Welcome to IIT Guwahati

PH101: PHYSICS-I

Lecture 1

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1. Analytical (Classical) mechanics

(Up to **Mid-Sem Exam**; Of 50% marks)

Prof. P. PouloseProf. Girish Setlur

2. Relativity

3. Quantum mechanics

(For End-Sem Exam; Of 50% marks)

Evaluations

Quiz-I of 10% marks on 27th August 2018 (*tentatively*)

Mid-Semester Exam of 40% (as per institute time table)

Quiz-II of 10 marks (Dates will be announced later)

End-Semester exam of 40% (as per institute time table)

Course Web Page:

http://www.iitg.ac.in/physics/fac/padmakumarp/Courses/PH101/JulyNov2018.htm

PHYSICS-I (PH101) July-Nov, 2018

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Lecture Notes & Tutorial Assignments

Syllabus

PH101: Physics - I (2-1-0-6)

Calculus of variation: Fermat's principle, Principle of least action, Euler-Lagrange equations and its applications.

Lagrangian mechanics: Degrees of freedom, Constraints and constraint forces, Generalized coordinates, Lagrange's equations of motion, Generalized momentum, Ignorable coordinates, Symmetry and conservation laws, Lagrange multipliers and constraint forces.

Hamiltonian mechanics: Concept of phase space, Hamiltonian, Hamilton's equations of motion and applications.

Special Theory of Relativity: Postulates of STR. Galilean transformation. Lorentz transformation. Simultaneity. Length Contraction. Time dilation. Relativistic addition of velocities. Energy momentum relationships.

Quantum Mechanics: Two-slit experiment. De Broglie's hypothesis. Uncertainty Principle, wave function and wave packets, phase and group velocities. Schrödinger Equation. Probabilities and Normalization. Expectation values. Eigenvalues and eigenfunctions.

Applications in one dimension: Infinite potential well and energy quantization. Finite square well, potential steps and barriers - notion of tunnelling, Harmonic oscillator problem zero point energy, ground state wavefunction and the stationary states.

Books

Text Books:

- **1. Introduction to Classical Mechanics by Takwale R and Puranik P** (McGraw Hill Education, 1 st Ed., 2077) **.**
- **2. Classical mechanics by John Taylor** (University Science, 2005).
- 3. Quantum Physics of Atoms, Molecules, Solids, Nuclei and Particles by
	- R. Eisbergand R, Resnick [f ohn-Wiley, 2nd Ed., 2006).

References:

- 1. A Student's Guide to Lagrangians and Hamiltonians by Patrick Hamill (Cambridge University Press, 1st edition, 2013).
- 2. Theoretical Mechanics by M. R. Spiegel (Tata McGraw Hill, 2008).
- 3. The Feynman Lectures on Physics, Vol. Iby R. P. Feynman, R. B. Leighton, and M.Sands, [Narosa Publishing House, 1998J.

Intro. Classical Mechanics, David Morin (Cambridge)

Layout of mechanics course

- •Mathematical concepts of partial differentiation and coordinate systems.
- •Constraints, degree's of freedom and generalized coordinates.
- •Challenges with unknown nature of constrain forces in Newtonian Mechanics
- • D'Alembert's Principle of virtual work to remove the constrain forces from analysis.
- •Lagrange's equation: An alternative to Newton's law
- •Variational method and Lagrange's equation from variational principle
- •Hamiltonian equations of motion

Analytical mechanics

Introduction of new concepts of mechanics beyond Newton's law:Largangian and Hamiltonian equations

Why this is important?

- \Box Making the analysis easier, in particular complex dynamical situations with imposed constrains/conditions.
- \Box More general concepts extendable to other modern area of physics like quantum mechanics, field theory etc.

Review of certain mathematical concepts

Key to understand classical mechanics

Total Differential: Function of one variable

$$
y = f(x) \text{ is a function of one variable } x
$$
\n
$$
f'(x) = \frac{dy}{dx} = Lt \frac{\Delta y}{\Delta x} = Lt \frac{f(x + \Delta x) - f(x)}{\Delta x}
$$
\n
$$
dy = [f'(x)] dx
$$
\n
$$
y = f(x)
$$
\n
$$
\Delta x
$$

- •• Infinitesimal change of y around certain point (x) =(rate of change of y around the point) (magnitude of change in x)
- •• At stationary points (A,B,C) , y does not changes $\lbrack dy = 0 \rbrack$ even if x is changed infinitesimally,

which implies that at those points $f'(x) = 0$.

Partial differential: function of more than one variables

 $f(x, y)$ depends on two independent variables x and y.

Example: Height (f) of a hill as function of position coordinate (x, y) .

 \Box The rate of change (slope) in the 'x' direction, when y
remains constant is denoted by remains constant is denoted by

$$
(\frac{\partial f}{\partial x})_y = Lt \frac{f(x + \Delta x, y) - f(x, y)}{\Delta x}
$$

$$
\Delta x \to 0
$$

 \Box The rate of change in the 'y' direction, when x remains constant is denoted by constant is denoted by

$$
(\frac{\partial f}{\partial y})_x = Lt \frac{f(x, y + \Delta y) - f(x, y)}{\Delta y}
$$

$$
\Delta y \to 0
$$

• Change in height if I walk in the 'x' direction [keeping 'y' fixed] by $\langle dx'$?

$$
[df]_{dx} = \left(\frac{\partial f}{\partial x}\right)dx
$$

==(rate of change in $'x'$ direction)(amount of change in x

• Similarly,
$$
[df]_{dy} = \left(\frac{\partial f}{\partial y}\right)dy
$$

Change in height if I go in the arbitrary direction so that x' changes by $'dx'$ and 'y' also changes by ' dy'

$$
df = \left(\frac{\partial f}{\partial x}\right)dx + \left(\frac{\partial f}{\partial y}\right)dy = \left[df\right]_{dx} + \left[df\right]_{dy}
$$

• Generalization for a function which depends on several variables $f(x_1, x_2, x_3, \ldots, x_n)$

$$
df = \left(\frac{\partial f}{\partial x_1}\right)dx_1 + \left(\frac{\partial f}{\partial x_2}\right)dx_2 + \dots + \left(\frac{\partial f}{\partial x_n}\right)dx_n = \sum_{i=1}^{\infty} \left(\frac{\partial f}{\partial x_i}\right)dx_i
$$

Partial differential (Examples)

$$
\Box \qquad f(x,y) = a x^2 + b y
$$

$$
\frac{\partial f}{\partial x} = 2 \text{ a x} \qquad \qquad \frac{\partial f}{\partial y} = 2 \text{ b y}
$$

2

 \Box \Box $f(x,y) = a x^2y + b$

$$
\frac{\partial f}{\partial x} = 2 \text{ a x y} \qquad \qquad \frac{\partial f}{\partial y} = ax2
$$

 \Box \Box f(x,θ) = a x Sin(θ) + b θ²

$$
\frac{\partial f}{\partial x} = a \sin(\theta) \qquad \frac{\partial f}{\partial \theta} = a \times \cos(\theta) + 2b \theta
$$

Differentiation of function of functions

 $f(x, y)$ is such that x and y are function of another variable say, u. We wish to find the derivative $\frac{df}{dx}$ $du^{\boldsymbol{\cdot}}$

Example:
$$
f = xy + \ln y^2
$$

\n(we say, f depends x & y explicitly;
\nf depends u implicitly!)
\nLet, $x = a \cos u$ and $y = a \sin u$
\nHow to calculate $\frac{df}{du}$?

Method 1: Direct substitution Step 1: $f = (a \cos u)(a \sin u) + \ln(a \sin u)^2$ Step 2: Find $\frac{df}{du}$

Example

Distance of the Projectile from the origin,

$$
S(x, y) = \sqrt{x^2 + y^2}
$$

But

$$
x(t) = u_0 \cos(\theta) t
$$

&

$$
y(t) = u_0 \sin(\theta) t - \frac{1}{2}gt^2
$$

Chain rule of partial differential

Method 2: Chain rule

You know,
$$
df = (\frac{\partial f}{\partial x})dx + (\frac{\partial f}{\partial y})dy
$$

$$
\frac{df}{du} = \left(\frac{\partial f}{\partial x}\right) \frac{dx}{du} + \left(\frac{\partial f}{\partial y}\right) \frac{dy}{du}
$$

Find the First differentials individually

$$
\frac{\partial f}{\partial x}, \quad \frac{\partial f}{\partial y}, \quad \frac{dx}{du}, \quad \frac{dy}{du}
$$

and then substitute in the above relation.

Chain rule of partial differential

Generalization for a function depends on several variables $f(x_1, x_2, x_3,..., x_n)$ and the variables are function of another set of variables, Let, x_i $(u_1, u_2, ..., u_n)$

$$
df = \left(\frac{\partial f}{\partial x_1}\right) dx_1 + \left(\frac{\partial f}{\partial x_2}\right) dx_2 + \dots + \left(\frac{\partial f}{\partial x_n}\right) dx_n = \sum_{i=1}^n \left(\frac{\partial f}{\partial x_i}\right) dx_i
$$

$$
\frac{\partial f}{\partial u_1} = \left(\frac{\partial f}{\partial x_1}\right) \frac{\partial x_1}{\partial u_1} + \left(\frac{\partial f}{\partial x_2}\right) \frac{\partial x_2}{\partial u_1} + \dots + \left(\frac{\partial f}{\partial x_n}\right) \frac{\partial x_n}{\partial u_1} = \sum_{i=1}^n \left(\frac{\partial f}{\partial x_i}\right) \frac{\partial x_i}{\partial u_1}
$$

$$
\frac{\partial f}{\partial u_j} = \sum_{i=1}^n \left(\frac{\partial f}{\partial x_i}\right) \frac{\partial x_i}{\partial u_j}
$$

