.0)</td>
</tr>
<tr>
<td>6</td>
<td>3.36</td>
<td>3.43(1.85)</td>
<td>3.34(1.75)</td>
<td>4.50(23.0)</td>
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<tr>
<td>7</td>
<td>4.59</td>
<td>4.66(1.00)</td>
<td>4.61(0.97)</td>
<td>2.56(16.3)</td>
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<tr>
<td>8</td>
<td>6.08</td>
<td>6.18(1.25)</td>
<td>6.02(1.25)</td>
<td>3.00(22.0)</td>
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<tr>
<td>9</td>
<td>6.71</td>
<td>6.60(2.35)</td>
<td>6.66(2.00)</td>
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<td>10</td>
<td>9.01</td>
<td>—</td>
<td>8.74(1.8)</td>
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<td>11</td>
<td>9.43</td>
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<td>12</td>
<td>10.09</td>
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</tbody>
</table>

“Hybrid Time-Frequency Blind Source Separation Towards Ambient System Identification of Structures”. Hazra et al. (2011)
SPARSE BSS

$f_{4}^{6,0}$

$f_{5}^{6,0}$

$f_{5}^{6,12}$

$F_{5}^{6,12}(\omega)$

$F_{4}^{6,12}(\omega)$
Assuming that the coefficients at the last scale level \((j = s)\) are mono-component:

\[
f_{k,i}^{s,v}(t) = A_{il} e_{k,l}^{s,v}(t)
\]

\[
\frac{f_{k,q}^{s,v}}{f_{k,r}^{s,v}} = \frac{A_{ql} e_{k,l}^{s,v}(t)}{A_{rl} e_{k,l}^{s,v}(t)} = \frac{A_{ql}}{A_{rl}} = |\hat{a}_{ql}|
\]

Line of orientation yields the mode shape coefficient