PH-101: Relativity and Quantum Mechanics

Special Theory of Relativity (5 Lectures)

Text Book:1. An Introduction to Mechanics

Author: Danieal Kleppner & Robert Kolenkow

2. Introduction to Special Relativity

Author: Robert Resnick

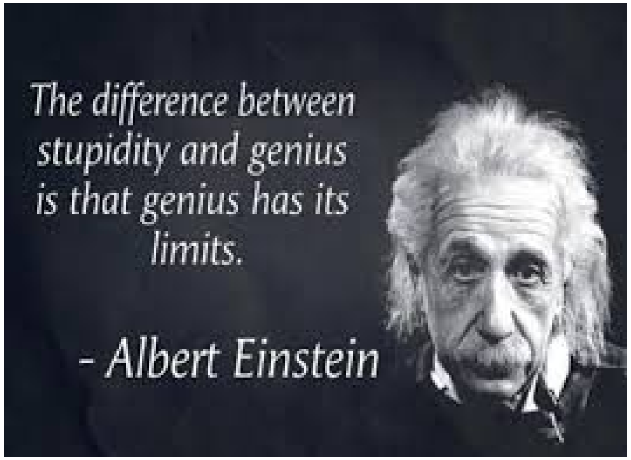
This Week: Thursday(Today), Friday(Tomorrow)

Next Week: Monday, Thursday

Quantum Mechanics (9 Lectures)

Group-II, IV

$$E = mc^2$$


The difference between stupidity and genius is that genius has its limits.
- Albert Einstein


Nagasaki
Hiroshima

Some facts

Einsteinain World

Macroscopic world of our ordinary experiences, the speed **u** of any object is alwasy much less than **C**.

$$C = 3 \times 10^8 \ m/s$$

Example: artificial satelite: u/c = 0.000027,

Sound speed: u/c = 0.000001

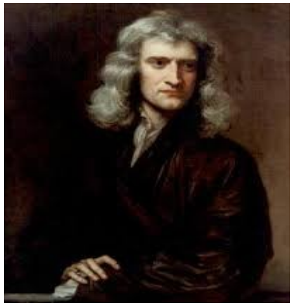
Newtonian World

Microscopic world:

Electron moving in 10 MeV potential difference: u/c = 0.9988

Electron moving in 40 MeV potential difference: Newtonian u/c =1.9976

Experimental u/c =0. 9999



Newtonian World?

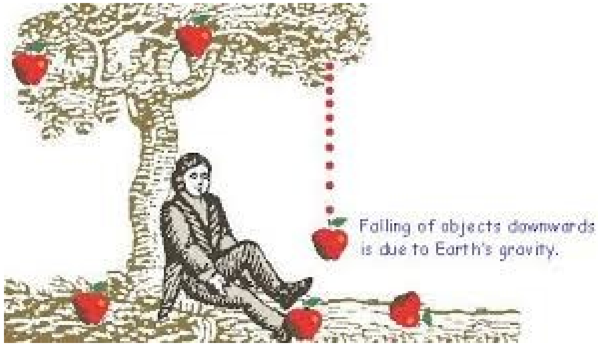
$$m\dot{v} = f \;\; ; \;\; I\dot{\omega} = N$$

January 4, 1643,

March 31, 1727

Newton's laws of Mechanics in Macroscopic world

You know the Law and applied it in various physical situations



Foundation of Newtonian Mechanic was so robust that nobody has dared to say that this is wrong or has to be modified.

Euler, D'Alembert, Lagrange, Hamilton, Laplace

Generalization of mechanics, Mathemtical Structure

Farady, Coulomb, ohm, Maxwell

Electromagnetism

Long time has passed

230 year after

March 14, 1879,

April 18, 1955,

Is Newton's world valid for any velocity?

Certainly Not!!

100 year after

was bothered about more on the "Principles" base on which those Laws are defined.

September, 2016

Symmetry Principles

Lot of open problems

Enough time has passed,

IT COULD BE YOUR WORLD!!

Try to convince you: It is "Einsteinian world"!!

Year 1905: Einstein, motivated by a desire to gain deeper insight into the nature of electromagnetism, push forward the idea of special theory of relativity

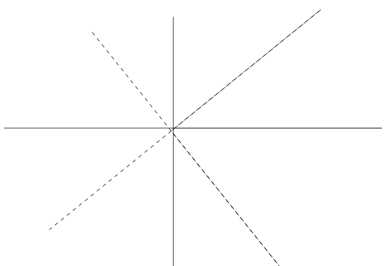
Where is the velocity U

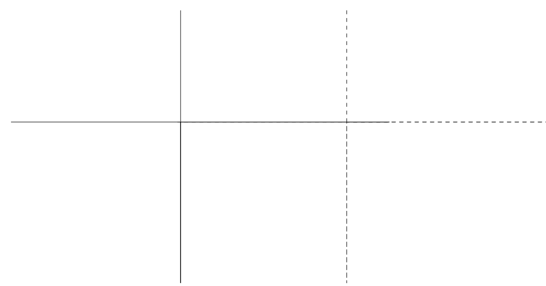
Symmetries

Ω

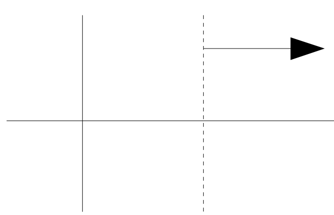
Any “physical law” should be invariant under some special set of transformations.

1) Rotation and translation of the given corodiante system





2) Transformation from one observer (S) to other(S') who is moving with constant velocity with respect to S.



Linear Transformations

Rotation and Translation

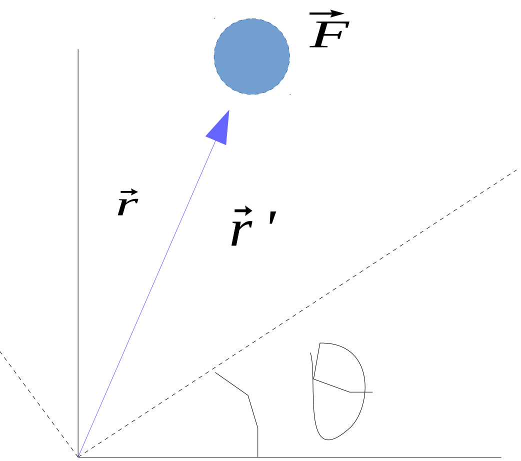
Rotation in x-y plane

Tutorial problem

$$x' = x\cos\theta + y\sin\theta$$
$$y' = -x\sin\theta + y\cos\theta$$
$$z' = z$$

Translation along x

$$x' = x + a$$
$$y' = y$$
$$z' = z$$

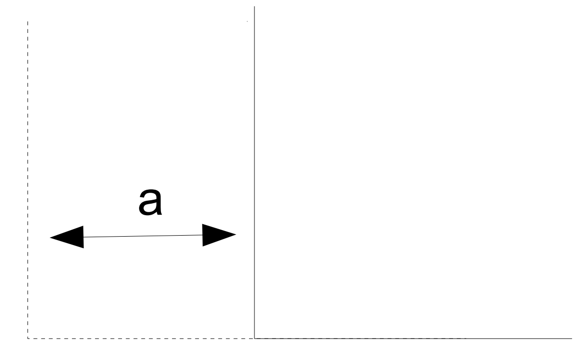


We do not touch “t” in Galilean transformation

Newton's Law is invariant under space rotaion

$$\frac{d^2\vec{r}}{dt^2} = \frac{d^2\vec{r}\,'}{dt^2}$$

$$m\frac{d^2\vec{r}}{dt^2} = \vec{F}(r) \qquad m\frac{d^2\vec{r}\,'}{dt^2} = \vec{F}\,'(r')$$



Linear transformation: Rotation

Length is invariant under rotation and translation:

$$ds^2 = dx^2 + dy^2 + dz^2 = dx'^2 + dy'^2 + dz'^2 = ds'^2$$

Galilean Transformation(GT)

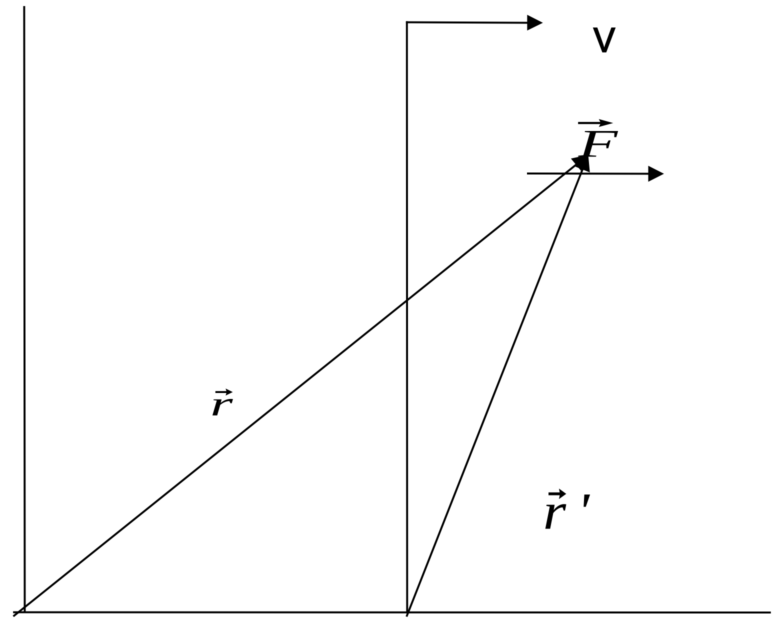
"Space" and "Time"

Motion is along x-direction

$$\vec{r}\,' = \vec{r} - \vec{v}\,t$$

$$x' = x - vt,\quad y' = y,\quad z' = z$$

We do not touch "t" in Galilean transformation



Newton's Law is invariant under GT

$$\frac{d^2\vec{r}}{dt^2} = \frac{d^2\vec{r}\,'}{dt^2}$$

$$m\frac{d^2\vec{r}}{dt^2} = \vec{F}(\vec{r}) \quad \rightarrow \quad m\frac{d^2\vec{r}\,'}{dt^2} = \vec{F}\,'(\vec{r}\,')$$

Inertial frames

Velocity addtion: u' = u - v

Linear transformation: Rotation

Not talking about: Non-linear transformation:

$$\vec{r}\,' = \vec{r} - \vec{v}\,t - \frac{1}{2}\vec{f}\,t^2 \quad \Rightarrow \quad m\frac{d^2\vec{r}\,'}{dt^2} = \vec{F}\,'(\vec{r}\,') + \vec{f}$$

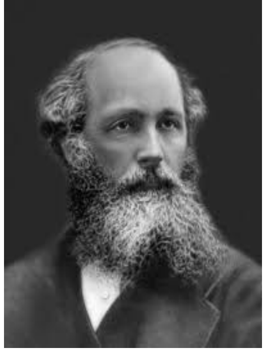
Depending on f: law changes

No mechanical experiments carried out entirely in one intertial frame can tell the observer what the motion of that frame is with respect to any other Inertial frame

There is no prefered reference frame

We can only speak about relative velocity: Galilean relativity

Electromagnetism and Galilean relativity!!

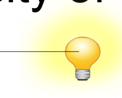


Another Piller: Maxwell's equation of electromagnetism

June 13, 1831- November 5, 1879,

v

c: velocity of light



$$\nabla \cdot E = 0, \nabla \cdot B = 0$$

E :Electric field

$$\nabla \times B = \frac{1}{c^2} \frac{\partial E}{\partial t}$$

B :Magnetic field

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

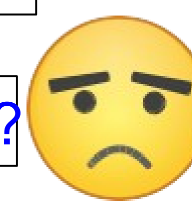
Relative velocity of light: c'= c-v

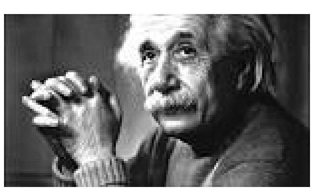
With respect to whom "c" is defined?

Michelson-Morley Experiment: could not prove the existence of any frame

Unfortunately these set of equtions are not invariant under GT

Why are we so obsessed with such an "invariance" property under GT?





"Laws" should not be dependent upon "who" is observing!

WHO?

Class of observes: moving with constant velocity relative to each other

Inertial observers

There is a problem!!

1) Electromagnetism?

2) Different relativity principle for Electromagnetism

2) Newtonian Mechanics?

Einstein did series of

1. "thought experiments": based on light and measurement of time!
2. measurement of time

Einstein was a Genius

worried about basic principle based on which laws are defined

1. Newton's law does not contain “velocity” explicity
2. Maxwell's equatioin contains velocity of light “c” explicitly

Since velocity of light, C, involves no reference to a medium,

C, must be universal constant!!

This simple but bold statement changes the very notion of “time”

How and why?

Follow Einstein's path: Do some thought experiments

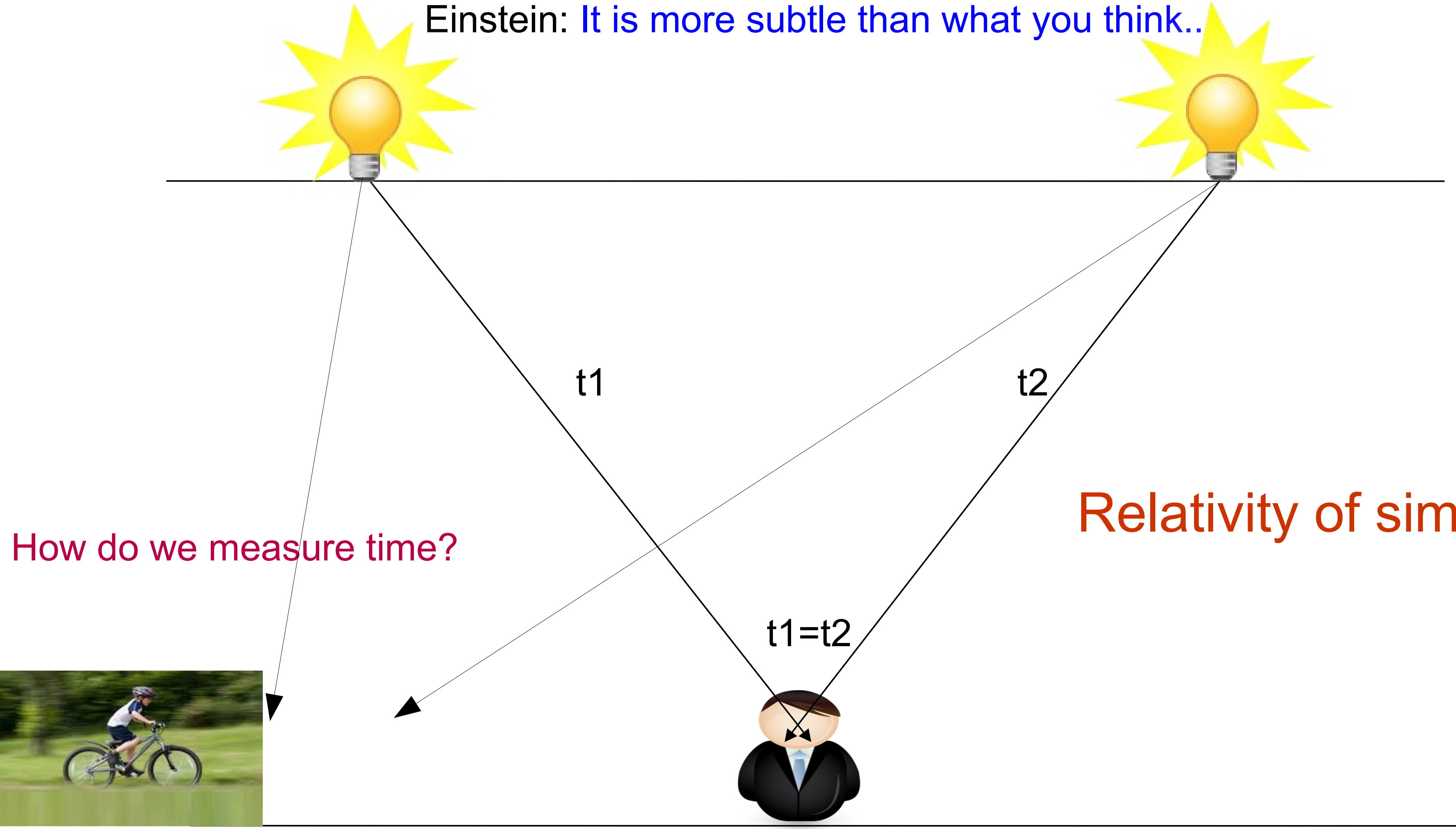
1. Velocity of light is finite
2. Usual Galileon relativity
3. Every observer has his own clock to measure time
4. Every observer has his own light detector

Newton's law does not talk about how do we measurement the time

Galileo: Time is sacred, it does not transfrom ...
Einstein: It is more subtle than what you think..

Relativity of simultaneity

How do we measure time?



Finite velocity of light plays crucial role in defining observer's “Time”

“Time” is observer dependent !!

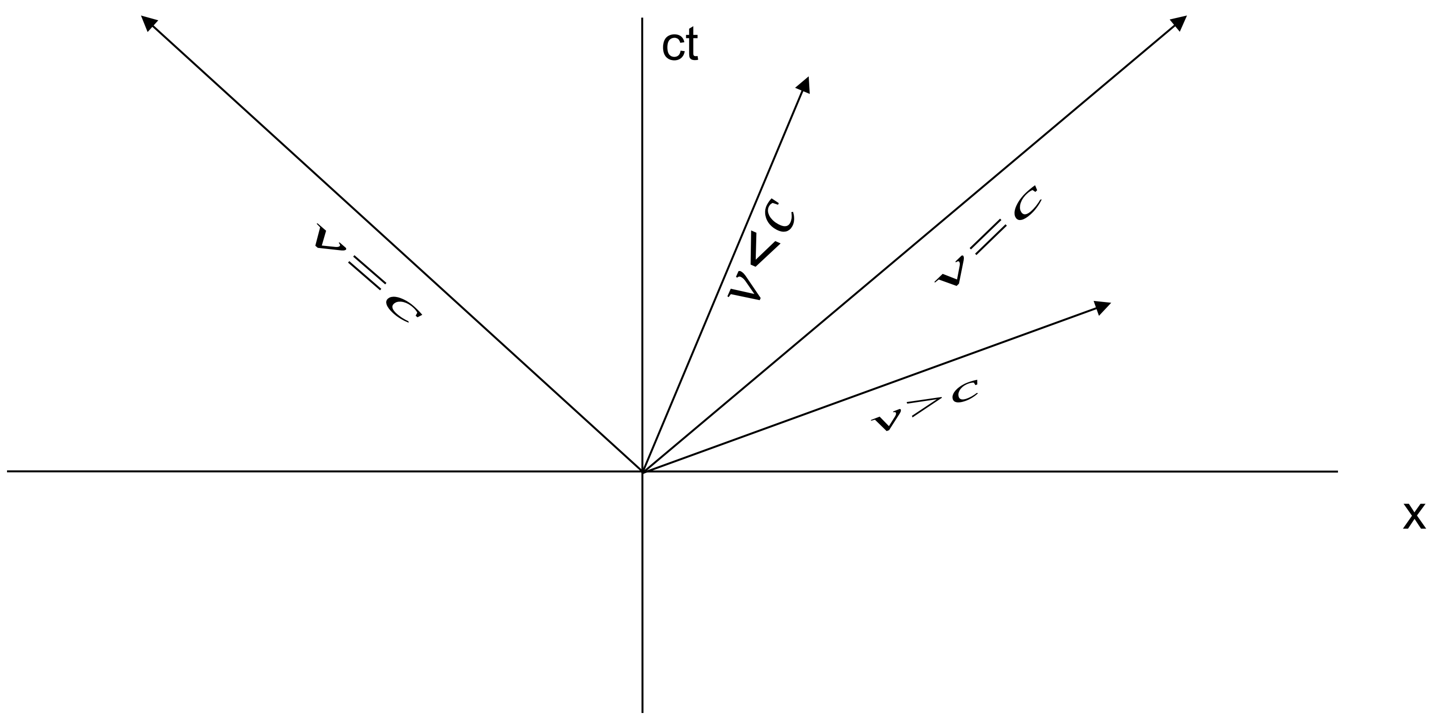
This can be very well explained in space and time diagram

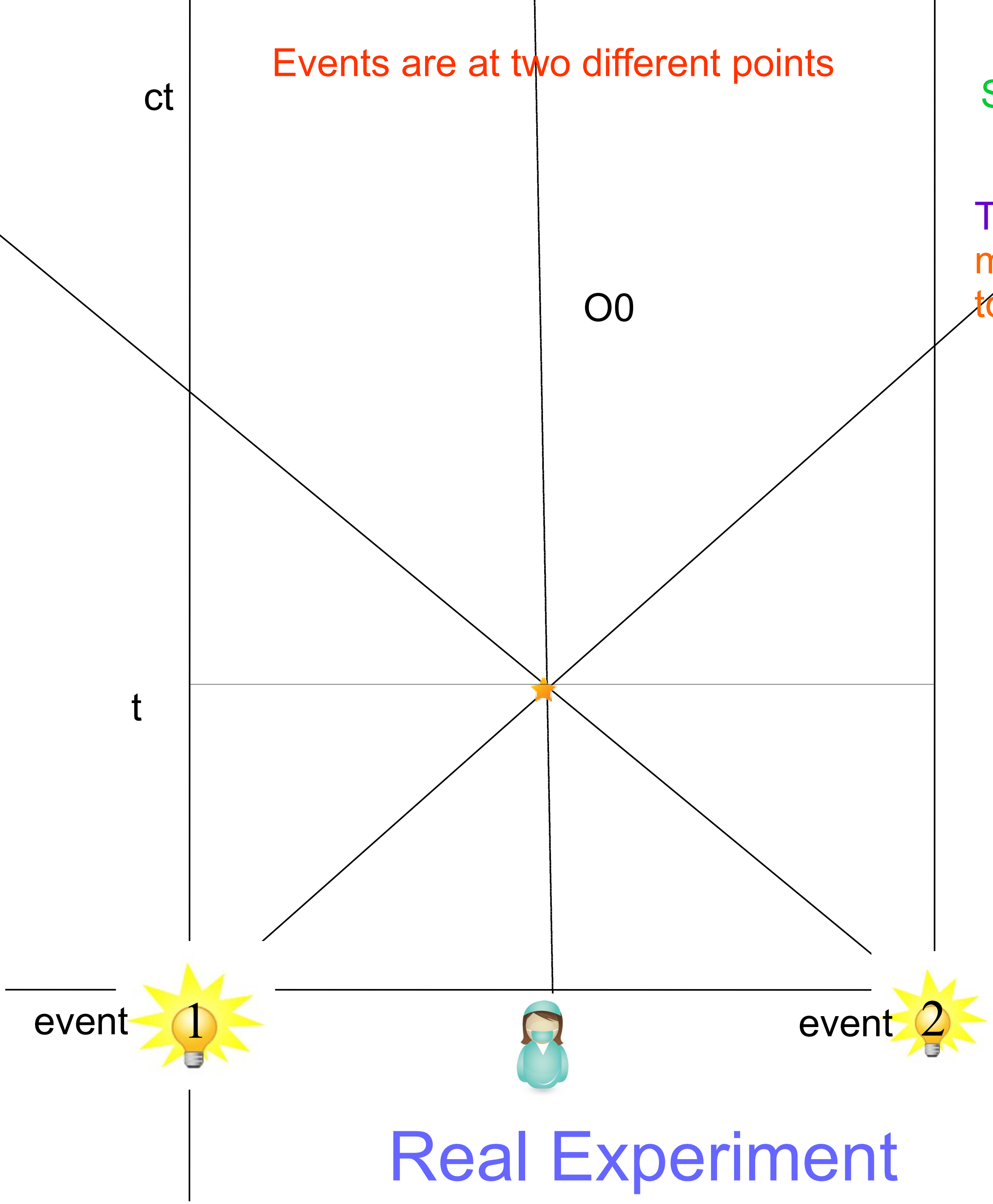
'Space and Time' diagram

Inertial observer: $x = vt$

$$x = \left(\frac{v}{c}\right) c\, t$$

Galilean relativity does not say anything about the magnitude of “v”





Real Experiment

Space" and "Time" diagram

Talking about a class of observers:
moving with constant velocity relative to each other

Inertial Observers

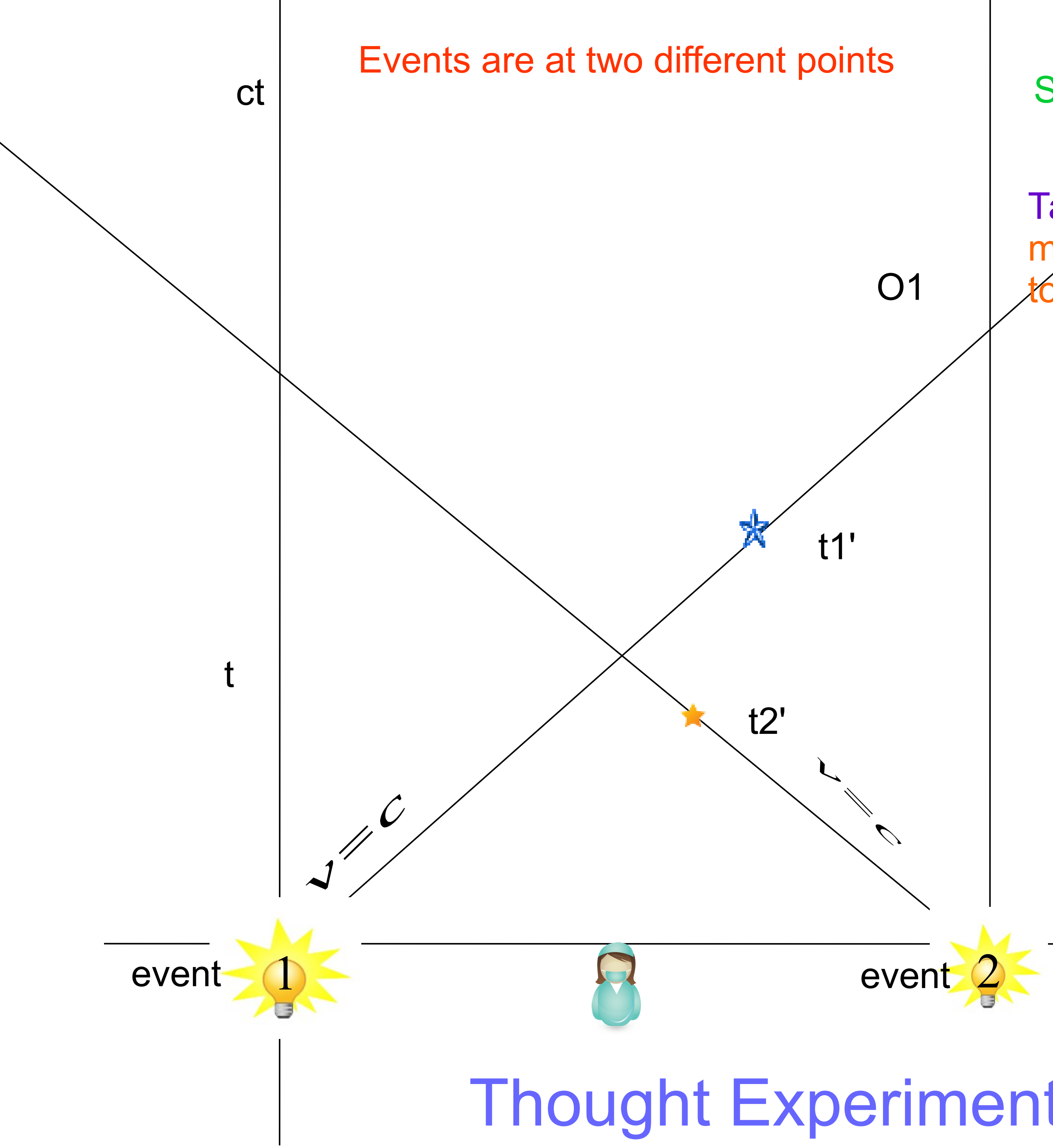
$$x = vt, \ x = \left(\frac{v}{c}\right) ct$$

$$x' = f(x, t), \ t' = g(t, x)$$

must be true

x

Events are at two different points



"Space" and "Time" diagram

Talking about a class of observers:
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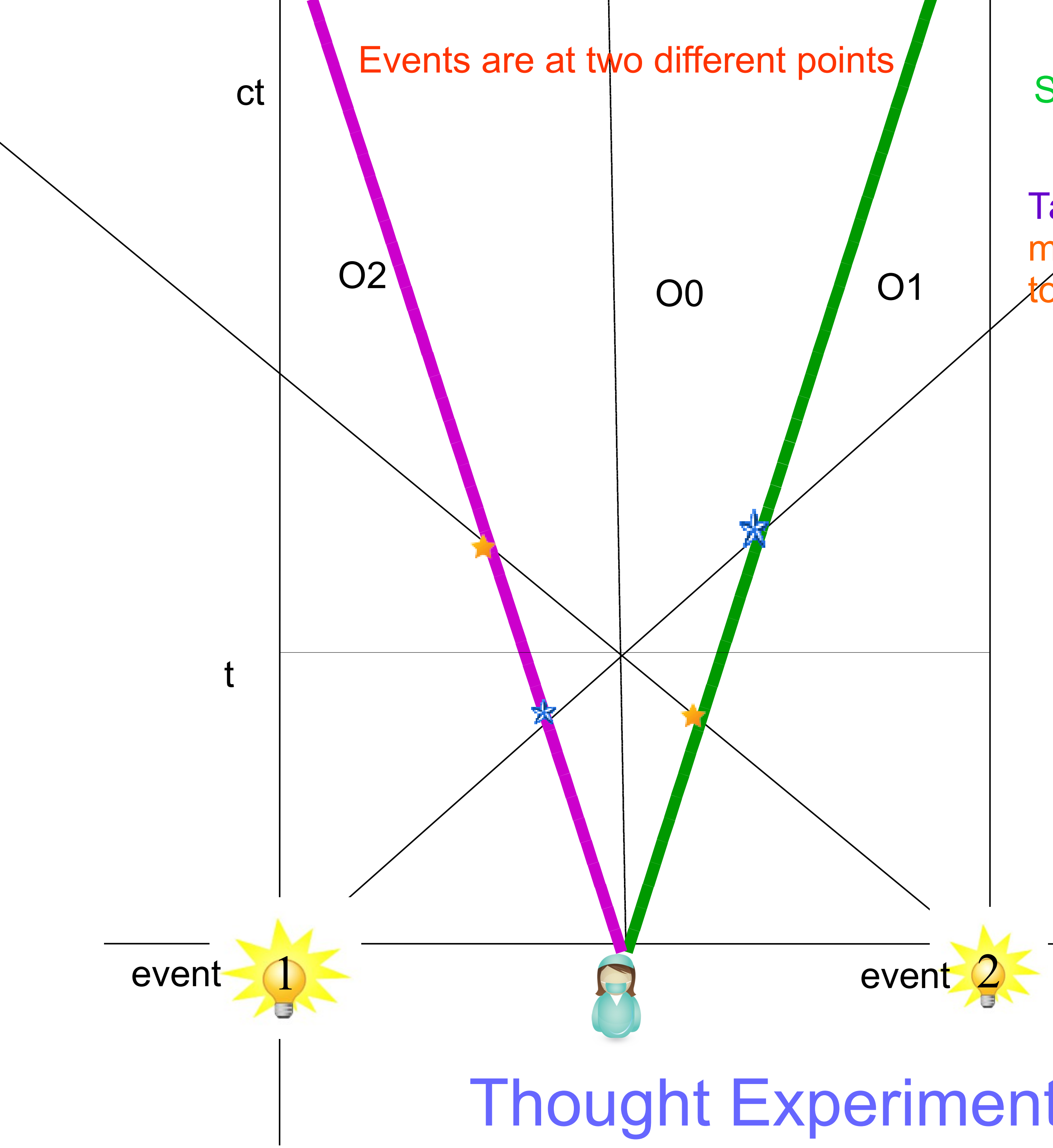
Inertial Observers

$$x = vt\,,\quad x = \left(\frac{v}{c}\right) ct$$

$$x' = f(x, t),\; t' = g(t, x)$$

must be true

Thought Experiment



“Space” and “Time” diagram

Talking about a class of observers:
moving with constant velocity relative to each other

Inertial Observers

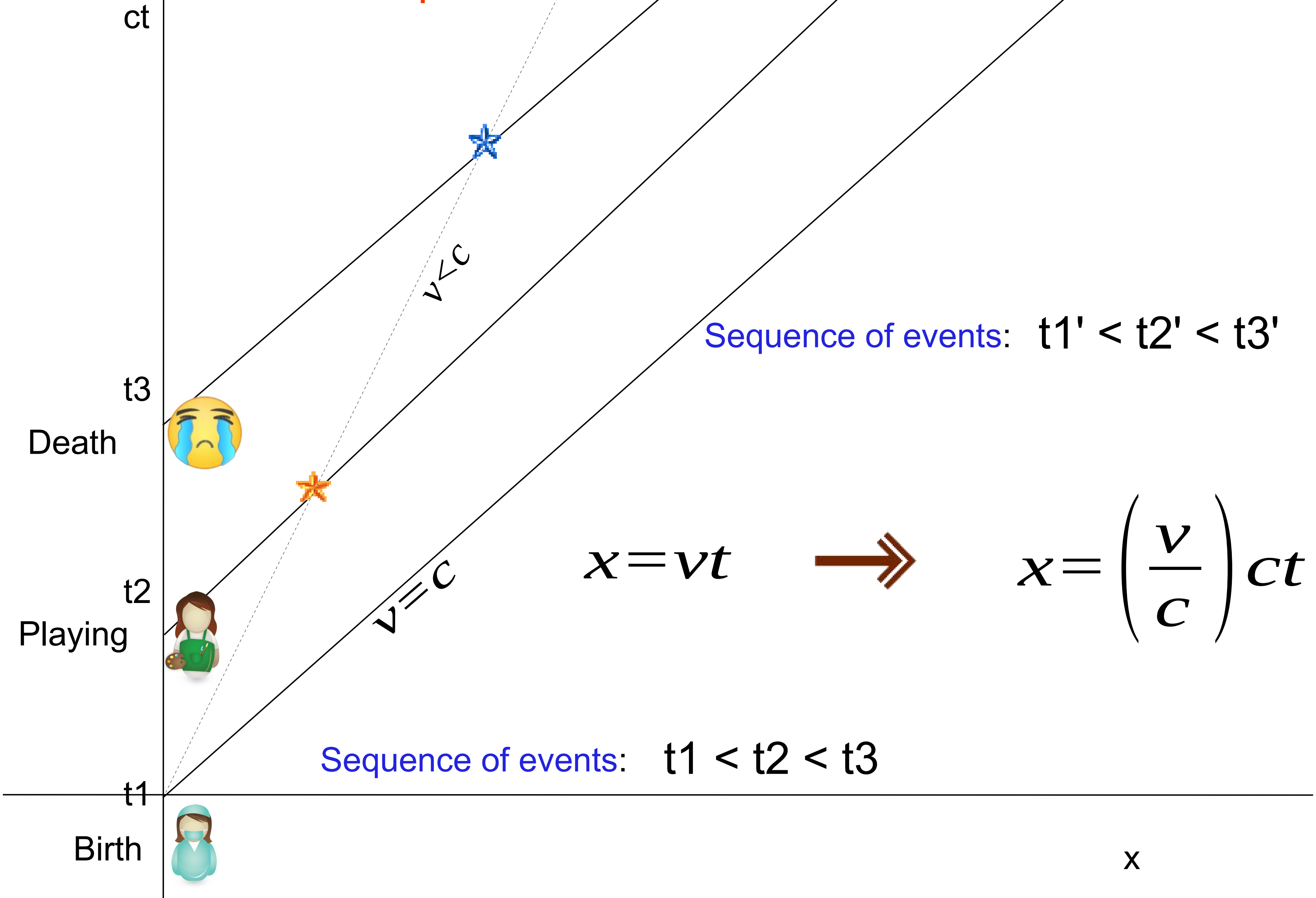
$$x = vt\,,\;\; x = \left(\frac{v}{c}\right) ct$$

$$x' = f(x, t),\; t' = g(t, x)$$

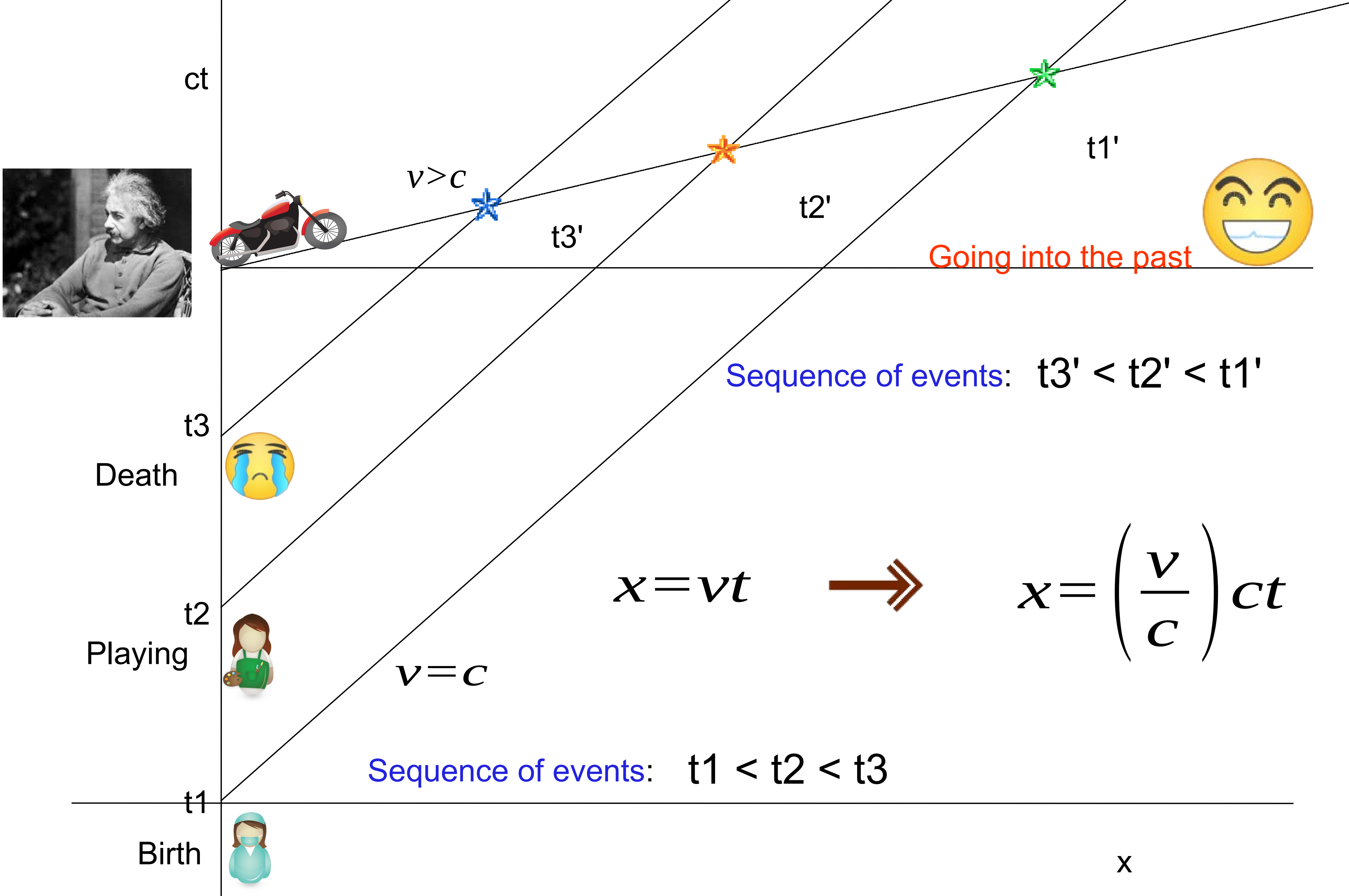
must be true

Thought Experiment

Events are at the same point



Thought Experiment



Thought Experiment

What we have obsered so far

1. Measurement of time does depened on the position of an event

2. There should exist a limiting velocity which is the velocity of light

3. Galiloe's idea of observer independence should be valid,
However, Galilean tranformation has to be modified

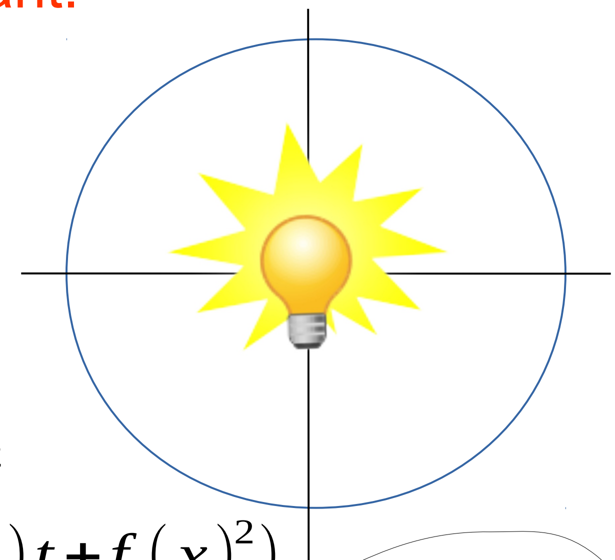
Einstein's postulates:

1) The laws of physics are the same in all intertial frame.
2) The speed of ligh in free space is same in all inerial frame.

What we have learnt: (x, t) & (x', t'): We must find out a **linear relation** among them keeping velocity of light constant.

Assuming:

$$x' = A(x - vt), t' = B(t - f(x))$$
$$y' = y, z' = z$$



$$x^2 + y^2 + z^2 = c^2 t^2$$
$$x'^2 + y'^2 + z'^2 = c^2 t'^2$$
$$A^2(x^2 - 2xvt + v^2t^2) + y^2 + z^2 = B^2c^2(t^2 - 2f(x)t + f(x)^2)$$
$$(A^2x^2 - B^2c^2f(x)^2) + 2t(-A^2xv + B^2c^2f(x)) + y^2 + z^2 = (B^2 - A^2\frac{v^2}{c^2})c^2t^2$$

$= x^2$ $\qquad = 0 \qquad = 1$

$$f(x) = \frac{A^2 v x}{B^2 c^2}$$

Consistent solution exists for:

A = B

$$A^2\left(1 - \frac{v^2}{c^2}\right) = 1$$

Lorentz Transformations

$$x'=A(x-vt),\ t'=B\left(t-\frac{vx}{c^2}\right)$$

$$y'=y,\ z'=z$$

$$A=B=\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}\equiv\gamma$$

$$x'=\gamma(x-vt),\ t'=\gamma\left(t-\frac{vx}{c^2}\right)$$

$$y'=y,\ z'=z$$

Newtonian Limit(non-relativistic)

$$\frac{v}{c}\ll 1$$

$$x'=(x-vt),\ t'=t$$

$$y'=y,\ z'=z$$

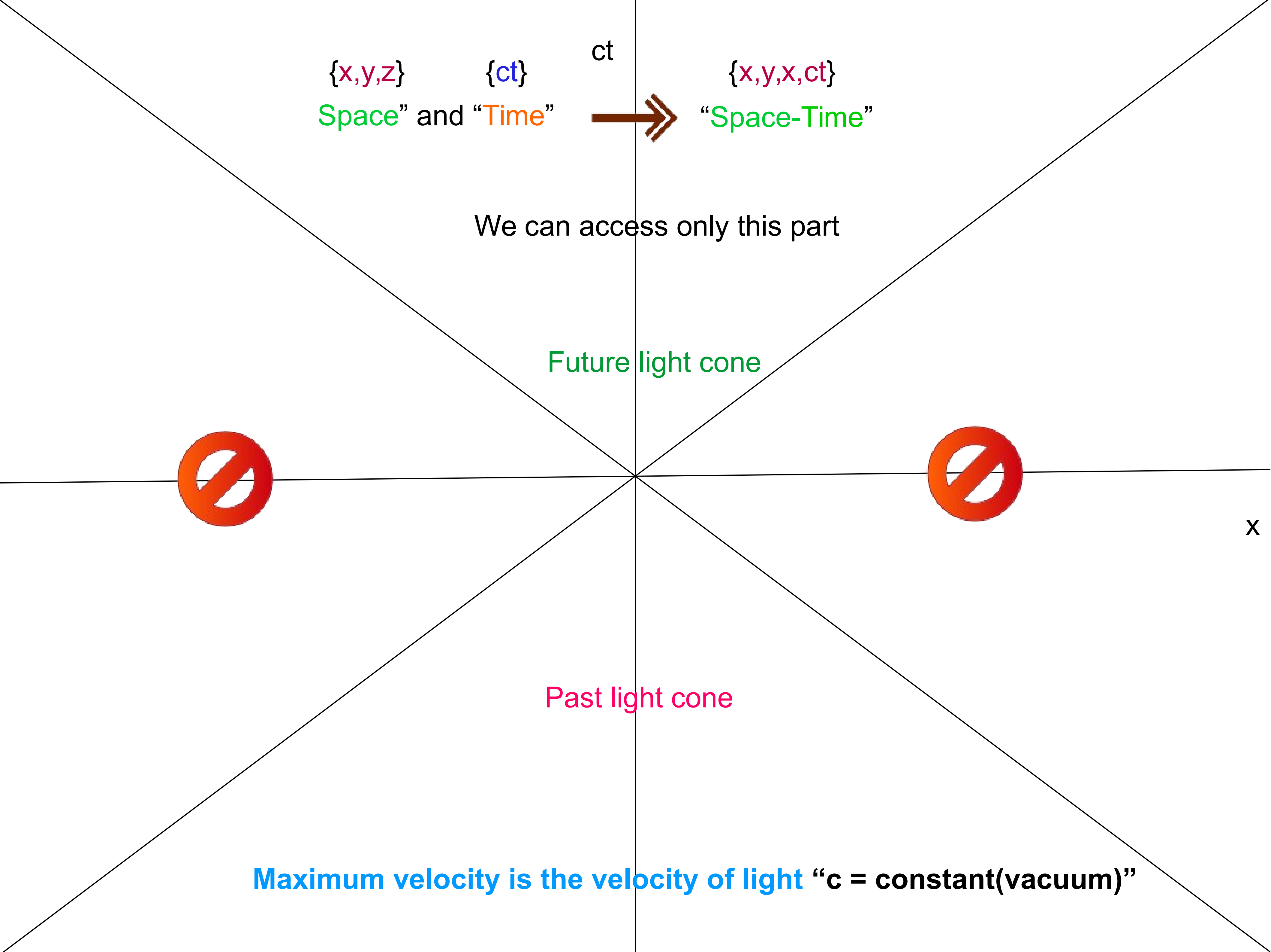
Since all ovservers are equivalent, the inverse transformation woule be:

$$x=\gamma(x'+vt'),\ t=\gamma\left(t'+\frac{vx'}{c^2}\right)$$

$$y=y',\ z=z'$$

Time and space get inter-connected

"Space and "Time" → "Space-time"



ct
{x,y,z}
{ct}
Space" and "Time"
{x,y,x,ct}
"Space-Time"
We can access only this part
Future light cone
x
Past light cone
Maximum velocity is the velocity of light "c = constant(vacuum)"

Offsprings

Time dilation (Moving clock runs slow!!)

Length Constraction (Running cat is safe for being shorter!!)

Relativistic Doppler effect

Time Dilation

Two events: emission and absorption of light at the same point

$$t_a = \gamma\left(t'_a + \frac{v x'}{c^2}\right)$$

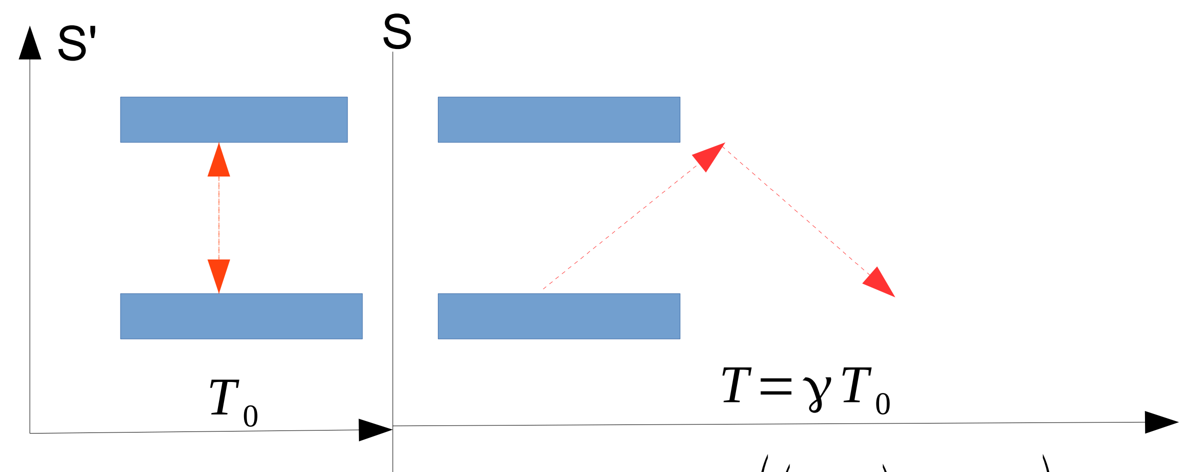
$$t_b = \gamma\left(t'_b + \frac{v x'}{c^2}\right)$$

$$t_b - t_a = \gamma\left(t'_b - t'_a\right)$$

$$T = \gamma T_0$$

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} > 1$$

$$T > T_0$$



$$T_0 = \frac{2L_0}{c}$$

$$T^2 v^2 = 4\left(\left(\frac{cT}{2}\right)^2 - L_0^2\right)$$

$$T = \frac{2L_0}{c\sqrt{1 - \frac{v^2}{v^2}}} = \frac{T_0}{\sqrt{1 - \frac{v^2}{v^2}}}$$

Moving clock runs slow

Length contraction

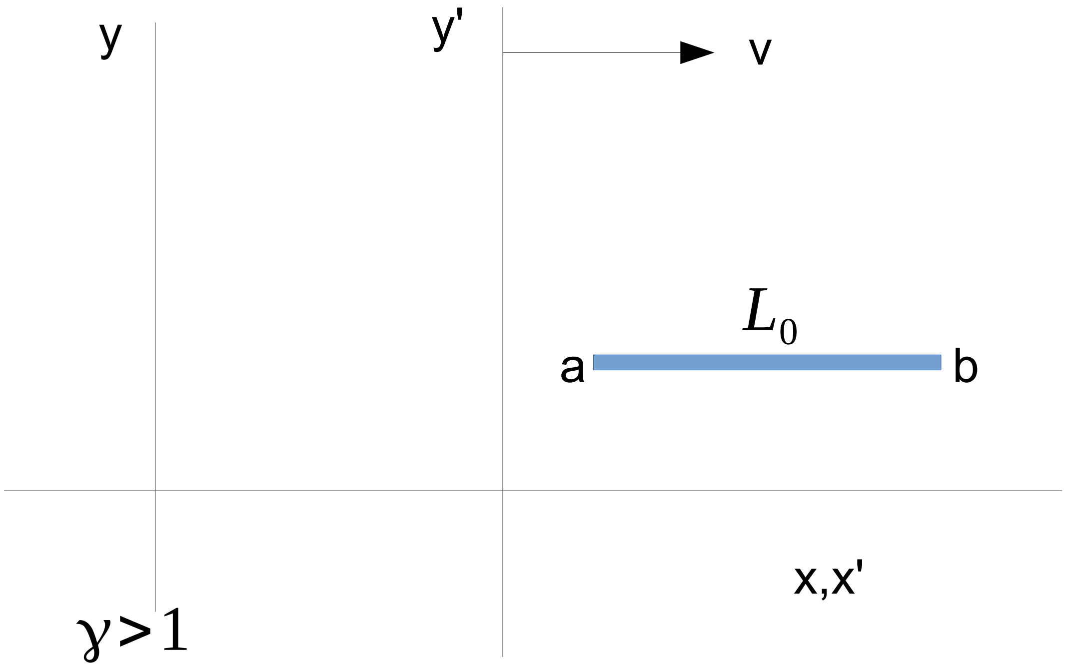
Before talking any such counter intutive things,
let us talk abou what do mean by lenght measurement

Length of a stick is the distance between its ends measured at the same instant of time

Length contraction

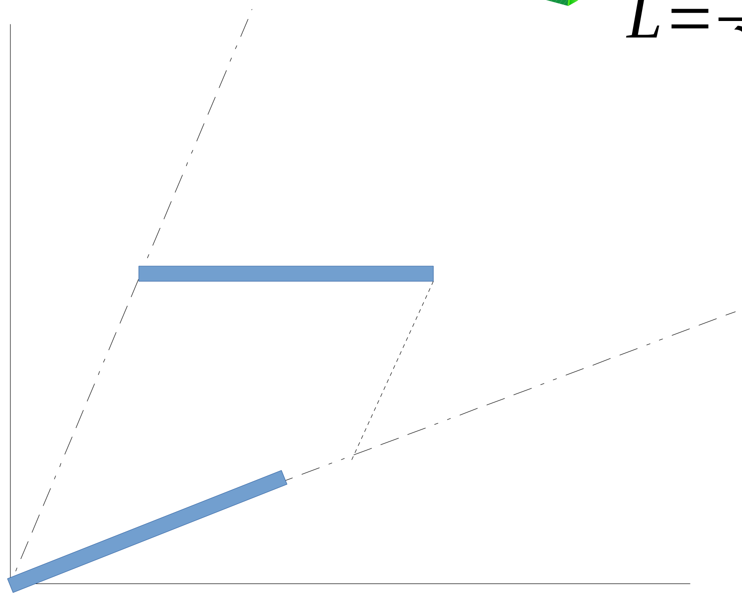
Condition: Length of a stick is the distance between its ends at the same instant of time

$$x'_a = \gamma(x_a - vt)$$
$$x'_b = \gamma(x_b - vt)$$



$$x'_b - x'_a = \gamma(x_b - x_a)$$
$$L_0 = \gamma L$$

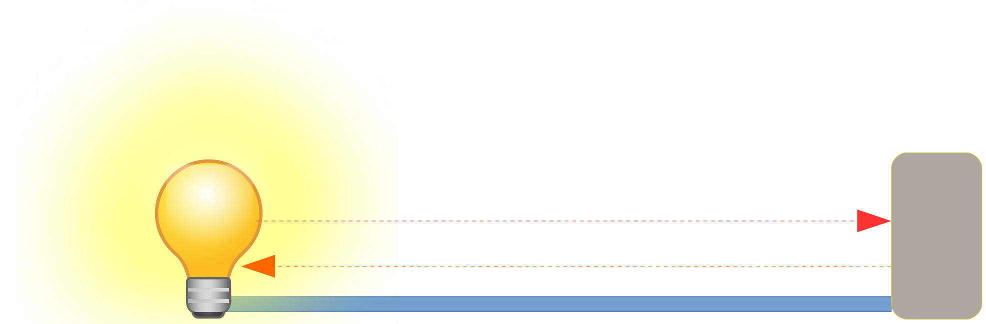
$$L = \frac{L_0}{\gamma} \qquad \gamma > 1$$



Therefore: $L < L_0$

Two events: emission and absorption of light at the same point

S'-frame

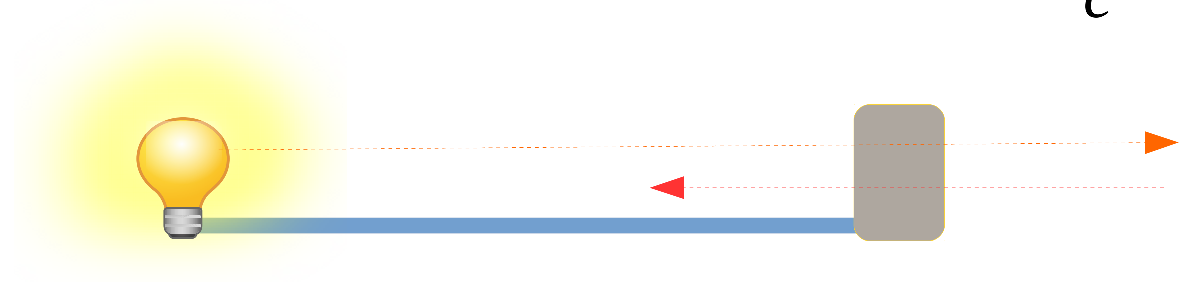


$$T_0 = \frac{2L_0}{c}$$

$$T_0 = t'_2 - t'_1$$

S-frame

$$\Delta t_1 = \frac{L}{c} + \frac{\Delta t_1 v}{c} \;;\; \Delta t_2 = \frac{L}{c} - \frac{\Delta t_2 v}{c}$$

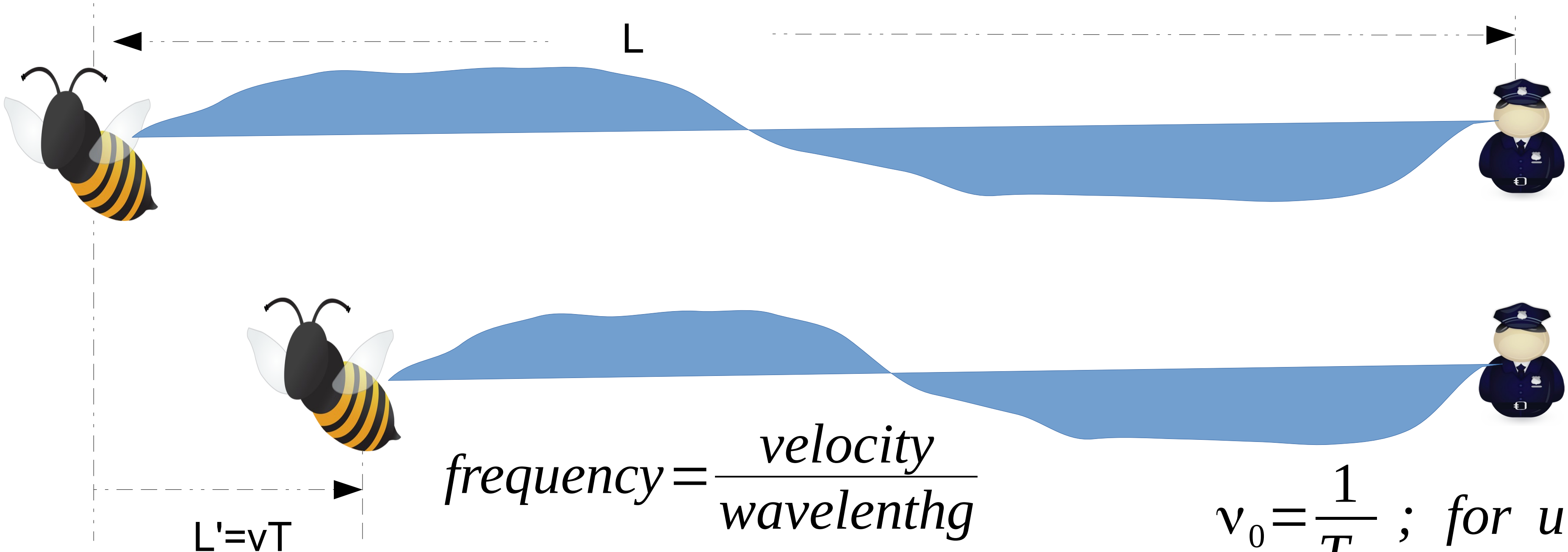


$$T = t_2 - t_1$$

$$T = \Delta t_1 + \Delta t_2 = \frac{L}{c-v} + \frac{L}{c+v} = \frac{2Lc}{(c^2 - v^2)}$$

We learnt: $T = \dfrac{T_0}{\sqrt{1 - \dfrac{v^2}{c^2}}} \Rightarrow \dfrac{2Lc}{(c^2 - v^2)} = \dfrac{2L_0}{c\sqrt{1 - \dfrac{v^2}{c^2}}} \Rightarrow L = \left(\sqrt{1 - \dfrac{v^2}{c^2}}\right) L_0$

The Doppler effect



$$frequency = \frac{velocity}{wavelenthg}$$

$$\nu_0 = \frac{1}{T_0}\ ;\ \ for\ \ u = 0$$

$$\nu_d = \frac{u}{L - L'} = \frac{u}{uT - vT} = \frac{1}{T}\frac{u}{u - v} = \frac{1}{T_0}\frac{u}{u - v}$$

(Newotonian)Non-relativistic

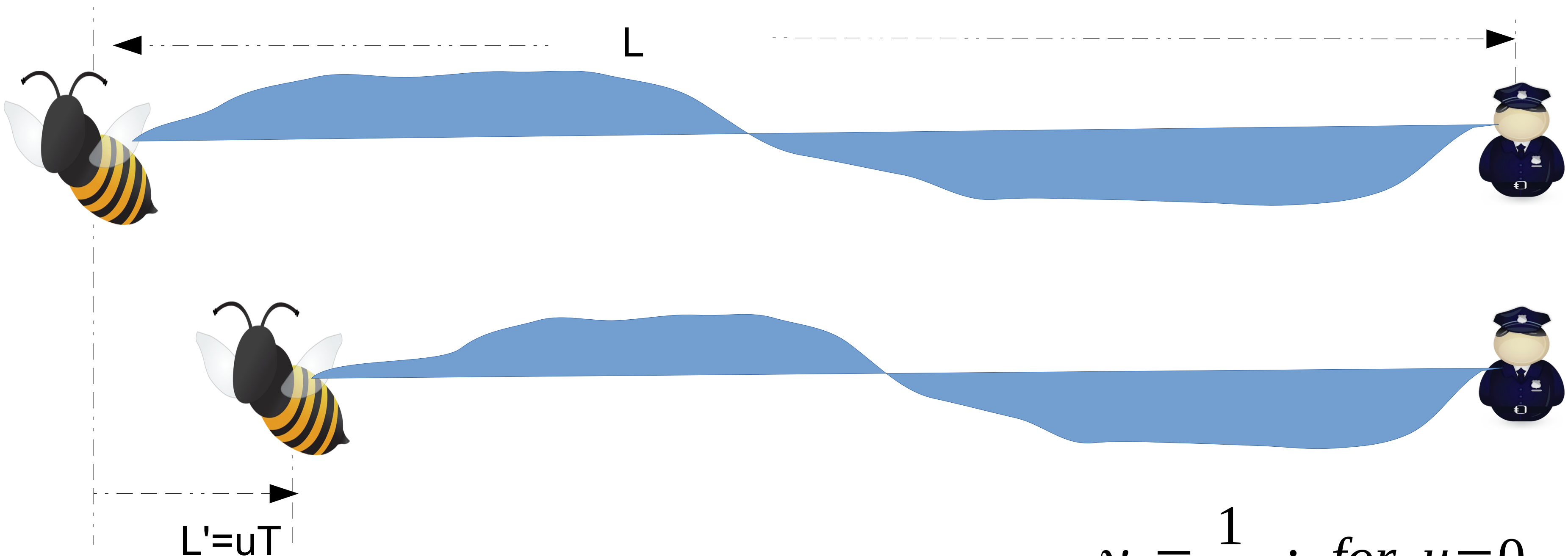
$$T = T_0$$

For light signal: u = c

$$\nu_d = \frac{1}{T}\frac{u}{u - v} = \frac{1}{\gamma T_0}\frac{c}{c - v} = \nu_0\sqrt{\frac{c + (+v)}{c + (-v)}}$$

Relativistic: $T = \gamma T_0$

The Doppler effect



$$\nu_0 = \frac{1}{T_0} \; ; \;\; for \;\; u = 0$$

$$\nu_d = \frac{u}{L - L'} = \frac{u}{uT - vT\cos\theta} = \frac{1}{T}\frac{u}{u - v\cos\theta}$$

Non-relativistic: $T = T_0$

$$\nu_d = \frac{1}{\gamma T_0}\frac{c}{c - v\cos\theta}$$

Relativistic: $T = \gamma T_0$

Navigation, GPS

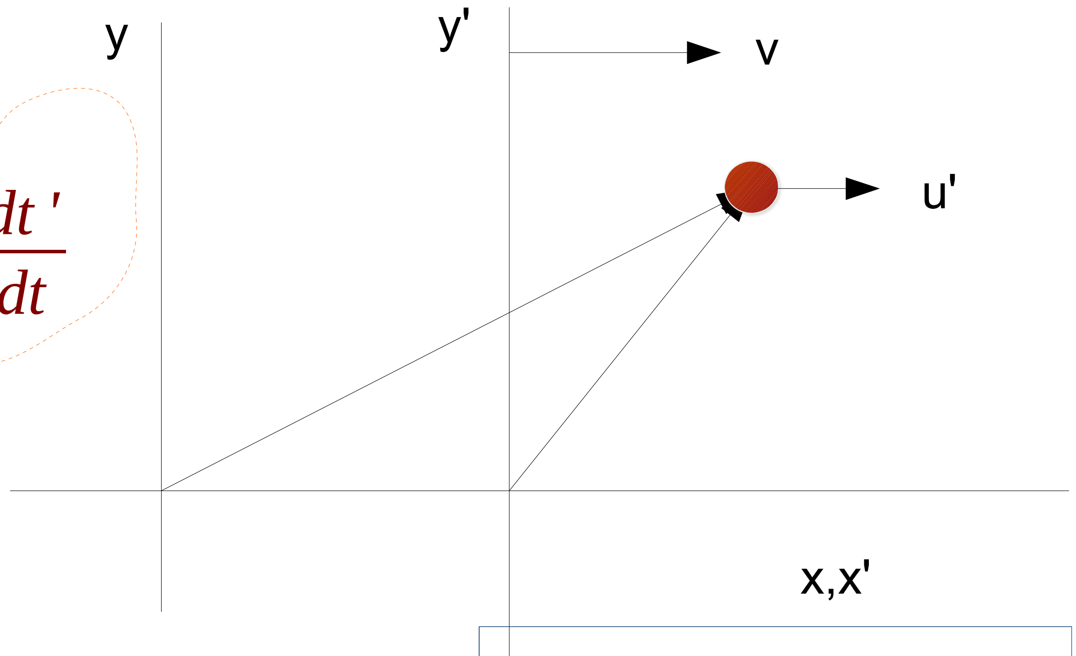
Velocity addition theorem

Assuming: u' is the velocity with respect to the moving frame

$$x' = \gamma(x - vt),\ t' = \gamma\left(t - \frac{vx}{c^2}\right)$$

$$\frac{dx}{dt} = u = \gamma \frac{d(x' + vt')}{dt} = \gamma \frac{d(x' + vt')}{dt'} \frac{dt'}{dt}$$

$$u = \gamma^2 (u' + v)\left(1 - \frac{vu}{c^2}\right)$$



$$u' = \frac{u - v}{1 - \frac{uv}{c^2}}$$

$$u = \frac{u' + v}{1 + \frac{u'v}{c^2}}$$

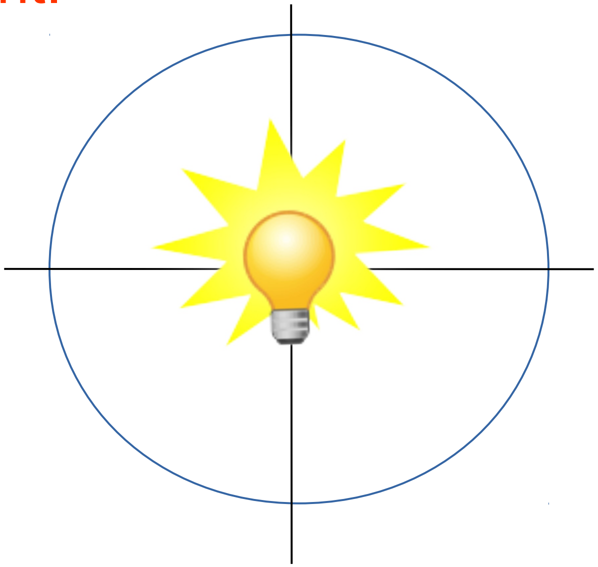
$$u_{max} = c$$

Lorentz Transformation: Invariance property

What we have learnt: (x, t) & (x', t'): We must find out a linear relation among them keeping velocity of light constant.

Assuming: $$x' = A(x - vt), t' = B\left(t - \frac{vx}{c^2}\right)$$

$$y' = y, z' = z$$



$$x^2 + y^2 + z^2 = c^2 t^2$$
$$x'^2 + y'^2 + z'^2 = c^2 t'^2$$

$$A^2\left(1 - \frac{v^2}{c^2}\right) = 1$$

Let us understand how do we derive "A"

If you think little deep, actually we demand:

$$S'^2 = -c^2 t'^2 + x'^2 + y'^2 + z'^2 = -c^2 t^2 + x^2 + y^2 + z^2 = S^2$$

For light, S =S'=0

Lorentz Transformation: Invariance property

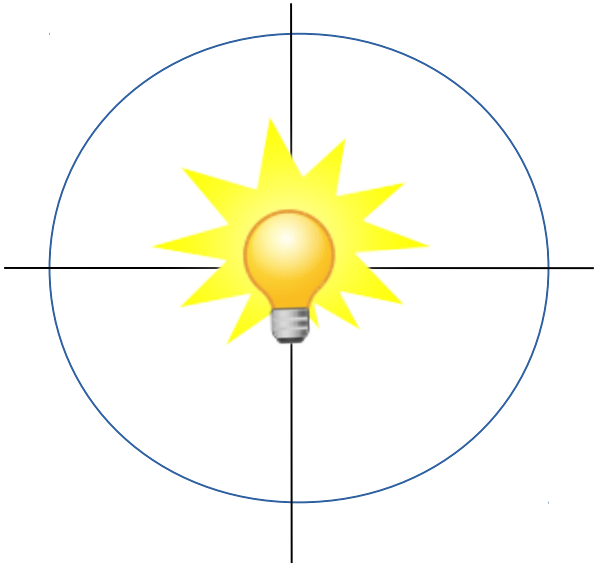
$$x' = \gamma(x - vt), t' = \gamma\left(t - \frac{vx}{c^2}\right)$$

$$y' = y, z' = z$$

$$S^2 = -c^2t^2 + x^2 + y^2 + z^2$$
$$S'^2 = -c^2t^2 + x^2 + y^2 + z^2$$
$$S^2 = S'^2$$

Tranformation is linear in (t, x, y, z)

Invariant



$$dx' = \gamma(dx - v\,dt), dt' = \gamma\left(dt - \frac{v\,dx}{c^2}\right)$$

$$dy' = dy, dz' = dz$$

Invariant quantities:

$$dS^2 = -c^2dt^2 + dx^2 + dy^2 + dz^2 = dS'^2$$

$$d\tau^2 = dt^2 - \frac{dx^2 + dy^2 + dz^2}{c^2} = dt^2\left[1 - \frac{1}{c^2}\left(\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2\right)\right]$$

$$\Rightarrow \quad = dt^2\left(1 - \frac{u^2}{c^2}\right)$$

Invariant time: proper-time

Newtonian

Rotation+Galileon

$$dx' = dx\cos\theta + dy\sin\theta$$
$$dy' = -dx\sin\theta + dy\cos\theta$$
$$dz' = dz$$

$$dx' = dx - v\,dt\,,\; dy' = dy\,,\; dz' = dz$$

$$dt' = dt$$

$$ds^2 = dx^2 + dy^2 + dz^2 = ds'^2$$

Distance: measured at same instant of time

Time t: Invariant under all observers

dt

With respect to which we measure physical variation

Einsteinian

Rotation+ Lorentz

$$x' = x\cos\theta + y\sin\theta$$
$$y' = -x\sin\theta + y\cos\theta$$
$$z' = z$$

$$dx' = \gamma(dx - v\,dt),\, dt' = \gamma\left(dt - \frac{v\,dx}{c^2}\right)$$

$$dy' = dy\,,\, dz' = dz$$

$$ds^2 = -c^2 dt^2 + dx^2 + dy^2 + dz^2 = ds'^2$$

Proper time τ: Invariant under all observers

$d\tau$?

Concept of ovserver independent time (proper time)

Why do need to define such a quantity?

Newtonina dynamics: we use time (t) as a parameter(observer independent): measure all the dynamical 'changes” of state of a system (position, momentum,...)

Now you know: Newtonin time is not invariant any more!!

Define invariant time(proper time)

$$d\tau^2 = dt^2 - \frac{dx^2 + dy^2 + dz^2}{c^2} = dt^2\left[1 - \frac{1}{c^2}\left(\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2\right)\right]$$

$$= dt^2\left(1 - \frac{u^2}{c^2}\right)$$

$$d\tau = dt\sqrt{1 - \frac{u^2}{c^2}} = dt'\sqrt{1 - \frac{u'^2}{c^2}} = d\tau'$$

u: velocity of the particle under study

Newtonian world

3-dimensionl Eucledian space

$$S^2 = x^2 + y^2 + z^2$$

$$dS^2 = dx^2 + dy^2 + dz^2$$

Definitio: three-position

$$r_i \equiv (x\ , y\ , z),\ r_1 = x,\ r_2 = y,\ r_3 = z$$

$$dr_i \equiv (dx\ , dy\ , dz),$$
$$dr_1 = dx,\ dr_2 = dy,\ dr_3 = dz$$

$$r^2 = \boldsymbol{r} \cdot \boldsymbol{r} = r_1^2 + r_2^2 + r_3^2$$
$$= x^2 + y^2 + z^2$$

Einstein's world

4-dimensionl Einsteinina space-time

$$S^2 = -c^2 t^2 + x^2 + y^2 + z^2$$

$$dS^2 = -c^2 dt^2 + dx^2 + dy^2 + dz^2$$

Definitio: Four-position

$$\Re_i \equiv (ict\ , x\ , y\ , z),$$
$$\Re_1 = ict,\ \Re_2 = x,\ \Re_2 = y,\ \Re_3 = z$$

$$d\,\Re_i \equiv (i\,c\,dt\ , dx\ , dy\ , dz),$$
$$d\,\Re_1 = ic\,dt,\ d\,\Re_1 = dx,\ d\,\Re_2 = dy,\ d\,\Re_3 = dz$$

$$\Re^2 = \boldsymbol{\Re} \cdot \boldsymbol{\Re} = \Re_1^2 + \Re_2^2 + \Re_3^2 + \Re_4^2$$
$$= -c^2 t^2 + x^2 + y^2 + z^2$$

Definitions: Position, Momentum, Acceleration

Newtonian	Relativistic
Time (t) as a parameter with respect to which we define all the changes.	Proper time as a parameter with respect to which we define all the changes.
Definition: 3-vectors	Definition: 4-vectors
3-Position: $r_i \equiv (x,y,z), r_1 = x \ldots$	4-Position: $\Re_\mu \equiv (ict,\ x,\ y,\ z),\ \Re_1 = ict \ldots$
3-momentum: $p_i = m_0 \frac{dr_i}{dt} \equiv m_0 \left(\frac{dx}{dt}, \frac{dy}{dt}, \frac{dz}{dt} \right)$	4-momentum: $\wp_\mu = m_0 \frac{d\Re_\mu}{d\tau} \equiv m_0 \left(ic\frac{dt}{d\tau}, \frac{dx}{d\tau}, \frac{dy}{d\tau}, \frac{dz}{d\tau} \right)$
3-acceleration: $a_i = \frac{d_2 r^i}{dt^2} \equiv \left(\frac{d^2x}{dt^2}, \frac{d^2y}{dt^2}, \frac{d^2z}{dt^2} \right)$	4-acceleration: $a_\mu = \frac{d^2\Re_\mu}{d\tau^2} \equiv \left(ic\frac{d^2t}{d\tau^2}, \frac{d^2x}{d\tau^2}, \frac{d^2y}{d\tau^2}, \frac{d^2z}{d\tau^2} \right)$
$\frac{dp_i}{dt} = F_i$	$\frac{d\wp_\mu}{d\tau} = f_\mu$

“dot” product of vectors

What you know

Definitio: three-vectors

3-Position: $r_i \equiv (x, y, z)\ ;\ r_1 = x \ldots$

$$r^2 = \boldsymbol{r}\cdot\boldsymbol{r} = \sum_{i=1}^{3} r_i r_i = r_1^2 + r_2^2 + r_3^2$$

$$= x^2 + y^2 + z^2$$

Any 3-vector (say 3-momentum):

$$p_i = m_0 \frac{dr_i}{dt} \equiv m_0 \left(\frac{dx}{dt}, \frac{dy}{dt}, \frac{dz}{dt} \right)$$

$$p^2 = \boldsymbol{p}\cdot\boldsymbol{p} = \sum_{i=1}^{3} p_i p_i = p_1^2 + p_2^2 + p_3^2$$

$$= m_0^2 \left(\dot{x}^2 + \dot{y}^2 + \dot{z}^2 \right)$$

$$\boldsymbol{A} \equiv A_i \equiv (A_1, A_2, A_3)$$

In a same way

Definitio: four-vectors

4-Position: $\Re_\mu \equiv (i\,ct, x, y, z),\ r_1 = i\,ct \ldots$

$$\Re^2 = \boldsymbol{\Re}\cdot\boldsymbol{\Re} = \sum_{\mu=1}^{4} \Re_\mu \Re_\mu = \Re_1^2 + \Re_2^2 + \Re_3^2 + \Re_4^2$$

$$= -c^2 t^2 + x^2 + y^2 + z^2$$

Any 4-vecto (say 4-momentum):

$$\wp_\mu = m_0 \frac{d\,\Re_\mu}{d\tau} \equiv m_0 \left(ic\frac{dt}{d\tau}, \frac{dx}{d\tau}, \frac{dy}{d\tau}, \frac{dz}{d\tau} \right)$$

$$\wp^2 = \boldsymbol{\wp}\cdot\boldsymbol{\wp} = \sum_{i=1}^{4} \wp_i \wp_i = \wp_1^2 + \wp_2^2 + \wp_3^2 + \wp_4^2$$

$$= m_0^2 \left(-c^2 t'^2 + x'^2 + y'^2 + z'^2 \right)$$

$$A^\mu \equiv (i A_1, A_2, A_3, A_4)$$

Let us look at term by term of 4-momentum

Momentum 4-vector (say 4-momentum): $\wp_\mu = m_0 \frac{dr_\mu}{d\tau} \equiv m_0 \left(ic\frac{dt}{d\tau}, \frac{dx}{d\tau}, \frac{dy}{d\tau}, \frac{dz}{d\tau} \right)$

$$d\tau = dt\sqrt{1-\frac{v^2}{c^2}} = d\tau'$$

$$\wp_1 = i m_0 c \frac{dt}{d\tau} = i\frac{m_0 c}{\sqrt{1-\frac{v^2}{c^2}}} = \left(\frac{i}{c}\right) m c^2 = i\, p_t$$

$$\wp_2 = m_0 \frac{dx}{d\tau} = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}\frac{dx}{dt} = m v_x = p_x$$

$$\wp_3 = p_y \;;\; \wp_4 = p_z$$

$$m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

Relativistic energy

$$\wp_\mu = (i\,p_t, p_x, p_y, p_z) \;;\; \wp^2 = \sum_{i=1}^{4} \wp_i \wp_i = -m^2c^2 + p_x^2 + p_y^2 + p_z^2$$

Let us closely look at the first component of momentum

Momentum 4-vector (say 4-momentum): $\wp_\mu = m_0 \frac{dr_\mu}{d\tau} \equiv m_0 \left(ic\frac{dt}{d\tau}, \frac{dx}{d\tau}, \frac{dy}{d\tau}, \frac{dz}{d\tau} \right)$

$$d\tau = dt\sqrt{1-\frac{v^2}{c^2}} = d\tau'$$

$$\wp_1 = i m_0 c \frac{dt}{d\tau} = i\frac{m_0 c}{\sqrt{1-\frac{v^2}{c^2}}} = \left(\frac{i}{c}\right) m c^2 \quad = \frac{i}{c}\left(m_0 c^2 + \frac{1}{2} m_0 v^2 \ldots\right)(\text{for } v \ll c)$$

KE : Kinetic energy

$$\delta\wp_1 = \wp_1(v) - \wp_1(0) = \left(\frac{i}{c}\right)\frac{1}{2} m_0 v^2 = \left(\frac{i}{c}\right)(KE)$$

Rest mas energy

Einstein's Total relativistic energy = E $= \left(m_0 c^2 + \frac{1}{2} m_0 v^2 \ldots\right) = m c^2$

$$\wp_\mu = \left(i\frac{E}{c}, p_x, p_y, p_z\right) \;;\; \wp^2 = \sum_{i=1}^{4} \wp_i \wp_i = -\left(\frac{E^2}{c^2}\right) + p_x^2 + p_y^2 + p_z^2$$

$Energy \equiv Mass$

$$\wp_\mu = \left(i\frac{E}{c}, p_x, p_y, p_z\right) \equiv \left(i\frac{E}{c}, \boldsymbol{p}\right)$$

$$d\tau = dt\sqrt{1-\frac{v^2}{c^2}} = d\tau'$$

Relativistic energy

$$E = mc^2$$

Relativistic Mass

$$m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\wp^2 = \sum_{i=1}^{4} \wp_i \wp_i = -\left(\frac{E^2}{c^2}\right) + p_x^2 + p_y^2 + p_z^2 = -m^2 c^2 + p_x^2 + p_y^2 + p_z^2$$

Relativistic energy of a particle of momentum p, and mass m_0

$$\wp^2 = -\left(\frac{E^2}{c^2}\right) + p_x^2 + p_y^2 + p_z^2 = -m^2 c^2 + p_x^2 + p_y^2 + p_z^2$$

$$= m^2\left(-c^2 + v_x^2 + v_y^2 + v_z^2\right)$$

$$= -\frac{m_0^2}{1 - \frac{v^2}{c^2}}\left(c^2 - v_x^2 - v_y^2 - v_z^2\right)$$

$$= -m_0^2 c^2$$

4-momentum
Massive particle:

$$\wp^2 = -m_0^2 c^2 \quad \Rightarrow \quad E^2 = p^2 c^2 + m_0^2 c^4$$

Relativistic energy of a massless particle: Photon

$$m_0 = 0 \quad \Rightarrow \quad \wp^2 = 0 \quad \Rightarrow \quad E = p\,c$$

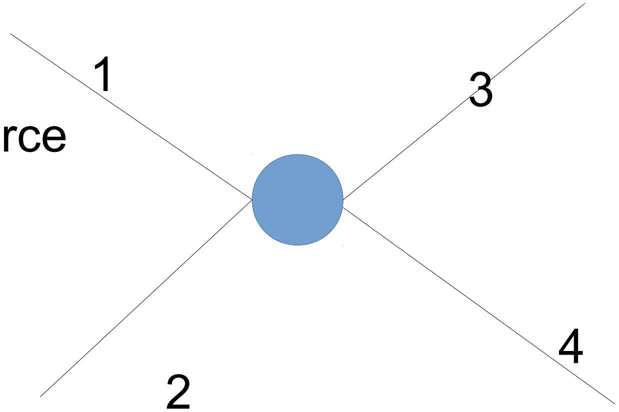
$$\wp_\mu = \left(i\frac{E}{c}, \boldsymbol{p}\right) = \left(i\,p, \boldsymbol{p}\right) = \left(i|\boldsymbol{p}|, \boldsymbol{p}\right)$$

NO Newtonian analog

Conservation Law

$$\frac{d\wp_\mu}{d\tau} = f_\mu = 0$$

If there is no external force



$$\wp_\mu^1 + \wp_\mu^2 = \wp_\mu^3 + \wp_\mu^4$$

$$E^1 + E^2 = E^3 + E^4$$

$$m_1 \boldsymbol{u_1} + m_2 \boldsymbol{u_2} = m_3 \boldsymbol{u_3} + m_4 \boldsymbol{u_4}$$