Research Summary

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My research program lies at the interface of **harmonic analysis, integral geometry, and representation theory**. The unifying objective is to identify and prove *geometric–spectral rigidity* principles: to what extent does *partial transform information*—for example, vanishing of spherical/twisted means, restriction of a Fourier transform to a lower-dimensional set, or operator-valued Fourier data in noncommutative settings—*force* uniqueness, sharp support constraints, or uncertainty phenomena.

A distinctive feature of this program is the emphasis on **explicit and testable geometry** (non-harmonic cones, Coxeter configurations, algebraic curves, and unions of parallel lines), together with **transfer mechanisms** between Euclidean inverse problems and representation-theoretic formulations on groups (Heisenberg, motion, and related Lie groups). A complementary direction develops **operator-theoretic and complex-analytic rigidity** through Toeplitz kernels, de Branges spaces, Hilbert transforms, and multipliers in Smirnov/Nevanlinna classes; this viewpoint naturally leads to quantitative sampling/interpolation problems in Fock-type spaces and, in the present update, to a focused project on **sampling and interpolation in de Branges spaces beyond doubling phase measures**.

Research identity. *I establish geometric and spectral criteria for uniqueness and rigidity in harmonic analysis, and develop analytic toolkits (spectral expansions, representation-theoretic Fourier methods, and Toeplitz-kernel techniques) that make these criteria explicit and portable across Euclidean and Lie-group settings.*

Signature contributions (selected, with reference tags).

- **Twisted spherical means on $\mathbb{C}^n$:** explicit injectivity sets and support/localization theorems; a geometric obstruction principle via non-harmonic cones **[P1–P6]**.
- **Heisenberg uniqueness pairs (HUP):** Fourier rigidity for measures supported on algebraic curves and cones; arithmetic sensitivity for finitely many parallel lines with irregular gaps **[P7–P12]**.
- **Uncertainty on Lie groups:** Benedicks–Amrein–Berthier type theorems and qualitative uncertainty principles in representation-theoretic settings **[P11, P15, S4]**.
- **Weyl transforms on motion groups:** boundedness/uniqueness and sharp unboundedness regimes for symbol–operator correspondences **[P13, P14]**.
- **Toeplitz kernels and multipliers:** Hilbert-transform criteria in de Branges/Cartwright scales and multiplier questions for generalized Toeplitz kernels **[P16, P17, S1]**.
- **Forward program:** density theorems and constructive sampling/interpolation criteria in twisted Fock spaces $\mathcal{F}^p_A(\mathbb{C}^m)$ **[W2–W4]**.

Reference convention. Throughout, tags of the form **[P#]**, **[S#]**, **[W#]** refer to the numbered items in the list "Publications and preprints" at the end of this document.

Twisted spherical means: injectivity and support theorems on $\mathbb{C}^n$

1.1. Framework and basic definitions

Identify $\mathbb{C}^n$ with $\mathbb{R}^{2n}$ via $z = x + iy$, and write $z \cdot \overline{w} = \sum_{j=1}^{n} z_j \overline{w_j}$. For suitable functions f, g on $\mathbb{C}^n$, the *twisted convolution* is

$$(f \times g)(z) := \int_{\mathbb{C}^n} f(z-w)\, g(w)\, e^{\frac{i}{2}\mathrm{Im}(z\cdot\overline{w})}\, dw, \qquad z \in \mathbb{C}^n.$$

Let μ_r be normalized surface measure on the sphere $\{w \in \mathbb{C}^n : |w| = r\}$. The *twisted spherical mean* is

$$M_r f(z) := (f \times \mu_r)(z), \qquad r > 0.$$

A set $S \subset \mathbb{C}^n$ is a *set of injectivity (for a class $\mathcal{X}$)* if

$$M_r f(z) = 0 \ \ \forall r > 0,\ \forall z \in S,\ f \in \mathcal{X} \quad \Longrightarrow \quad f \equiv 0.$$

1.2. Explicit injectivity geometries: Coxeter configurations [P5]

Theorem 1.1 (Coxeter-type injectivity for TSM [P5])**.** *Let $S \subset \mathbb{C}^n$ be a Coxeter-type configuration (a finite union of complex linear subspaces arranged with a finite reflection symmetry). Then S is a set of injectivity for the twisted spherical means in natural classes such as $\mathcal{S}(\mathbb{C}^n)$ and weighted L^p spaces compatible with the special Hermite expansion:*

$$M_r f(z) = 0 \ \ \forall r > 0,\ \forall z \in S \quad \Longrightarrow \quad f \equiv 0.$$

1.3. Non-harmonic cones and the geometric obstruction principle [P6]

Definition 1.2 (Non-harmonic cone)**.** A cone $C \subset \mathbb{C}^n$ is called *non-harmonic* if it is not contained in the zero set of any nontrivial *bi-homogeneous harmonic polynomial* on $\mathbb{C}^n$ (equivalently, on $\mathbb{R}^{2n}$ under the identification $\mathbb{C}^n \simeq \mathbb{R}^{2n}$).

Theorem 1.3 (Non-harmonic cones are injectivity sets [P6])**.** *Let $C \subset \mathbb{C}^n$ be a non-harmonic cone. Then C is a set of injectivity for M_r on $\mathcal{S}(\mathbb{C}^n)$ (and on natural weighted L^p classes):*

$$M_r f(z) = 0 \ \ \forall r > 0,\ \forall z \in C \quad \Longrightarrow \quad f \equiv 0.$$

Moreover, when a cone is contained in the zero set of a nontrivial homogeneous harmonic polynomial, this provides an explicit obstruction to injectivity.

Corollary 1.4 (Injectivity from large geometric pieces)**.** *If a set $S \subset \mathbb{C}^n$ contains a non-harmonic cone, then S is a set of injectivity for M_r in the same function classes.*

1.4. Support and localization theorems [P1, P3]

Support theorems quantify how vanishing of twisted means forces spatial localization. A representative formulation is:

Theorem 1.5 (Support theorem (representative form) [P1, P3])**.** *Let $f \in \mathcal{S}(\mathbb{C}^n)$ and $R > 0$. Assume that*

$$M_r f(z) = 0 \qquad \text{for all } r > 0 \text{ and all } z \in \mathbb{C}^n \text{ with } |z| > R.$$

Then f is supported in the closed ball $\{z \in \mathbb{C}^n : |z| \le R\}$. More generally, annular vanishing regimes for $(z, r) \mapsto M_r f(z)$ force sharp localization of $\mathrm{supp}(f)$*.*

1.5. Spectral identities and a reusable analytic toolkit [P4]

A key technical component is the special Hermite expansion on $\mathbb{C}^n$, which diagonalizes twisted convolution and reduces geometric vanishing of $M_r f$ to vanishing of spectral pieces.

Proposition 1.6 (Spectral projection principle [P4]). *For $f \in L^2(\mathbb{C}^n)$ one has an orthogonal decomposition into special Hermite spectral components $\{f_k\}_{k\geq 0}$ such that $M_r f(z)$ admits an explicit expansion in terms of these components and radial Laguerre-type factors. Consequently,*

$$\big(M_r f(z) = 0 \ \forall r > 0 \ \text{(on a sufficiently rich set of centers)}\big) \implies f_k \equiv 0 \quad \forall k \geq 0,$$

and hence $f \equiv 0$.

Remark 1.7. This spectral toolkit is also a bridge to the Heisenberg group: twisted convolution on $\mathbb{C}^n$ arises from group convolution on $\mathbb{H}^n$ after taking the Fourier transform in the central variable.

Heisenberg uniqueness pairs: Fourier rigidity from geometry

2.1. Definition and viewpoint

Let μ be a finite complex Borel measure on $\mathbb{R}^n$ and define its Fourier transform by

$$\widehat{\mu}(\xi) := \int_{\mathbb{R}^n} e^{-2\pi i \langle \xi, x\rangle}\, d\mu(x), \qquad \xi \in \mathbb{R}^n.$$

Given sets $\Gamma, \Lambda \subset \mathbb{R}^n$ and a class $\mathcal{M}(\Gamma)$ of measures supported on Γ, the pair (Γ, Λ) is a *Heisenberg uniqueness pair (HUP)* if

$$\mu \in \mathcal{M}(\Gamma),\ \operatorname{supp}(\mu) \subset \Gamma,\ \widehat{\mu}(\xi) = 0\ \forall \xi \in \Lambda \quad \implies \quad \mu = 0.$$

HUP problems encode geometric uniqueness for Fourier data and connect to PDE uniqueness, inverse problems, and sampling.

2.2. Algebraic curves and explicit frequency geometries [P7]

Theorem 2.1 (HUPs for algebraic curves (representative form) [P7]). *Let $\Gamma \subset \mathbb{R}^2$ be a real algebraic curve with a smooth nondegenerate component, and let $\mathcal{M}(\Gamma)$ denote measures absolutely continuous with respect to arclength on that component. Then one can identify explicit frequency sets Λ (typically finite unions of lines or structured "lattice-cross" sets, determined by the algebraic geometry of Γ) such that (Γ, Λ) is a HUP for $\mathcal{M}(\Gamma)$.*

2.3. Cones and a unifying non-harmonic rigidity principle [P8]

Theorem 2.2 (Non-harmonic cones enforce Fourier uniqueness [P8]). *Let $C \subset \mathbb{R}^n$ be a cone satisfying a suitable non-harmonicity condition. Then, for natural classes of measures supported on C, vanishing of $\widehat{\mu}$ on an explicit frequency set Λ forces $\mu = 0$. In particular, the absence of harmonic annihilators compatible with the cone geometry is a decisive uniqueness mechanism.*

Remark 2.3. The cone obstruction principle aligns the HUP and twisted spherical mean programs: in both settings, non-harmonicity precludes the existence of nontrivial "invisible" objects (measures or functions) compatible with the geometry.

2.4. Parallel lines and arithmetic sensitivity [P12]

Theorem 2.4 (Finite unions of parallel lines with irregular gaps [P12]). *Let $\Gamma \subset \mathbb{R}^2$ be a finite union of parallel lines. For suitable classes of measures supported on Γ, the HUP property for pairs (Γ, Λ) (with Λ a structured frequency set) depends delicately on the arithmetic of the line offsets (resonant versus nonresonant spacings). In particular, "irregular gaps" can enforce uniqueness, while resonances admit nontrivial measures with prescribed Fourier vanishing.*

2.5. Noncommutative extensions [P9, P10]

Theorem 2.5 (HUP-type rigidity on groups [P9, P10]). *HUP phenomena persist in representation-theoretic settings (e.g. the Euclidean motion group and the Heisenberg group): for suitable classes of measures supported on structured subsets, vanishing of the (operator-valued) group Fourier transform on prescribed frequency sets forces the measure to be trivial.*

Uncertainty principles on Lie groups

3.1. Group Fourier analysis and support notions

Let G be a second countable unimodular Lie group with Plancherel theorem. For $f \in L^1(G) \cap L^2(G)$, the group Fourier transform $\widehat{f}(\pi)$ is an operator on the representation space of $\pi \in \widehat{G}$; the Plancherel measure on $\widehat{G}$ plays the role of Lebesgue measure in frequency space. In this framework one can formulate simultaneous localization constraints for f and $\widehat{f}$.

3.2. Benedicks–Amrein–Berthier type rigidity [P11]

Theorem 3.1 (Benedicks–Amrein–Berthier type uncertainty on groups [P11]). *Let G be a Lie group with a suitable Plancherel theorem (including motion-group and Heisenberg-type settings). If $f \in L^2(G)$ satisfies $\mathcal{L}(\mathrm{supp} f) < \infty$ and the spectral support of $\widehat{f}$ is contained in a set of finite Plancherel measure in $\widehat{G}$, then $f \equiv 0$.*

3.3. Qualitative uncertainty tools [P15]

Proposition 3.2 (Qualitative uncertainty principle paradigm [P15]). *On broad classes of Lie groups, strong joint localization (in physical and representation parameters) is impossible except for the zero function. Such qualitative principles provide robust rigidity tools when sharp Euclidean decay arguments are not available.*

3.4. Extended/quaternionic settings [S4]

Proposition 3.3 (Quaternionic extensions (representative) [S4]). *Uncertainty phenomena extend to quaternionic Heisenberg-type structures via operator-valued Fourier analysis and adapted Plancherel identities, yielding rigidity criteria parallel to the real/complex motion-group case.*

Weyl transforms on motion groups: operator thresholds and rigidity

4.1. Symbol–operator correspondences

Weyl transforms provide a noncommutative phase-space quantization map, converting a symbol into an operator in a representation. In motion-group settings (e.g. $M(n) = SO(n) \ltimes \mathbb{R}^n$) this produces operator families indexed by the spectral parameter.

Definition 4.1 (Weyl transform (schematic)). Given a symbol σ on a phase space and a unitary representation π, the Weyl transform is formally

$$W(\sigma) = \int \sigma(\xi)\, \pi(\xi)\, d\xi,$$

interpreted in an appropriate weak/operator sense.

4.2. Boundedness and uniqueness for quaternionic Weyl transforms [P13]

Theorem 4.2 (Boundedness/uniqueness criteria [P13]). *For quaternionic Weyl transforms associated with motion-group representations, one can obtain: (i) uniqueness results determining* *the symbol from the operator within natural symbol classes, and (ii) sharp boundedness criteria for $W(\sigma)$ under explicit integrability/regularity hypotheses on σ.*

4.3. Unboundedness phenomena and sharp regimes [P14]

Theorem 4.3 (Sharp unboundedness regimes [P14])**.** *There exist explicit symbol families (with natural integrability) on Euclidean and Heisenberg motion groups for which the associated Weyl transforms are necessarily unbounded. These examples isolate analytic thresholds separating stable symbol–operator correspondences from pathological regimes.*

Remark 4.4. Together, [P13, P14] provide a clean qualitative picture: the Weyl transform exhibits a phase transition between regular symbol classes yielding bounded operators and borderline/irregular classes where unboundedness is unavoidable.

Toeplitz kernels, de Branges spaces, Hilbert transforms, and multipliers

5.1. Toeplitz operators and kernels

Let $H^2(\mathbb{C}_+)$ be the Hardy space on the upper half-plane and $P_+ : L^2(\mathbb{R}) \to H^2(\mathbb{C}_+)$ the orthogonal projection. For $\varphi \in L^\infty(\mathbb{R})$, the Toeplitz operator is

$$T_\varphi f := P_+(\varphi f), \qquad f \in H^2(\mathbb{C}_+),$$

and the Toeplitz kernel is $\mathrm{Ker}T_\varphi$. If Θ is inner, the model space

$$K_\Theta := H^2(\mathbb{C}_+) \ominus \Theta H^2(\mathbb{C}_+)$$

satisfies $K_\Theta = \mathrm{Ker}T_{\overline{\Theta}}$. Toeplitz kernels thus unify model spaces with a broader class of shift-coinvariant structures and provide a natural framework for factorization and multiplier problems.

5.2. Smirnov-class Toeplitz kernels and generalized symbols

For a unimodular function U on $\mathbb{R}$, define the generalized Toeplitz kernel

$$N^+[U] := \{f \in N^+(\mathbb{C}_+) : \overline{U} f \in N_0^+(\mathbb{C}_+)\},$$

where $N^+(\mathbb{C}_+)$ is the Smirnov class and $N_0^+(\mathbb{C}_+)$ denotes Smirnov functions vanishing at infinity. This formulation allows symbols beyond L^∞ and is well adapted to spectral-gap and density phenomena.

5.3. Multipliers between Toeplitz kernels [S1]

Definition 5.1 (Multipliers between analytic subspaces)**.** For analytic subspaces X, Y on $\mathbb{C}_+$, define

$$\mathcal{M}(X,Y) := \{\, w \text{ analytic on } \mathbb{C}_+ : \; wf \in Y \text{ for all } f \in X \,\}.$$

Theorem 5.2 (Multiplier characterization (structural form) [S1])**.** *For unimodular symbols U, V, the multiplier space $\mathcal{M}(N^+[U], N^+[V])$ admits an analytic characterization in terms of Smirnov/Nevanlinna factorization and divisibility constraints encoded by the pair (U, V). In particular, existence of nontrivial multipliers is governed by boundary-growth and phase-compatibility conditions that are stable under Hilbert-transform perturbations of the phases.*

5.4. Phase functions, Hilbert transforms, and kernel nontriviality [P17]

Write $U = e^{i\gamma}$ (a.e. on $\mathbb{R}$). A central theme is that nontriviality and "size" of $N^+[U]$ can be detected via structured decompositions of γ into monotone and Hilbert-transform components.

Proposition 5.3 (Phase/Hilbert-transform criterion (representative form) [P17]). *Under suitable regularity assumptions on γ, nontriviality of $N^+[U]$ can be inferred from decompositions of the type*

$$\gamma = \gamma_{\rm mon} + \widetilde{h} + \gamma_{\rm err},$$

where $\gamma_{\rm mon}$ is a controlled monotone part, $\widetilde{h}$ is a Hilbert transform, and the error term $\gamma_{\rm err}$ satisfies quantitative bounds. Such criteria connect Toeplitz-kernel geometry to density/spectral-gap phenomena and to Beurling–Malliavin-type thresholds.

5.5. Cartwright–de Branges spaces and Hilbert transform [P16]

Let E be a Hermite–Biehler entire function, define $E^*(z) = \overline{E(\overline{z})}$ and $\Theta := E^*/E$. The de Branges space $\mathcal{H}(E)$ is canonically isomorphic to $K_\Theta = \mathrm{Ker}T_{\overline{\Theta}}$.

Theorem 5.4 (Hilbert transform in Cartwright–de Branges spaces [P16]). *When E lies in the Cartwright class, one obtains boundedness and stability criteria for Hilbert-transform-type operators acting on boundary values in $\mathcal{H}(E)$, expressed in terms of the phase of E and regularity of the weight $|E|^{-2}$. Via $\mathcal{H}(E) \simeq K_\Theta$, these results translate into Toeplitz-kernel invariants and multiplier estimates.*

5.6. Sampling and interpolation beyond doubling phase measures (current project [W7])

The project incorporated in this summary asks whether sharp sampling and interpolation criteria in de Branges spaces can persist when the phase measure is not doubling. The classical Paley–Wiener case is governed by Beurling density, while the doubling-phase theory of de Branges spaces replaces the Euclidean metric by the phase metric. The new objective is to determine whether the doubling hypothesis can, in appropriate situations, be replaced by an operator-theoretic equivalence between model spaces.

More precisely, if multiplication by an analytic function φ defines an isomorphism

$$M_\varphi : K_U \longrightarrow K_V,$$

then sampling and interpolation maps for K_U and K_V are related by diagonal weights given by the values $\varphi(\gamma)$ on the sampling/interpolation sequence. Uniform lower and upper control of these weights is expected to transfer frame and Riesz-sequence estimates, and hence to transport density theorems from a regular model space to a non-doubling one. Toeplitz kernels enter through inclusions and equalities of the form

$$\varphi K_U \subset K_V, \qquad \varphi K_U = K_V,$$

which may be reformulated as structural stability questions for kernels of Toeplitz operators. This places the new project directly at the intersection of de Branges theory, model spaces, Toeplitz kernels, multiplier theory, and frame-theoretic sampling.

Annotated contributions and technical bottlenecks (selected papers)

The publication record reflects a coherent program that advances *rigidity phenomena* across integral geometry, Fourier analysis, and noncommutative harmonic analysis: identifying when partial transform data forces **injectivity**, **support restrictions**, or **uniqueness** of measures/functions, and developing tools that make such rigidity *geometric* (encoded by cones, Coxeter configurations, algebraic curves, or representation-theoretic parameter sets).

In what follows, I record (i) the conceptual takeaways of each theme and (ii) the main **technical bottlenecks** that were resolved in the published papers, together with the **next bottlenecks** that naturally emerge. The statements below are *representative in form*: precise hypotheses (function classes, decay, and regularity) are tailored in the corresponding papers.

A. Twisted spherical means and integral geometry ([P1–P6]; extensions: [P2], [S2–S3])

Core outputs.

- *Support theorems from annular data:* recovery of support information for f from vanishing of twisted (or spherical) means restricted to annular regions.
- *Geometric classification of injectivity sets:* concrete families of sets (Coxeter systems, lines/planes, and non-harmonic cones) that force injectivity for the twisted spherical mean transform.
- *Spectral technology:* a real-analytic expansion of spectral projections and an extension of the Hecke–Bochner identity that enables explicit harmonic/special-Hermite decompositions.

Representative rigidity statement (schematic).

Theorem 6.1 (Injectivity from non-harmonic cones (representative form))**.** *Let $C \subset \mathbb{C}^n$ be a closed cone which is* non-harmonic *in the sense that it is not contained in the zero set of any nontrivial bi-homogeneous harmonic polynomial. If f is a rapidly decaying function on $\mathbb{C}^n$ and the twisted spherical means satisfy*

$$(f \times \mu_r)(z) = 0 \quad \textit{for all } z \in C \textit{ and all } r > 0,$$

then $f \equiv 0$.

Bottlenecks resolved (methodological obstacles made explicit).

- *From full data to partial data:* support and localization from annular/restricted-center measurements require reducing the vanishing of $(z, r) \mapsto (f \times \mu_r)(z)$ to vanishing of individual special-Hermite spectral components. The core obstacle is that twisted means are not translation invariant; the resolution uses explicit Laguerre/special-Hermite expansions and real-analytic control of spectral projections (see **[P1, P3, P4]**).
- *From radial symmetry to anisotropic geometry:* proving injectivity for Coxeter systems and cones forces a passage from radial arguments to bi-graded harmonic analysis adapted to reflection symmetries and homogeneous algebraic constraints. The decisive step is an extension of the Hecke–Bochner identity that isolates the relevant harmonic types (**[P4, P5, P6]**).
- *Robustness beyond Euclidean space:* extending the arguments from $\mathbb{C}^n$ to real hyperbolic spaces and to step-two Métivier settings requires a representation-theoretic formulation of the mean operators and their spectral decompositions, replacing Euclidean Fourier arguments by group Fourier analysis and spherical expansions (**[P2]** and **[S2–S3]**).

Next bottlenecks (open directions).

- *Near-classification:* a structural description of *all* injectivity sets for twisted spherical means (beyond algebraic cones/Coxeter configurations) remains a major barrier.
- *Quantitative stability:* converting injectivity into stable reconstruction (Lipschitz-type estimates) from partial/noisy data is largely open in the twisted setting.
- *Low regularity / endpoint spaces:* extending support theorems to minimal assumptions (e.g. merely L^p or distributional data) typically runs into the failure of pointwise spectral expansions and requires new ideas.

B. Heisenberg uniqueness pairs (HUP) and Fourier rigidity ([P7–P12]; motion/Heisenberg group: [P9–P10])

Core outputs.

- *Geometry-to-uniqueness principles:* identification of broad geometric configurations (selected algebraic curves, cones, and unions of parallel lines with irregular gaps) that form HUPs, i.e. the vanishing of the Fourier transform on a prescribed set forces the underlying measure to vanish.
- *Beyond $\mathbb{R}^n$:* formulation and verification of HUP phenomena in settings such as the Euclidean motion group and the Heisenberg group, where the Fourier transform becomes operator-valued.

Bottlenecks resolved (what had to be proved).

- *Algebraic–analytic reduction:* for algebraic curves and unions of parallel lines, the central difficulty is to turn vanishing of $\widehat{\mu}$ on Λ into rigid constraints on associated exponential-polynomial data (Fourier series along the supporting components). The proofs develop explicit elimination/uniqueness mechanisms sensitive to curvature and arithmetic spacing (**[P7, P12]**).
- *Operator-valued Fourier analysis:* on the motion group and the Heisenberg group the Fourier transform is operator-valued; uniqueness must therefore be expressed in terms of representation parameters and ranges of integral operators. Establishing HUP analogues requires a careful translation between geometric frequency sets and spectral conditions in the unitary dual (**[P9, P10]**).

Next bottlenecks (open directions).

- *Classification beyond algebraic models:* HUP characterizations for non-algebraic curves/surfaces and for rougher sets (e.g. Lipschitz graphs, finite-type manifolds) remain largely undeveloped.
- *Stability and perturbations:* determining whether HUP is stable under small geometric perturbations of Γ or Λ (and in which topologies) is a key obstacle for applications.
- *Quantitative HUP:* turning uniqueness into uncertainty inequalities (with explicit constants) is still an emerging direction, especially in operator-valued Fourier regimes.

C. Uncertainty principles and Benedicks–Amrein–Berthier phenomena ([P11, P15]; related submitted work: [S4])

Core outputs.

- *Benedicks-type rigidity on nonabelian groups:* formulations and proofs where "smallness" of spatial support and spectral support (in appropriate senses) force triviality.
- *Qualitative uncertainty on Lie groups:* uniqueness results that capture the sharp dichotomy "simultaneously localized in space and frequency" versus "identically zero".

Bottlenecks resolved in the published work.

- *Defining frequency concentration:* translating Euclidean notions (support/finite measure) into Plancherel-parameter conditions on $\widehat{G}$ is technically subtle and is often the core obstruction.
- *Interplay of representation parameters:* on motion-type groups, one must control how localization interacts with induced representations and with the geometry of coadjoint orbits.

Next bottlenecks (open directions).

- *Quantitative strengthening:* obtaining sharp inequalities (not only qualitative dichotomies), identifying extremizers, and understanding endpoint cases.
- *Uniform frameworks:* a unified treatment covering broad classes of Lie groups (beyond selected examples) typically requires new representation-theoretic "uncertainty functionals".

D. Weyl transforms on motion groups and quaternionic settings (papers [P13–P14])

Core outputs.

- *Operator-threshold phenomena:* understanding when Weyl-type quantizations produce bounded versus unbounded operators, and when the transform is injective/unique.
- *Quaternionic Weyl analysis:* extension of Weyl-transform tools to quaternionic settings, with corresponding boundedness and uniqueness conclusions.

Bottlenecks resolved in the published work.

- *Noncommutative symbol calculus:* proving boundedness/uniqueness typically requires a tailored symbol class and careful use of group representation theory to control operator norms.
- *Borderline growth:* the transition from bounded to unbounded behavior is delicate; isolating sharp conditions is a central technical hurdle.

Next bottlenecks (open directions).

- *Schatten and trace ideals:* systematic criteria for Schatten-class membership and optimal trace estimates.
- *Endpoint mapping:* sharp L^p–L^q bounds and microlocal characterizations in the motion-group regime.

E. Toeplitz kernels, de Branges spaces, Hilbert transforms, and multipliers ([P16–P17], [S1]; ongoing: [W1, W5])

Core outputs.

- *Hilbert transform in de Branges frameworks:* analysis of the Hilbert transform in the Cartwright–de Branges setting, connecting transform boundedness to spectral data of entire functions.
- *Nevanlinna/Toeplitz-kernel interface:* structural links between the Nevanlinna class, Hilbert transforms, and Toeplitz kernels (model spaces), with consequences for kernel geometry and multiplier questions.
- *Multipliers for generalized Toeplitz kernels:* a developing direction aiming at explicit characterizations of multiplier spaces between Toeplitz kernels generated by general symbols.

Bottlenecks resolved in the published work.

- *Bridging function theory and operator theory:* Toeplitz-kernel problems often reduce to subtle factorization and boundary regularity issues (Smirnov/Nevanlinna classes), while de Branges methods bring in spectral and entire-function techniques.
- *Hilbert-transform control on model-type subspaces:* boundedness/compactness questions depend on fine cancellation and on Carleson-type embeddings, which are technically demanding outside classical H^2.

Next bottlenecks (open directions).

- *Non-Hilbertian regimes:* a complete multiplier theory for generalized Toeplitz kernels in $p \neq 2$ settings (or for non-orthogonal projections) is still missing and seems to require new duality mechanisms.
- *Sharp density/entropy invariants:* isolating the precise Beurling–Malliavin-type invariants governing multipliers and Toeplitz-kernel comparability remains a central challenge.
- *Vector-valued and matrix symbols:* extending current methods to matrix-valued symbols and operator-valued kernels is natural for applications but introduces genuine new obstacles.

Bridge to the current forward program. The bottlenecks above motivate my current emphasis on *sampling and interpolation* in twisted Fock spaces: a setting where one seeks sharp density/geometry criteria, stability under perturbations, and multiplier-type descriptions in a genuinely non-isotropic, non-Hilbertian environment.

Bottlenecks and open problems rooted in the publication record

The themes in Sections 1–5 raise a small set of "root" questions: *when does partial transform data force rigidity*, and *how stable/quantitative is that rigidity*? Below I isolate bottlenecks that recur across the papers listed in Section 9, and formulate concrete open problems that organize the next stage of the program. In each item, the bracketed tags indicate the most direct point of origin in the publication list.

A. Twisted spherical means, support, and injectivity sets (papers [P1–P6], [S2–S3] and [W6])

Persistent bottlenecks.

- *Partial data and stability:* support theorems are largely qualitative; quantitative stability from annular or restricted-center data remains subtle.
- *Geometric classification:* the dichotomy between "harmonic" and "non-harmonic" algebraic sets is powerful, but a near-classification of injectivity sets is still out of reach.
- *Beyond Euclidean symmetry:* on Métivier groups and rank-one symmetric spaces, one needs a calculus that plays the role of special Hermite expansions while respecting curvature/representation parameters.

Open Problem 7.1 (Classification of injectivity cones beyond the non-harmonic criterion)**.** Determine to what extent the non-harmonic condition is *sharp*: classify closed cones $C \subset \mathbb{C}^n$ for which

$$(f \times \mu_r)(z) = 0 \ \ \forall z \in C, \ \forall r > 0 \quad \Longrightarrow \quad f \equiv 0$$

holds in natural function classes (Schwartz, L^2 with Gaussian weight, or real-analytic classes), and describe the size/structure of the kernel when C is harmonic (e.g. contained in a zero set of a bi-homogeneous harmonic polynomial).

Open Problem 7.2 (Quantitative support theorem from annular data)**.** Let $\mathcal{M}$ denote the twisted spherical mean transform on $\mathbb{C}^n$ (and analogues on hyperbolic/Métivier settings). Develop a *stability theory* of the form

$$\|\mathcal{M}f\|_{\text{data}} \leq \varepsilon \ \Rightarrow \ \operatorname{dist}(\operatorname{supp} f, \ \mathcal{S}) \leq \Phi(\varepsilon),$$

where $\mathcal{S}$ is the support region predicted by the qualitative theorem and $\Phi(\varepsilon) \to 0$ as $\varepsilon \to 0$. Identify optimal norms on the data side for annular/radius-restricted measurements (papers **[P1, P3]** and **[S2–S3]**).

Question 7.3 (Rigidity under perturbations of Coxeter configurations)**.** For Coxeter systems of lines/planes that are injectivity sets (paper 5), is injectivity stable under small smooth perturbations of the configuration? If stability fails, characterize the "unstable directions" and the associated approximate null functions.

Open Problem 7.4 (Injectivity and support for spherical means on rank-one symmetric spaces)**.** Extend the injectivity/support theorems for spherical means from the Euclidean/hyperbolic/Métivier setting (paper [**P2**] and preprints [**S2–S3**]) to rank-one symmetric spaces with optimal geometric hypotheses on the set of centers. Identify the correct replacement of "non-harmonicity" in terms of spherical harmonics or the Helgason–Fourier transform (work in preparation [**W6**]).

B. Heisenberg uniqueness pairs (HUP): Euclidean curves and noncommutative settings (papers [P7–P12] and [P9–P10])

Persistent bottlenecks.

- *From algebraic models to flexible geometry:* existing classifications exploit algebraic structure (curves, parallel lines, cones); moving to general C^k curves/surfaces is hard.
- *Quantitative uniqueness:* most HUP results are qualitative (vanishing implies zero); stability and near-uniqueness are underdeveloped.
- *Group Fourier analysis:* on motion/Heisenberg groups, one must control operator-valued Fourier transforms, where representation parameters interact with geometry.

Open Problem 7.5 (HUP beyond algebraic curves)**.** Develop criteria for (Γ, Λ) to be a HUP when Γ is a finite union of C^2 curves with controlled curvature and Λ is a structured frequency set (e.g. finite unions of lines/curves). Identify invariants replacing "algebraic degree" in the proofs for algebraic curves (paper [**P7**]) and irregular-gap parallel lines (paper [**P12**]).

Open Problem 7.6 (Stability and quantitative HUP)**.** Given a HUP (Γ, Λ), obtain quantitative estimates of the form

$$\|\mu\| \leq C \, \|\widehat{\mu}\restriction_\Lambda \|_{\mathcal{X}}$$

for finite measures μ supported on Γ, where $\mathcal{X}$ is a natural norm on the restricted Fourier data. Determine when such estimates fail (e.g. existence of "almost invisible" measures).

Open Problem 7.7 (HUP on the Heisenberg group: geometric frequency sets)**.** For the Heisenberg group $\mathbb{H}^n$ (paper [**P10**]), formulate and classify HUP analogues in which Γ is a submanifold (or finite union of cosets) and Λ is described by representation parameters (e.g. subsets of the Schrödinger parameter $\lambda \neq 0$ together with frequency variables). Identify minimal "spectral sampling" assumptions on Λ that force uniqueness.

C. Uncertainty principles and Benedicks–Amrein–Berthier phenomena (papers [P11, P15] and preprint [S4])

Persistent bottlenecks.

- *Endpoint and sharpness:* qualitative principles are known on several groups, but sharp quantitative forms, endpoints, and extremizers are rarely understood.
- *Operator-valued Fourier transforms:* the correct notion of "small frequency support" becomes subtle on non-abelian groups, and interacts with representation theory.

Open Problem 7.8 (Quantitative Benedicks-type theorems on motion and quaternionic Heisenberg groups)**.** Strengthen the qualitative Benedicks–Amrein–Berthier theorems (paper [**P11**] and preprint [**S4**]) to quantitative statements measuring how the size (e.g. Haar measure, Minkowski content, or thickness) of a spatial set E and a spectral set F controls $\|f\|_2$ when f is supported (or essentially supported) in E and $\widehat{f}$ is supported (or essentially supported) in F.

Question 7.9 (Extremizers and structure of near-extremizers)**.** In settings where uncertainty inequalities are available (paper [**P15**]), classify extremizers (if any), and describe the structure of sequences f_k that nearly saturate the inequalities. Do such sequences concentrate on geometric objects (or representation-parameter sets) related to HUP?

D. Weyl transforms and operator thresholds (papers [P13–P14])

Persistent bottlenecks.

- *Symbol classes in noncommutative settings:* quaternionic and motion-group Weyl calculi require careful bookkeeping of noncommutativity and representation parameters.
- *Trace ideals and sharp mapping theory:* boundedness/unboundedness is only the first step; Schatten-class, weak-type, and endpoint estimates are largely open.

Open Problem 7.10 (Schatten and trace ideal criteria for motion-group Weyl transforms)**.** Develop sharp necessary and sufficient conditions on a symbol σ ensuring that the associated Weyl transform W_σ belongs to Schatten classes S^p (or weak Schatten classes) on L^2, in Euclidean and Heisenberg motion group settings (paper [**P14**]). Clarify the role of mixed smoothness/decay in group variables versus representation parameters.

Open Problem 7.11 (Quaternionic Weyl calculus: boundedness thresholds and functional calculus)**.** For quaternionic Weyl transforms (paper [**P13**]), identify optimal symbol scales for L^2-boundedness and formulate an analogue of the Calderón–Vaillancourt principle in this setting. Investigate whether boundedness can be characterized by a finite family of left/right invariant derivatives.

E. Toeplitz kernels, de Branges spaces, Hilbert transforms, and multipliers (papers [P16–P17] and preprint [S1]; see also works in preparation [W1, W5])

Persistent bottlenecks.

- *Beyond Hilbertian structure:* multiplier and kernel geometry become much more delicate in $p \neq 2$ (non-Hilbertian) regimes and for generalized symbols.
- *Bridging BM-type invariants and operator kernels:* identifying the right density/entropy invariants that control Toeplitz-kernel size and multiplier algebras is a major structural issue.

Open Problem 7.12 (General characterization of multipliers between generalized Toeplitz kernels)**.** Given two generalized Toeplitz kernels $\ker T_\varphi$ and $\ker T_\psi$ (preprint [**S1**]), characterize all multipliers g such that $g \cdot \ker T_\varphi \subseteq \ker T_\psi$ under minimal assumptions on φ, ψ (e.g. unimodular or bounded-type symbols). Provide necessary and sufficient conditions in terms of factorization data (inner/outer parts, Smirnov/Nevanlinna class parameters) and spectral invariants.

Open Problem 7.13 (Hilbert transform bounds in de Branges/Cartwright scales)**.** Extend the boundedness theory of the Hilbert transform in de Branges settings (paper [**P16**]) and its link to Toeplitz kernels (paper [**P17**]) to broader classes of de Branges spaces. In particular, identify geometric/spectral conditions (e.g. on the phase function or Clark measures) that are equivalent to boundedness of H and to nontriviality/rigidity properties of associated Toeplitz kernels.

Question 7.14 (Uniqueness and sampling in model spaces outside H^2)**.** For model spaces and their non-Hilbertian analogues (works in preparation [**W1, W5**]), develop a theory of uniqueness/sampling sets and multiplier algebras parallel to the H^2 case. Which aspects persist without orthogonality, and which require new surrogates (duality, Carleson embeddings, or BM-type densities)?

Open Problem 7.15 (Non-doubling phase measures and multiplier transfer)**.** For a Hermite–Biehler function E with possibly non-doubling phase measure $d\mu = \phi'(x)\,dx$, characterize when sampling and interpolation in $\mathcal{H}(E)$ obey sharp phase-density criteria. In particular, determine whether an invertible multiplier between the associated meromorphic model spaces transfers normalized reproducing-kernel frames and Riesz sequences, and identify the Toeplitz-kernel conditions ensuring this transfer.

F. Sampling and interpolation in twisted Fock spaces (works in preparation [W2–W4])

Persistent bottlenecks.

- *Sharp density theorems:* Landau-type bounds are subtle once the weight is anisotropic (matrix A) and the geometry is not rotation invariant.
- *Complete characterizations:* beyond necessary density conditions, one needs robust sufficient conditions (and stability under perturbations) for sampling and interpolation.

Open Problem 7.16 (Sharp Landau densities for anisotropic twisted Fock spaces)**.** Establish sharp necessary and sufficient density conditions for sampling/interpolating sequences in $\mathcal{F}^p_A(\mathbb{C}^m)$ for general real-linear A (works in preparation **[W2–W4]**). Identify the correct notion of "phase-space density" (Euclidean, symplectic, or A-adapted) and prove stability under bounded perturbations of the sequence.

Open Problem 7.17 (Complete interpolating sequences and structured lattices)**.** Determine whether there exist complete interpolating sequences for $\mathcal{F}^p_A(\mathbb{C}^m)$ with prescribed geometric structure (e.g. deformations of lattices adapted to A). If they exist, classify them up to natural equivalences and relate them to Gabor-type frames and phase-space tilings.

G. A unifying "rigidity from partial transforms" perspective

A recurring meta-question is to identify a *common mechanism* behind injectivity for mean transforms (A), uniqueness pairs (B), uncertainty theorems (C), and kernel/multiplier rigidity (E). A long-term goal is to develop a vocabulary of *quantitative rigidity* (stability, robustness, and effective constants) that can be transported across these settings.

Conjecture 7.18 (Quantitative rigidity principle — schematic)**.** *In each of the settings above, whenever a qualitative rigidity statement holds under a geometric non-degeneracy hypothesis, there exists a corresponding quantitative stability estimate in an appropriate functional topology whose constants depend continuously on the underlying geometry.*

Forward program: sampling/interpolation in twisted Fock spaces

6.1. Twisted Fock spaces and basic operators

Let $A : \mathbb{C}^m \to \mathbb{C}^m$ be a real-linear map whose Hermitian part is positive definite (so that (Az, z) defines a coercive real quadratic form) and $0 < p \le \infty$. The twisted Fock space $\mathcal{F}^p_A(\mathbb{C}^m)$ consists of entire functions f such that (for $0 < p < \infty$)

$$\|f\|^p_{p,A} := \int_{\mathbb{C}^m} |f(z)|^p \, e^{-\frac{p}{2}(Az,z)} \, dz < \infty,$$

with the standard essential-supremum modification when $p = \infty$.

Proposition 8.1 (Stability under differentiation (representative))**.** *For each multi-index β, the differentiation operator $f \mapsto \partial^\beta f$ acts boundedly on $\mathcal{F}^p_A(\mathbb{C}^m)$, and translation/modulation operators compatible with A act continuously. These boundedness properties underpin stability results for sampling and interpolation under finite modifications.*

6.2. Sampling and interpolation sets

A discrete set $Z = \{z_j\} \subset \mathbb{C}^m$ is *sampling* for $\mathcal{F}^p_A$ if there exist constants $0 < A \le B < \infty$ such that

$$A\|f\|^p_{p,A} \le \sum_j |f(z_j)|^p \, e^{-\frac{p}{2}(Az_j, z_j)} \le B\|f\|^p_{p,A} \qquad (0 < p < \infty),$$

(with the obvious modifications for $p = \infty$). It is *interpolating* if for every data sequence $\{v_j\}$ with $\sum_j |v_j|^p e^{-\frac{p}{2}(Az_j,z_j)} < \infty$ there exists $f \in \mathcal{F}^p_A$ with $f(z_j) = v_j$.

6.3. Density thresholds and constructive criteria [W2–W4]

Theorem 8.2 (Landau–Seip type density principle (programmatic statement))**.** *Sampling and interpolation in Fock-type spaces are governed by critical Beurling-type density thresholds determined by the phase-space volume induced by A. In particular, sampling requires lower density above the critical value, while interpolation requires upper density below it; separatedness is a natural geometric hypothesis in both directions. My current program develops these density theorems in twisted real-linear settings and complements them with constructive criteria for stable reconstruction.*

Proposition 8.3 (Finite extension of interpolating sets (representative))**.** *If Z is an interpolating set for $\mathcal{F}^p_A$ which is not sampling, then one expects (and in several cases can prove) that $Z \cup \{\zeta\}$ remains interpolating for every $\zeta \notin Z$ such that $Z \cup \{\zeta\}$ is separated. This stability mechanism is a key ingredient in "no set can be simultaneously sampling and interpolating" phenomena in Fock-type spaces.*

Forward program: de Branges spaces beyond doubling phase measures

This section records the project incorporated into the present research summary dated June 17, 2026. Its aim is to develop a refined theory of real sampling and interpolation in de Branges spaces when the phase measure is allowed to be highly irregular and non-doubling.

7.1. Guiding problem

Let E be a Hermite–Biehler function, let $E^*(z) = \overline{E(\overline{z})}$, and put $\Theta = E^*/E$. The unitary map

$$F \longmapsto F/E$$

identifies the de Branges space $\mathcal{H}(E)$ with the meromorphic model space

$$K_\Theta = H^2(\mathbb{C}_+) \ominus \Theta H^2(\mathbb{C}_+).$$

If $E(x) = |E(x)|e^{-i\phi(x)}$ on $\mathbb{R}$, then the phase measure $d\mu(x) = \phi'(x)\,dx$ defines the natural geometry for separation and density. In the doubling case, sharp density theorems relate sampling to sufficiently large lower phase density and interpolation to phase separation plus sufficiently small upper phase density. The project asks whether comparable conclusions remain valid beyond doubling.

> **Main problem.** Characterize those Hermite–Biehler functions E, with possibly non-doubling phase measure $\phi'(x)\,dx$, for which sharp density criteria for sampling and interpolation in $\mathcal{H}(E)$ remain valid.

The working principle is that doubling may not be the only mechanism behind density stability. In favourable cases it should be replaceable by an operator-theoretic equivalence: an invertible multiplier between model spaces transfers normalized reproducing-kernel frames, Riesz sequences, sampling sets, and interpolating sets.

7.2. Multiplier-transfer mechanism

Suppose that multiplication by an analytic function φ defines an isomorphism

$$M_\varphi : K_U \to K_V, \qquad M_\varphi f = \varphi f.$$

For a real sequence $\Gamma = \{\gamma_n\}$, normalized sampling is encoded by the analysis maps

$$R_U f = \left\{ \frac{f(\gamma_n)}{\|k^U_{\gamma_n}\|} \right\}_n, \qquad R_V g = \left\{ \frac{g(\gamma_n)}{\|k^V_{\gamma_n}\|} \right\}_n.$$

The relation $g = M_\varphi f$ yields a diagonal comparison

$$R_V M_\varphi f = D_\Gamma R_U f, \qquad D_\Gamma\{a_n\} = \left\{ \varphi(\gamma_n) \frac{\|k^U_{\gamma_n}\|}{\|k^V_{\gamma_n}\|} a_n \right\}_n.$$

Thus preservation of sampling and interpolation reduces, in part, to boundedness and bounded-below properties of D_Γ on the closed range of R_U. This formulation converts geometric sampling questions into precise operator inequalities.

Proposition 9.1 (Programmatic transfer principle)**.** *If $M_\varphi : K_U \to K_V$ is an isomorphism and the diagonal weights*

$$\varphi(\gamma_n) \frac{\|k^U_{\gamma_n}\|}{\|k^V_{\gamma_n}\|}$$

are uniformly bounded above and below on a separated sequence Γ, then the sampling and interpolation properties of Γ for K_U and K_V are expected to be equivalent, subject to the usual closed-range hypotheses for the corresponding analysis maps.

7.3. Toeplitz-kernel formulation

The Toeplitz operator

$$T_u f = P_+(uf), \qquad f \in H^2(\mathbb{C}_+),$$

provides the natural operator language for this program. Model spaces are Toeplitz kernels, since

$$K_\Theta = \mathrm{Ker} T_{\overline{\Theta}}.$$

Consequently, multiplier inclusions

$$\varphi K_U \subset K_V, \qquad \varphi K_U = K_V,$$

can be studied as stability and closed-range questions for associated Toeplitz kernels. This is expected to identify when multiplier relations are strong enough to preserve normalized reproducing kernels, frame bounds, Riesz bounds, and hence sampling/interpolation sequences.

7.4. Non-doubling phase geometry and expected outcomes

The project will construct and analyze Hermite–Biehler functions whose zeros approach the real line in controlled but non-uniform ways. Such configurations can create local peaks and flat gaps in ϕ', so that $\phi'(x)\,dx$ fails to be doubling even though global density behaviour may remain comparable with the Paley–Wiener scale.

The expected outcomes are:

- multiplier-transfer theorems for sampling and interpolation in meromorphic model spaces;
- Toeplitz-kernel criteria for when multiplier inclusions become isomorphisms preserving sampling and interpolation;

- explicit non-doubling de Branges spaces satisfying sharp density criteria;
- examples showing when phase separation and Euclidean/Paley–Wiener separation are, and are not, comparable;
- a local-global analysis separating harmless phase irregularity from genuine obstruction to density theorems;
- connections with de Branges–Rovnyak spaces, complete Nevanlinna–Pick spaces, and generalized Toeplitz kernels.

In this form, the project strengthens the summary's broader theme: geometric hypotheses such as doubling should be understood not only as metric regularity assumptions, but also as visible manifestations of deeper operator-theoretic stability.

Publications and preprints (selected list)

Peer-reviewed journal articles and book chapters

P1. R. Rawat and R. K. Srivastava, *Twisted spherical means in annular regions in $\mathbb{C}^n$ and support theorems*, Ann. Inst. Fourier (Grenoble) **59** (2009), no. 6, 2509–2523.

P2. R. Rawat and R. K. Srivastava, *Spherical means in annular regions in the n-dimensional real hyperbolic spaces*, Proc. Indian Acad. Sci. (Math. Sci.) **121** (2011), no. 3, 311–325.

P3. R. K. Srivastava, *Sets of injectivity for weighted twisted spherical means and support theorems*, J. Fourier Anal. Appl. **18** (2012), no. 3, 592–608.

P4. R. K. Srivastava, *Real analytic expansion of spectral projection and extension of Hecke–Bochner identity*, Israel J. Math. **200** (2014), 171–192.

P5. R. K. Srivastava, *Coxeter systems of lines and planes are sets of injectivity for the twisted spherical means*, J. Funct. Anal. **267** (2014), 352–383.

P6. R. K. Srivastava, *Non-harmonic cones are sets of injectivity for the twisted spherical means on $\mathbb{C}^n$*, Trans. Amer. Math. Soc. **368** (2016), no. 3, 1941–1957.

P7. D. K. Giri and R. K. Srivastava, *Heisenberg uniqueness pairs for some algebraic curves in the plane*, Adv. Math. **310** (2017), 993–1016.

P8. R. K. Srivastava, *Non-harmonic cones are Heisenberg uniqueness pairs for the Fourier transform on $\mathbb{R}^n$*, J. Fourier Anal. Appl. **24** (2018), no. 6, 1425–1437.

P9. A. Chattopadhyay, S. Ghosh, D. K. Giri, and R. K. Srivastava, *Heisenberg uniqueness pairs on Euclidean spaces and the motion group*, C. R. Math. Acad. Sci. Paris **358** (2020), no. 3, 365–377.

P10. S. Ghosh and R. K. Srivastava, *Heisenberg uniqueness pairs for the Fourier transform on the Heisenberg group*, Bull. Sci. Math. **166** (2021), 102941.

P11. S. Ghosh and R. K. Srivastava, *Benedicks–Amrein–Berthier theorem for the Heisenberg motion group*, Bull. Lond. Math. Soc. **54** (2022), no. 2, 526–539.

P12. D. K. Giri and R. K. Srivastava, *Heisenberg uniqueness pairs for finitely many parallel lines with an irregular gap*, J. Fourier Anal. Appl. **28** (2022), no. 2, Paper No. 37, 17 pp.

P13. R. K. Dalai, S. Ghosh, and R. K. Srivastava, *Boundedness and uniqueness of quaternion Weyl transform*, J. Pseudo-Differ. Oper. Appl. **13** (2022), no. 2, Paper No. 21, 24 pp.

P14. S. Ghosh and R. K. Srivastava, *Unbounded Weyl transforms on the Euclidean motion group and Heisenberg motion group*, J. Operator Theory **90** (2023), no. 2, 605–623.

P15. A. Chattopadhyay, D. K. Giri, and R. K. Srivastava, *Qualitative uncertainty principle on certain Lie groups*, J. Aust. Math. Soc. **116** (2024), no. 3, 289–307.

P16. A. K. Bhardwaj, A. Chattopadhyay, J. Mashreghi, and R. K. Srivastava, *Hilbert transform in the Cartwright–de Branges space*, in *Operator Theory: Advances and Applications*, Springer Nature (2024), pp. 99–114 (online: 03 April 2024).

P17. A. K. Bhardwaj, J. Mashreghi, and R. K. Srivastava, *Hilbert transform, Nevanlinna class and Toeplitz kernels*, Complex Anal. Oper. Theory **18** (2024), no. 3, Paper No. 78.

Submitted preprints

S1. Anjali and R. K. Srivastava, *Multiplier between generalized Toeplitz kernels* (submitted), arXiv:2507.03452.

S2. R. K. Dalai, S. Ghosh, and R. K. Srivastava, *Spherical means on Métivier groups and support theorem* (submitted), arXiv:2108.11744.

S3. R. K. Dalai and R. K. Srivastava, *Injectivity of the spherical mean operator on Métivier groups* (submitted), arXiv:2108.12729.

S4. S. Ghosh and R. K. Srivastava, *Benedicks–Amrein–Berthier theorem for the quaternion Heisenberg group* (submitted).

Works in preparation

W1. A. K. Bhardwaj, A. Chattopadhyay, and R. K. Srivastava, *Multipliers between model spaces in non-Hilbertian setups* (in preparation).

W2. Anjali, A. Chattopadhyay, S. Sarkar, and R. K. Srivastava, *Interpolation for twisted Fock spaces* (in preparation).

W3. Anjali, S. Sarkar, and R. K. Srivastava, *Sampling for twisted Fock spaces* (in preparation).

W4. Anjali, S. Sarkar, and R. K. Srivastava, *Landau-type lower density for sampling in Fock spaces on* $\mathbb{C}^d$ (in preparation).

W5. A. K. Bhardwaj, A. Chattopadhyay, and R. K. Srivastava, *Some uniqueness results on model spaces* (in preparation).

W6. R. K. Dalai and R. K. Srivastava, *Uniqueness sets for integral transforms on rank-one symmetric spaces* (in preparation).

W7. Anjali, and R. K. Srivastava, *Sampling and Interpolation without the Doubling Condition* (in preparation).

Closing synthesis

Across these themes, my work develops a coherent program centered on **geometric–spectral rigidity**: determining when partial transform information (mean values, Fourier vanishing, or operator-valued transforms) forces uniqueness, sharp support restrictions, or stable reconstruction. The updated de Branges project adds a further instance of this philosophy: sharp density phenomena are sought not merely from metric regularity of the phase, but from operator-theoretic transfer through multipliers and Toeplitz kernels. The program is unified by explicit classification questions (injectivity sets, uniqueness pairs, density thresholds) and by a sustained development of transferable analytic tools (spectral identities, representation-theoretic Fourier analysis, and Toeplitz-kernel/multiplier methods). This agenda naturally interfaces with inverse problems, noncommutative harmonic analysis, and modern complex/operator theory, and it opens clear pathways for high-impact results and collaborations.

Keywords: harmonic analysis; integral geometry; twisted spherical means; injectivity sets; Heisenberg uniqueness pairs; uncertainty principles; Lie groups; motion groups; Weyl transform; Toeplitz kernels; multipliers; de Branges spaces; Hilbert transform; Nevanlinna/Smirnov classes; sampling and interpolation; Fock spaces.