Computing nearest stable matrices

†Nicolas Gillis *Punit Sharma
†Department of Mathematics and Operational Research, University of Mons, Belgium
*Department of Mathematics, Indian Institute of Technology Delhi, India
*Email- punit.sharma@maths.iitd.ac.in

Abstract

The stability of a continuous linear time-invariant (LTI) system $\dot{x} = Ax + Bu$, where $A \in \mathbb{R}^{n,n}$, $B \in \mathbb{R}^{n,m}$, solely depends on the eigenvalues of the matrix A. Such a system is stable if all eigenvalues of A are in the closed left half of the complex plane and those on the imaginary axis are semisimple. It is essential to know that when an unstable LTI system becomes stable, i.e., when it has all eigenvalues in the stability region, or how much it has to be perturbed to be on this boundary. For control systems, this *distance to stability* is well-understood. This is the converse problem of the *distance to instability*, where a stable matrix A is given, and one looks for the smallest perturbation that moves an eigenvalue outside the stability region.

In this talk, I will talk about the *distance to stability* problems for LTI control systems. Motivated by the structure of dissipative-Hamiltonian systems, we define the *DH matrix*: a matrix $A \in \mathbb{R}^{n,n}$ is said to be a DH matrix if $A = (J - R)Q$ for some matrices $J, R, Q \in \mathbb{R}^{n,n}$ such that J is skew-symmetric, R is symmetric positive semidefinite and Q is symmetric positive definite. We will show that a system is stable if and only if its state matrix is a DH matrix. This results in an equivalent optimization problem with a simple convex feasible set. We propose new algorithms to solve this problem. Finally, we show the effectiveness of our method compared to the other approaches and to several state-of-the-art algorithms.

These ideas can be generalized to get reasonable approximate solutions to some other nearness problems for control systems like, distance to stability for descriptor systems, distance to positive realness, and minimizing the norm of static feedback.