ON CERTAIN SEMIGROUPS OF TRANSFORMATIONS THAT PRESERVE A PARTITION

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ABSTRACT. Let X be a nonempty set, and let $\mathcal{T}_X$ be the full transformation semigroup on X under composition of functions. For a partition $\mathcal{P}$ of X, we consider the semigroup $T(X, \mathcal{P}) = \{f \in \mathcal{T}_X \mid \forall X_i \in \mathcal{P} \; \exists X_j \in \mathcal{P}, \; X_i f \subseteq X_j\}$, the subsemigroup $\Sigma(X, \mathcal{P}) = \{f \in T(X, \mathcal{P}) \mid Xf \cap X_i \neq \emptyset \; \forall X_i \in \mathcal{P}\}$, and the group of units $S(X, \mathcal{P})$ of $T(X, \mathcal{P})$. We first characterize the elements of the semigroup $\Sigma(X, \mathcal{P})$. For a permutation f of finite set X, we next observe that whether there exists a nontrivial partition $\mathcal{P}$ of X such that $f \in S(X, \mathcal{P})$. We then characterize and enumerate the idempotents of the semigroup $\Sigma(X, \mathcal{P})$ for an arbitrary X and finite X, respectively. We also characterize the elements of the group of units $S(X, \mathcal{P})$. We finally count the number of elements of $T(X, \mathcal{P})$, $\Sigma(X, \mathcal{P})$, and $S(X, \mathcal{P})$ for a finite set X.

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