On Characterization of 2-Path Signed Graphs

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A graph G provided with a signature $\sigma : E(G) \to \{1, -1\}$ is called a *signed graph*, and it is denoted by $\Sigma = (G, \sigma)$. By the *sign* of a subgraph of Σ we mean the product of the sign of the edges in it and Σ is *balanced* if each cycle in it has sign 1. The 2-*path signed graph* of $\Sigma = (G, \sigma)$ is $\Sigma\#\Sigma = (G^2, \sigma')$ and for any edge uv in $\Sigma\#\Sigma$ the sign $\sigma'(uv)$ is -1 whenever in every uv-path of length 2 in Σ all edges are negative. Given a signed graph $\sum = (G, \sigma)$, the function $\mu : V(G) \to \{-1, 1\}$ defined by

$$\mu(v) = \begin{cases} +1, & \text{if } v \text{ is isolated;} \\ \prod\limits_{u \in N(v)} \sigma(uv), & \text{otherwise;} \end{cases}$$

is known as the *canonical marking* of Σ, where $N(v)$ is the open neighborhood of v in G. A signed graph Σ is said to be *canonically consistent* if every cycle in it has mark 1, where by mark of a subgraph of Σ we mean the product of the mark of the vertices in it. In this article, we have studied how the balanced and canonically consistent property of the signed graph $\Sigma = (C_n, \sigma)$ is transfered to its 2-path signed graph, where C_n is the cycle on n vertices.

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