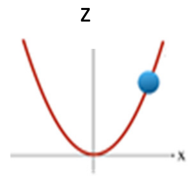
PH101 **Tutorial#3** **July – Nov. 2018**

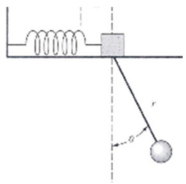
Department of Physics, IIT Guwahati. Due on: 13th Aug, 2018

I. For the following systems (a) obtain the constraint relations, (b) determine the degrees of freedom (DOF), (c) setup the Lagrangian in terms of generalized coordinates, and (d) obtain the Euler-Lagrange (E-L) equation of motion. Ignore frictional forces, unless otherwise mentioned! Always, make a sketch of the system, marking the origin as O, (w.r.to which the potential energy is defined), the Cartesian axes and the choice of generalized coordinates chosen.

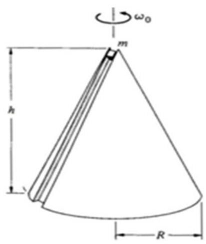
1. A bead of mass, m, sliding on a parabolic wire (kept vertical), $z = \alpha x^2$ under gravity.



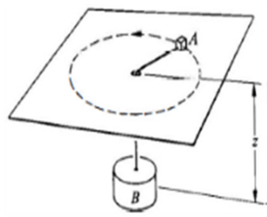
2. A mass M is confined to move on the x-axis under action of a spring of spring-constant, k, and equilibrium length, l. A small mass, m hanging from M is free to oscillate on the vertical plane under gravity.



3. A solid cone of height h and radius R is free to rotate about the vertical z-axis. At time t=0 small mass m starts sliding down from the apex of the cone along a straight groove cut on its surface, under gravity. Let the moment of inertia of the cone about the z-axis is I. If at time, t=0, the angular velocity of the cone is ω_0, find the angular velocity of the cone when the mass m leaves it. Also, show that the angular momentum of the system along the z-axis is conserved.



4. A particle of mass M (marked A) is confined to move the horizontal *x-y* plane. Another mass m (marked B) which is free to move on the vertical *z*-axis is tied to the mass A through a light string of length l.



[Homework to students]

II. Formulate the Lagrangian for the following systems.

a) A projectile of mass, m, moving on the vertical $xy -$ plane under gravity. (Obtain L in terms of both Cartesian and plane polar coordinates. Which of the two is *more insightful* in this case?!)

b) A bead of mass, m, moving on parabolic wire, given by, $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, where a and b are the semi-major and semi-minor axes, respectively. (Ignore gravity)

c) A point mass m constrained to move on the surface of a fixed gravitating solid sphere of mass M and radius R.