FOURIER UNIQUENESS SETS FOR THE HYPERBOLA

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In this talk, we discuss the *"Heisenberg uniqueness pair" (HUP)* for the hyperbola. The notion of Heisenberg uniqueness pair introduced in the article "Heisenberg uniqueness pairs and the Klein-Gordon equation, Ann. of Math. (2) 173 (2011), no. 3, 1507-1527" by Hedenmalm and Montes-Rodríguez.

Let Γ be a smooth curve or finite disjoint union of smooth curves in the plane and Λ be any subset of the plane. Let $\mathcal{X}(\Gamma)$ be the space of all finite complex-valued Borel measures in the plane which are supported on Γ and are absolutely continuous with respect to the arc length measure on Γ. Let $\mathcal{AC}(\Gamma, \Lambda) = \{\mu \in \mathcal{X}(\Gamma) : \hat{\mu}|_\Lambda = 0\}$, then (Γ, Λ) is said to be a *Heisenberg uniqueness pair* if $\mathcal{AC}(\Gamma, \Lambda) = \{0\}$. In this case, since Λ *determine* the measures $\mu \in \mathcal{X}(\Gamma)$, we say that Λ is a *"Fourier uniqueness set"* for Γ.

In this talk we present the following result. Let Γ be the hyperbola $\{(x, y) \in \mathbb{R}^2 : xy = 1\}$ and Λ_β^θ be the lattice-cross in $\mathbb{R}^2$ is defined by

$$\Lambda_\beta^\theta := ((\mathbb{Z} + \{\theta\}) \times \{0\}) \cup (\{0\} \times \beta\mathbb{Z}),$$

where $\theta = 1/p,$ for some $p \in \mathbb{N}$, and β is a positive real. Then $\mathcal{AC}\left(\Gamma, \Lambda_\beta^\theta\right) = \{0\}$ if and only if $0 < \beta \leq p$.

References

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