On M-POLYNOMIAL of the Third Type of Hex-derived Network

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Let $G = (V, E)$ be a simple connected graph, where $V = V(G)$ denotes the vertex set and $E = E(G)$ denotes the edge set which contains unordered pairs of vertices. The degree of a vertex $u \in V(G)$ in a graph G is the number of vertices that are adjacent to u and is denoted by $d(u)$ [1].

Chemical Graph Theory (CGT) is a branch of mathematical chemistry that unifies graph theory and chemistry. To form a graph of a given chemical compound, each atom is represented by a vertex and each chemical bond between the atoms is represented by an edge. The relationship between graph theory and chemistry and their several chemical applications has been investigated in [2, 3].

In the field of CGT, a *topological index* is a real number which is correlated with the various physical properties, biological activities and chemical reactivities of a molecular graph. Generally, the topological indices are characterized into degree-based topological indices [4], distance-based topological indices [5], degree and distance-based topological indices [6] and counting related topological indices [7]. All categories of topological indices are useful in predicting the physical, chemical, biological and other properties of chemical structure.

In recent trends, topological indices of the structures are calculated by using polynomials, rather than calculating them directly by their respective basic formulas. In the literature, many such polynomials have been defined, some of which are as follows: the M-polynomial [8], the matching polynomial [9], the Clar covering polynomial (also known as the Zhang-Zhang polynomial) [10], the Schultz polynomial [11], the Tutte polynomial [12], the Hosoya polynomial [13], etc. Amid all of these polynomials, the M-polynomial is used to calculate several degree-based topological indices.

The Hexagonal network of dimension n and its properties are discussed in [14]. Hex-derived networks (HDNs) have a wide range of applications in pharmaceutical sciences, electronics and communication networks. In 2008, type 1 Hex-derived network of dimension n ($HDN1[n]$) and type 2 Hex-derived network of dimension n ($HDN2[n]$) (where n denotes the number of vertices in a side of the HDN structure) are constructed from the Hexagonal network of dimension n [15]. After that in 2017, based on the structure of the Hexagonal network, Raj and George [16] have designed a new network called Hex-derived network of type 3 of dimension n (denoted as $HDN3[n]$).

In this talk, we determine a general form of M-polynomial for the third type of Hex-derived network of dimension n and hence calculate the related degree-based topological indices. We also assess the pictorial behavior of the M-polynomial and the related degree-based topological indices of this network for different dimensions.

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