DEPARTMENT OF MATHEMATICS
INDIAN INSTITUTE OF TECHNOLOGY GUWAHATI

Course: MA224: Real Analysis — **Quiz I**
Instructor: Rajesh Srivastava — **Date:** February 18, 2026
Duration: 1.5 hours — **Maximum Marks:** 10

Note: Answers lacking proper justification will not be awarded marks.

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1. (a) Let (X, d) be a discrete metric space and let (Y, ρ) be an arbitrary metric space. Is every function $f : X \to Y$ continuous? **[1]**

   (b) Suppose $f : (X, d) \to (Y, \rho)$ is a continuous function. Does it imply that f transforms every Cauchy sequence in X to a Cauchy sequence in Y? **[1]**

2. Let F be a non-empty closed set in a metric space (X, d). For each $n \in \mathbb{N}$, define $O_n = \{x \in X : d(x, F) < \frac{1}{n}\}$. Show that $F = \bigcap_{n=1}^{\infty} O_n$. **[2]**

3. Let T be a linear map from a normed linear space $(X, \|\cdot\|)$ to itself. If T is continuous at 0, show that there exists a constant $M > 0$ such that $\|T(x)\| \leq M\|x\|$ for every $x \in X$. **[2]**

4. Let $f : \mathbb{R} \to \mathbb{R}$ be a function that is continuous at 0, with $f(0) = 0$, and satisfies the subadditivity condition $f(x + y) \leq f(x) + f(y)$ for all $x, y \in \mathbb{R}$. Show that f is uniformly continuous on $\mathbb{R}$. **[2]**

5. Let (X, d) be a compact metric space. Show that, for every $\epsilon > 0$, there exists finitely many x_i's such that $X = \bigcup_{i=1}^{n} B_\epsilon(x_i)$. **[2]**

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