DEPARTMENT OF MATHEMATICS
INDIAN INSTITUTE OF TECHNOLOGY GUWAHATI

Course: MA224: Real Analysis | **MidSem**
Instructor: Rajesh Srivastava | March 01, 2026
Duration: 2.0 hours | **Maximum Marks:** 30

Note: Answers without adequate justification may receive little or no credit.

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1. (a) Is it possible to construct a bounded metric out of an unbounded metric? **[1]**
   (b) Let $(X, \|\cdot\|)$ be a complete normed linear space. Let $\{x_n\}$ be sequence X such that $\sum_{n=1}^{\infty} \|x_n\| < \infty$. Does it implies that $\sum_{n=1}^{\infty} x_n$ is convergent in X? **[1]**
   (c) Is it possible to cover the set $\{0, 1, \frac{1}{2}, \frac{1}{3}, \ldots, \frac{1}{n}, \ldots\}$ by finitely many open intervals of arbitrarily small length? **[1]**
   (d) Consider the set $\{f \in C[0,1] : f(0) = 0\}$. Is this set closed in $(C[0,1], \|\cdot\|_1)$? **[1]**

2. For $n \in \mathbb{N}$, let $f_n(t) = \dfrac{t}{n^2 + t^2}$. Show that f_n converges uniformly on $[0, \infty)$. Does the same conclusion hold on $\mathbb{R}$? **[3]**

3. Let $f : [1, \infty) \to \mathbb{R}$ be uniformly continuous. Show that there exists constant $M > 0$ such that $|\frac{f(x)}{x}| \leq M$ for all $x \in [1, \infty)$. **[3]**

4. Show, by constructing an explicit counterexample, that the continuous image of a complete metric space need not be complete. **[3]**

5. Show that $f(x) = \sqrt{|x|}$ is uniformly continuous on $\mathbb{R}$. **[3]**

6. Let $(0, 1) \to \mathbb{R}$ be uniformly continuous. Prove that both one-sided limits $\lim_{x \to 0+} f(x)$ and $\lim_{x \to 1-} f(x)$ exist (as finite real numbers). **[2]**

7. Let A and B be two compact sets in a metric space (X, d). Show that there exists $(x_o, y_0) \in A \times B$ such that $d(A, B) = d(x_o, y_o)$, where $d(A, B) = \inf_{x \in A, y \in B} d(x, y)$. Does the same conclusion hold if A and B are both closed only? **[4]**

8. Let $f : (1, \infty) \to (1, \infty)$ be a contraction mapping and define $g(x) = x^3 - f(x)$. Prove that $g : (1, \infty) \to \mathbb{R}$ is injective. Is g surjective as well? Justify your answer. **[4]**

9. For $f \in C^1[0, 1]$, define $\|f\| = \|f\|_1 + \|f\|_\infty$. Determine whether $(C^1[0, 1], \|\cdot\|)$ is a complete normed linear space. **[4]**

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