Application of MGGP in Predicting Bearing strength of a Strip Footing Resting on the Crest of a Marginal Soil Hillslope

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Abstract

A set of finite element investigations are performed to examine the maximum bearing strength of strip footings positioned on the crest of a cohesive-frictional marginal soil hillslope. In this regard, the influence of contributing geometrical and geotechnical parameters on the maximum bearing strength of the footing are illustrated. It is revealed that the nearness of slope face has negligible influence on the bearing strength of footing if it is located at a setback distance beyond six times the footing width. Further, using multi-gene genetic programming technique, a predictive relationship between the maximum bearing strength and the contributory factors is established and validated through relevant experimental findings. The hyper-parameters of the MGGP model are suitably optimized, as indicated by the coefficient of correlation attaining high magnitudes. A sensitivity analysis based on local perturbation is conducted to recognize the importance ranking of the contributory parameters. It is revealed that the friction angle of slope material predominantly influences the evaluation of maximum bearing strength for strip footing on slopes, followed by other contributing factors.

Keywords: *Strip footing on slope*, *Finite element analysis*, *Maximum bearing strength*, *MGGP*, *Sensitivity assessment*

1. Introduction

Maximum bearing strength of a footing is the ultimate load sustained without undergoing shear failure or exceeding the allowable limit of settlement. This entity is important for foundation engineers to design a suitable (safe and economic) foundation for variety of infrastructures. The maximum bearing strength depends on the shear strength parameters of the foundation soil (friction angle, φ, and cohesion, c) as well as controlled by the footing size, shape, and its embedment depth. Based on several simplified assumptions, Terzaghi (1943) pioneered an expression to assess the maximum bearing strength of strip footing on a semi-infinite horizontal ground, which was expressed in terms of c and φ and some bearing strength factors (BCFs), i.e. N_c, N_q and N_γ. Meyerhof (1951) extended Terzaghi's proposal by reckoning the failure line extending up to the ground level, thereby proposing a modified set of BCFs. Further, Skempton (1951) conducted field and laboratory investigations to assess N_c for finding the bearing strength of footings of various dimensions embedded in saturated clay. Thereafter, other investigators (Hansen 1970; Vesic 1973) had proposed general expressions for estimating bearing strength of shallow footings accommodating depth, shape, load inclination and compressibility effects in the bearing strength factors.

In urbanized hilly terrains, footings of infrastructures are often constructed on or near the slope crests. Such can be commonly observed in the foundations of electric transmission and telecom towers, bridge abutments, and overhead water tanks. In comparison to the flatter terrains, the maximum bearing strength of footings situated at the slope crest are lower owing to the partially developed passive zone underneath the footing due to its intersection with the slope face. The passive resistance reduces significantly for steeper slopes. In this regard, during the 1970s and 1980s, researchers have experimentally investigated the maximum bearing strength of strip footings placed near the crest of a sandy slope and furnished a set of modified BCFs (Shields *et al*. 1977; Bauer *et al*. 1981). For similar scenario, several researchers have identified that development of plastic zones is also affected by the footing shape (Castelli and Lentini 2012), whether strip, square or circular (Azzam and El-Wakil 2015; Acharyya and Dey 2017). The maximum bearing strength of footings on the slope comprising marginal soil and the related failure mechanism is also explored by few researchers (Leshchinsky 2015; Acharyya *et al*. 2020).

With the growing demands of populations, urbanization in the hilly terrains has been on the rise all over the world. The north-eastern provinces of India especially cover hilly areas supporting a large share of the building infrastructures (single or multi-storeyed) on or near the crest of the slopes. In case of common low-rise infrastructures, adoption of strip footing is mostly prevalent. The hill-slopes of north-east India predominantly comprises marginal soil whose strength properties are dictated by both cohesion and friction, and hence such soils are colloquially termed as cohesive-frictional or c-φ soils. It is important to understand the effect of slope inclination and setback distance on the normalized maximum bearing strength of strip footing positioned near slope face [given as $q_u/\gamma H_s$ where, H_s is the slope height of slope, γ is the unit weight of slope material, and q_u is the ultimate bearing capacity (or, UBC)]. This can be commonly achieved using a limit equilibrium analysis (LEA) by equilibrating the driving and resisting forces or moments (Castelli and Motta 2009). On the contrary, continuum analysis through finite element analysis (FEA) employing coupled stress-deformation approaches provide a better perspective (Chakraborty and Kumar 2013). Slope instability, triggered naturally or by some applied external force, is largely governed by the displacement of the slope face that plays an instrumental role in realistically deviating from the assessment of limit-equilibrium based safety factor. However, it is also to be noted that conducting FEA for assessing the stability of foundation on slopes for each and every isolated case is tedious and time-demanding. It would be prudent to rather develop a database of governing parameters and their quantitative influence on the stability, and employ the database to develop a set of mathematical expressions that could be easily used in simple calculating engines by any common applicant to gain the preliminary information on the stability of foundation on slopes.

In the present times, various soft computing or machine learning algorithms are utilized for this purpose. Although, jargon-wise, there are a plethora of learning techniques different from each other in their functionality and algorithms, yet all of them finally produces a set of simplified mathematical expressions that is commonly stated to be the architecture of the solution. In the current research, MGGP (or, multi-gene genetic programming) is used to generate the predictive relationship between the UBC of footings on the slope and the contributory parameters. MGGP is capable to overcome the restrictions of dimensionality, regression, and empirical analyses. It aids in the robust choice of algorithm parameters such the model performs as a best descriptor of variability while achieving reasonable predictability (Pattanaik *et al*. 2017; Mishra *et al*. 2017). Both genetic programming (GP) and MGGP has been used in several practical civil engineering problems namely Prediction of unsaturated hydraulic conductivity curve (Johari *et al*. 2006), unconfined compressive strength of soft soil (Narendra *et al*. 2006), settlements of foundation (Rezania and Javadi 2007), local scouring adjacent to hydraulic structures (Guven and Gunal 2008), cyclic stress-strain behaviour of sand (Shahnazari *et al.* 2010), lateral load capacity of piles (Gandomi and Alavi 2012), landslide induced displacements (Chen *et al*. 2016), assessment of pavement performance (Shahnazari *et al*. 2012) and uniaxial compressive strength of rock (Armaghani *et al.* 2018). It is noted that the MGGP has not yet been utilized for predicting the stability of footings on hillslopes.

This paper reports the devising of a MGGP-based predictive relationship to assess the maximum bearing strength of a strip footing located on a hillslope comprising marginal soil. The dataset for the MGGP model will be populated from finite element (FE) analysis of strip footing on slope crest. Along with the detailing of the developed FE model, a description of various parameters affecting the response of FE model is also provided. Further, the development of the MGGP-based predictive relation through the optimization of its various hyper-parameters and its performance is elucidated through various statistical evaluators. The developed MGGP-based predictive relation is validated against relevant experimental investigations. Finally, a local perturbation based sensitivity study is carried out to identify the importance ranking of the input parameters on the developed MGGP model.

2. Problem Statement and Adopted Methodologies

The present study focusses on evaluating the maximum bearing strength of a strip footing located at various positions on a hillslope (as depicted in Figure 1). The bearing strength is affected by several parameters, i.e. c and φ of the slope material, width of footing (B), setback distance (b, the distance between the slope face and nearer edge of the footing; expressed in terms of footing width as a ratio b/B), depth of embedment of footing (D_f; expressed as a ratio of embedment depth to footing width, i.e. D_f/B) and the slope inclination angle (β). The reasonable ranges of these parameters as adopted in the current research are provided in Table 1. In general, it is observed in many hillslopes in north-eastern parts of India (Sarma *et al*. 2014, 2018, 2020a), the slope material has very high cohesive strength and are unlikely to fail. Most of the failures have been noted in the slopes having cohesive strength below 80 kPa. In such slopes, the frictional characteristics remains mostly on the lower side, mostly the friction angle is below 25°. For slopes with higher sandy and silty content, the friction angle can get as high as 40°, while the corresponding cohesive strength remain on the lower side, mostly below 30 kPa. Accordingly, the ranges of shear strength parameters are decided as given in Table 1. Literature suggests that slopes with inclinations within the range 10°-40° are more likely to attract urbanization, making them more susceptible to fail (Sarma *et al.* 2020b). Due to paucity of open spaces, it is a common practice in hillslopes to construct infrastructures supported on strip or combined footings with larger length-to-width ratios. In such case, it is not expected to have a strip footing more than 2 m width (Acharyya and Dey, 2021), and hence, a range of 0.5 m (for wall foundations) to 2.0 m (for foundations of buildings and transmission towers) is considered in the present study. Excavation of pits for shallow foundations in hillslopes is always a risky task, and hence, it is not a customary practice to consider depth of open excavations of shallow foundations more than 2 m in the hillslopes. In case the foundation design remains unsatisfactory with the depth of excavation being less than 2 m, alternative foundations have to be explored. Hence, in compliance to the classification of shallow foundations (Murthy 2008), a normalized depth of embedment is considered up to the value of 1. In most of the instances, the infrastructure in the hillslopes are built by preparing a horizontal bed at the crest of the slope. The nearness of the infrastructure to the face of the slope would affect the performance and bearing strength of the adopted footing. Hence, to study this influence and identify the threshold beyond which the slope does not impart its effect on the bearing strength of the footing, a normalized setback distance of 0-10 is used. …

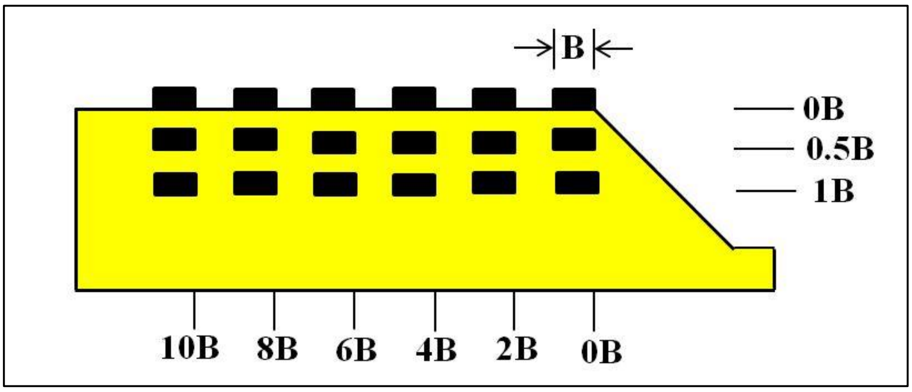


Fig. 1 Typical positions of strip footing in a marginal soil slope

Table 1 Ranges of geometrical and geotechnical parameters adopted in the study

	Parameters	MGGP notation	Range
Geotechnical parameters	c (kPa)	X_1	10-80
	φ (°)	X_2	10-40
Geometrical parameters	B (m)	X_3	0.5-2
	b/B	X_4	0-10
	β (°)	X_5	10-40
	D_f/B	X_6	0-1

In the present study, all the possible variations in the individual input parameters (c, φ, B, b/B, β, D_f/B) within their feasible engineering ranges are included for conducting the FE simulations. However, it is to be yet noted that the output parameter might be considered idealized as it is an outcome of the finite element simulations following a specific constitutive model and compatibility behavior. At the same time, even though the input parameters are considered in their corresponding ranges, yet each set of input parameters lead to a specific and deterministic output magnitude. In this process, the uncertainties or variabilities commonly encountered in the field, i.e. either epistemic or aleatoric, does not get incorporated in the analyses. There are several avenues by which variability and uncertainty can percolate in the real-life scenarios of foundations on hillslopes. In the present study, the hillslope is considered homogeneous in each analyses. However, the shear strength parameter generally exhibits spatial variability. Variability can emerge in the real field scenarios due to the actual behavior of the hillslope material, which is restrained in the present analysis by the usage of a specific constitutive model. In the present study, the slope inclination is also considered identical throughout the height of the slope; however, it is understood that the same might noticeably vary as natural hillslopes hardly follow a specific pattern of inclination throughout. These differences in the input parameters, added to the measurement uncertainties, can lead to a magnitude of the bearing capacity that might be different than the estimations provided by the FE simulation. It is possible to incorporate some of the variabilities in the analyses by adopting a stochastic analysis employing random variables or random fields for the relevant parameters. However, such adoption is significantly resource and computation demanding, which is beyond the scope of the present study. Thus, the findings of the present study of formulating MGGP models remain restrained by the deterministic solution of bearing capacity of strip footing on slopes within the reasonable engineering ranges of the input parameters.

2.1 Description of finite element modelling

In the present study, the FE modelling is adopted to generate the input-output database required for the development of the MGGP model. As mentioned earlier, the realistic option is collating a database from laboratory-scale or prototype experiments so that variabilities are imbibed in the dataset. However, conducting a very large number of such controlled experiments is resource demanding and infeasible in most cases. Hence, in order to generate the data pool required to develop the MGGP model, considering a plane-strain condition, a two-dimensional (2D) finite element (FE) framework is developed to numerically model and analyse the problem using PLAXIS-2D v2015.02 (PLAXIS 2015). Conforming to Figure 1, multiple numerical models were prepared. Any such numerical model should be free from boundary effects. In this case, the stress-deformation response of the footing and the slope face should be free from any interference from the left lateral and the bottom boundary. As illustrated in Figure 2, it is conceived that the pressure bulb representing the significant stress isobar (i.e. $0.1q$, where q is the failure stress), originating beneath a loaded strip footing, should not be intersected by the edges of the domain. Through this procedure, the optimum dimensions of the model are determined for various geometrical configurations. Figure 3 illustrates the application of boundary conditions, in which ‘standard fixity’ is applied. Accordingly, in the FE model, the lowermost boundary is considered rigid, thereby displacement fixities in all directions are employed at the bottom edge of the model. The far lateral edges are free to deform in the vertical direction, while it is restrained from horizontal deformation. For allowing unhindered deformation of the slope face, no fixities are given on the same.

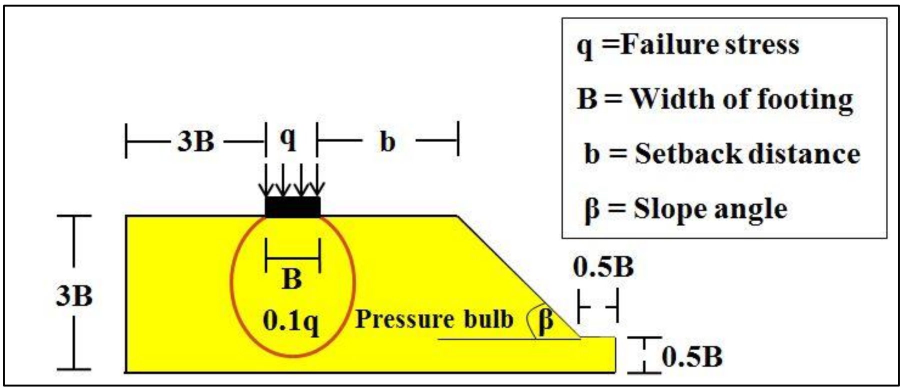


Fig. 2 Typical diagram exhibiting the choice of model dimension (not to scale)

For conducting FEA, sufficient number of finite elements are used to discretize the model domain. For obtaining as reasonable estimates of displacements and stresses as possible, 15-noded triangular elements comprising larger numbers of Gauss points and nodes are chosen. PLAXIS-2D program produces the FE meshes with the support of ‘robust triangulation technique’ employing automatic discretization as per pre-described meshing schemes, termed as ‘very fine’, ‘fine’, ‘medium’, ‘coarse’ and ‘very coarse’ as per their relative element size. As per necessity, according to the user-defined ‘mesh coarseness factor’, meshing from each predefined scheme can be progressively refined. The adopted mesh should be optimal and adequately refined to achieve the best possible numerical results. Figure 3 illustrates the typical meshing arising from the choice of the preliminary ‘fine’ meshing scheme, which is further refined at locations wherever stress concentration is anticipated to develop.

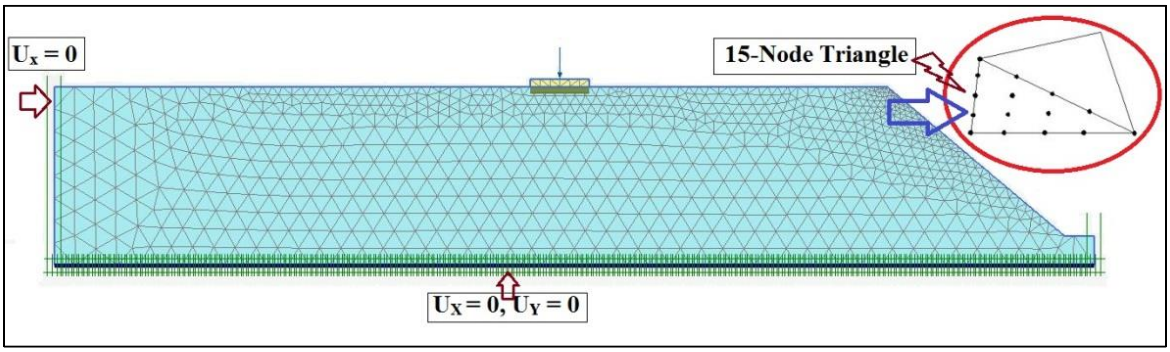


Fig. 3 Typical meshing scheme and boundary fixities employed on a FE model

In the current simulation, the constitutive behaviour of hillslope material is represented by Mohr-Coulomb, or MC, model (Naderi and Hataf 2013). It is an elastic-perfectly plastic model that is characterized using three strength parameters (φ, c, and the dilatancy angle ψ) and two stiffness parameters (v and E i.e. Poisson's ratio and Modulus of elasticity, respectively). The slope material is assumed following 'associated flow rule', described by the dilative coefficient $\eta = \psi / \varphi = 1$ (Drescher and Detournay 1993; Xiao-Li *et al*. 2017). Literature reveals that the maximum bearing strength is negligibly influenced by variation in soil unit weight i.e. γ (Acharyya and Dey 2017), and hence, a constant γ = 16 kN/m^3 is considered for the hillslope material. The rigid rough strip footing is considered made of M20 concrete, whose behaviour is considered following the linear elastic (LE) constitutive model, having γ_c = 25 kN/m^3, E_c = 22 GPa and v_c = 0.15, corresponding to the unit weight, modulus of elasticity and Poisson's ratio of concrete, respectively. Suitable interface element is provided at the concrete-soil boundary, whose stiffness matrix is developed using the Newton-Cotes integration points (Nasr 2014). The elastic modulus and shear strength parameters of the interface element are considered identical to that of the adjacent soil elements. The interface strength reduction factor (R_{inter}) are chosen to be 1, such that the strip footing exhibits a rough base without any slippage between the footing and soil nodes.

2.2 Description of multi-gene genetic programming (MGGP) technique

Genetic programming (GP) mimics the Darwinian principle of natural selection. It is a symbolic regression technique that generates programs to execute a problem, wherein the outcomes are revealed in terms of tree structures (Searson 2009, 2010). Random population of individuals are generated to attain high variability, which in a hierarchical tree structure comprises functions and terminals, thereby representing a mathematical expression. The function set consists of typical programming operations, typical mathematical functions, simple arithmetic operations, domain-specific operators, logical functions or any other necessary mathematical operators. The terminal set consists of arguments of the operators and comprises numerical and logical constants as well as variables. A standard GP tree, illustrating a simple function $f(x) = \sqrt{(3/x_1 + x_2)}$, is shown in Figure 4. The representative structure of GP consists of nodes, or elements, belonging to both terminal (Constants like 3 and variables like x_1, x_2) and functional sets (Mathematical operator).

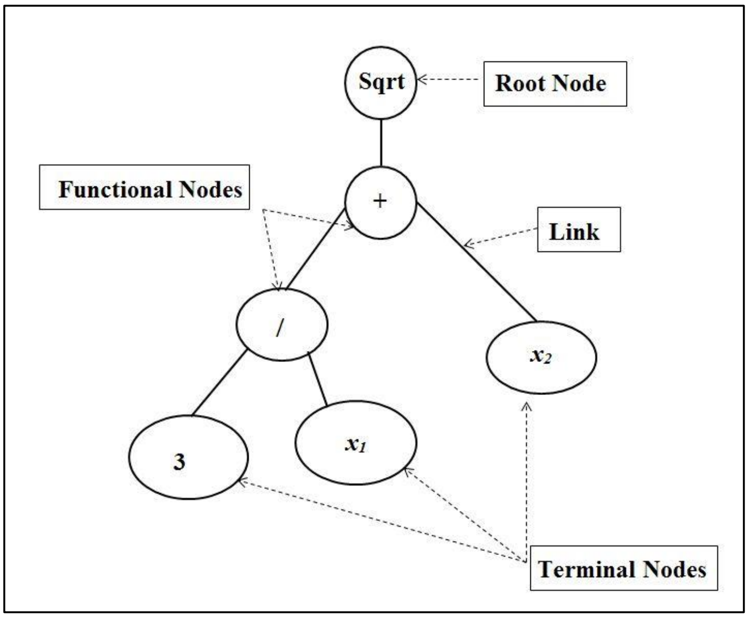


Fig. 4 A typical tree structure of a GP model representing a mathematical function

MGGP operates on scaled symbolic regression and is considered to be an advanced and robust variant of GP. It has the ability to amalgamate the model structure selection as in traditional GP with the capability of parameter estimation akin to classical regression techniques. In contrast to GP, every symbolic model in MGGP is a weighted linear arrangement of the outputs from a huge quantity of trees (Wang et al., 2016). A MGGP structure comprises gene, chromosome or gene-tree, individuals, fitness evaluator, evolutionary operators, gene interaction, hierarchical structure, parallelism and dynamic adaptation. Every tree in the stated population is represented as a 'gene'. A gene signifies a distinct program or solution to a problem, while a combination of multiple genes forms a chromosome or gene-tree. At the outset of the MGGP algorithm process, the maximum number of genes and maximum depth of trees are pre-set to constrain the complexity of the predictive model. One or multiple gene-trees are used by MGGP to calibrate the corresponding coefficients of the gene-tree structure with the aid of statistical regression techniques such as least squares, or others. An individual expression is characterized by multiple genes within a chromosome. Each individual in the population is assessed by the performance or evaluation of its constituent genes. At each step, the generated population is evaluated for its prediction capability. The fitness measure is taken into account for making a decision about the retention or modification of the said population. Fitness is primarily quantified by the error between the estimated and predicted magnitudes. If the desired fitness is not achieved, a series of evolutionary operations are conducted at the gene level, including reproduction, crossover and mutation, thereby allowing for the evolution of multiple programs (through replacement or generation) within each individual. This process is repeated until the desired fitness is achieved. An illustrative representation of the modelling steps is highlighted through Figure 5. Genes within an individual can interact and cooperate to mutually solve the problem. Thus, a hierarchical structure is created where genes interact at different levels of abstraction. In line with the weighted structure, MGGP can exploit parallelism and concurrency by executing multiple genes simultaneously or in parallel. MGGP enables dynamic adaptation of the number and composition of genes within individuals throughout evolution. As an end-product, MGGP produces a set of equations which are linear combinations of non-linear terms, and the same is developed without having a-priori functional structure. In a nutshell, in contrast to GP, MGGP offers a more robust approach to arrest the nonlinearity associated with a phenomenon (Noh et al., 2021).

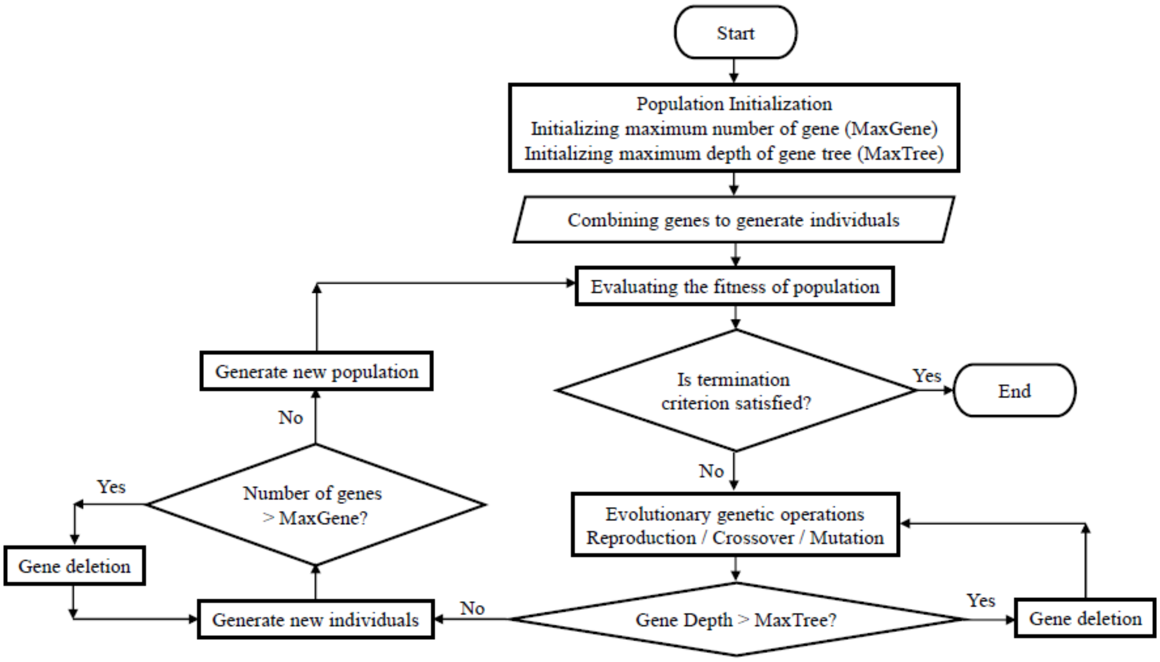


Fig. 5 An illustrative description of the MGGP algorithm

A standard multi-gene model is portrayed in Figure 6, which denotes a function $y = w_0 + w_1\left[8x_1 + \sin(3+x_2)\right] + w_2\left[\sqrt{5/x_2} + \cos(x_3)\right]$ comprising an output variable (y) and three dependent input variables (x_1, x_2 and x_3). The model comprises nonlinear algebraic and trigonometric operators, although it is linear with regard to the individual operators associated with the coefficients w_0, w_1 and w_2. The linear coefficients for each gene are assessed with aid of least squares method. A multitude of population and generations are verified to decipher the minimal error in the model. The permissible number of genes and the tolerable gene depth directly impacts the extent of the search space. The maximum number of genes and tree depth are decided by the user for a particular problem. The programs are executed until the tolerance is achieved in successive iterations. Fitness function assesses the generated equations to deliver the best expressions. The basic mathematical expression of MGGP is mentioned as per Hii *et al*. (2011):

$$Y = \sum_{i=1}^{n} d_i G_i + d_0 \tag{1}$$

where, d_i is the i^{th} weighting coefficient, G_i is the magnitude of i^{th} gene and function of input variables, d_0 is the bias, n is the number of genes and Y is the predicted output. …

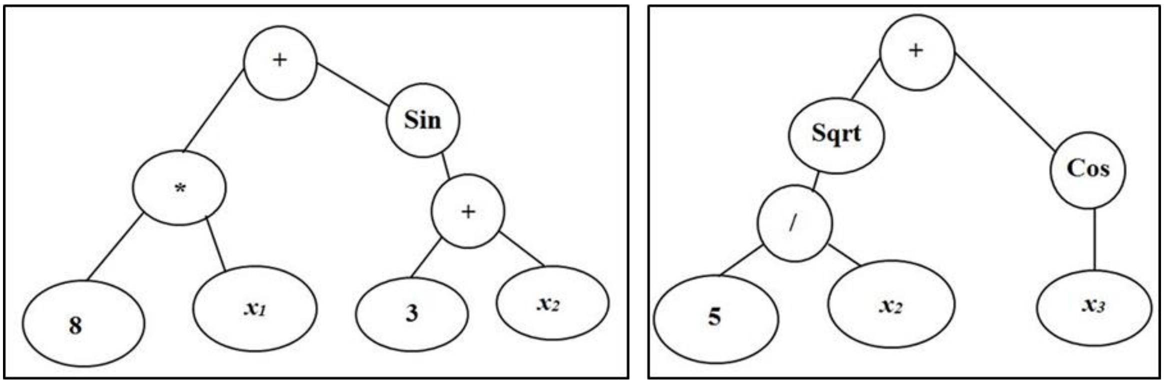


Fig. 6 A typical tree structure of a MGGP model representing a mathematical function

GPTIPS is an extensively used platform for the assessment of GP models. It is efficient in producing symbolic nonlinear models for predictor response data. GPTIPS is functioned by MGGP which contains the flexibility and capability to arrest nonlinear characteristics of GP with aid of 'classical linear least squares parameter estimation'. In the current investigation, multi-gene symbolic regression operator is utilized to minimize the difference (in terms of RMSE, i.e. root mean squared error) between the simulated and predicted normalized bearing capacities. Following a 70:30 ratio, the simulated datasets are randomly categorized into training and testing sets respectively (Pattanaik *et al*. 2017). The training data are utilized for genetic development, while the testing data checks the efficiency of the mathematical formulations obtained from MGGP. The performance of the MGGP tree structure is investigated with varying population, in which the coefficient of correlation (R^2) is utilized for the assessment of model prediction wherein the population corresponding to the peak R^2 is chosen as the optimum population.

3. Validation of Numerical Model

The developed FE model is validated against the findings from relevant laboratory experiments performed by Keskin and Laman (2013). In the said experimental investigation, the bearing strength of a strip footing positioned on a sandy slope was assessed as a function of setback distances, slope inclination angles and footing widths. For validation purpose, exact geometrical dimensions of the experimental setup are replicated in the FE model. The width of strip footing is taken as 70 mm resting on the slope crest with $b = 0$ (i.e. at the edge); the inclination of slope being 30°. In accordance to the experiment, φ and γ of the cohesionless soil are considered 41.8° and 17 kN/m^3, respectively.

Firstly, a mesh convergence study is performed for discerning the optimum mesh configuration for the analysis (as per Acharyya and Dey 2019). Figure 7 depicts that a global 'medium' meshing, having a non-dimensional element size of 0.07, leads to an optimum configuration for the simulation. As a consequence, for validation study as well as for rest of the numerical analyses, the same meshing arrangement is considered. Figure 8 depicts the tolerable agreement between numerically obtained pressure-settlement plot with that obtained from the experimental investigation, there by representing the overall accuracy of the generated numerical model.

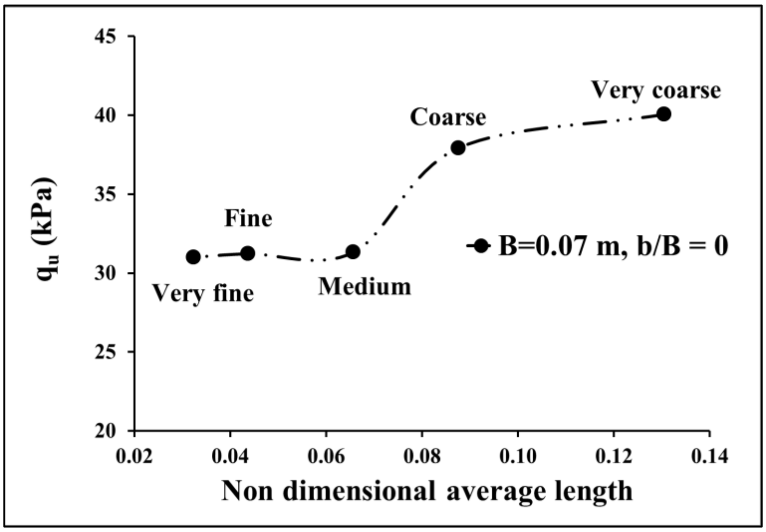


Fig. 7 Optimal mesh configuration from mesh convergence curve

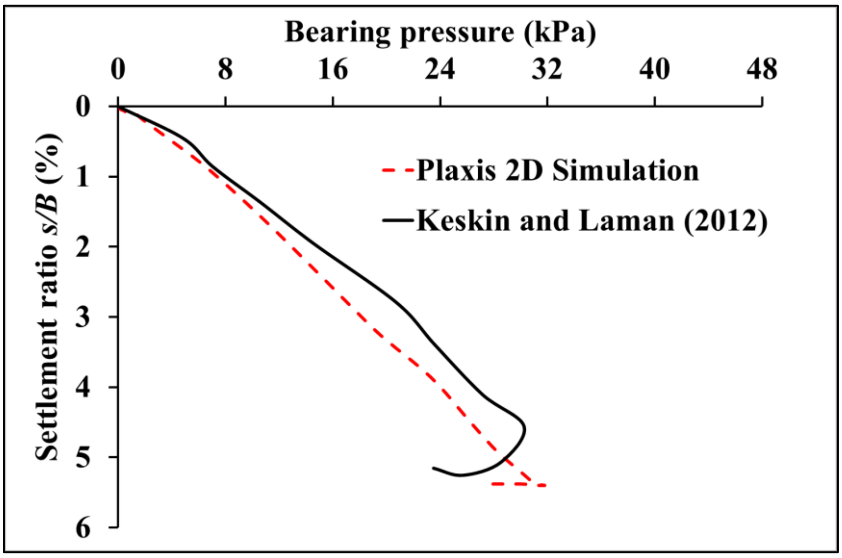


Fig. 8 Validation of FE model with respect to experimental investigations

4. Results and Discussions

4.1 Parametric investigation using finite element modelling

In the current research, the earlier-stated parameters affecting the bearing strength of strip footing on slopes are varied to identify their influence on the bearing strength assessment. Figure 9 depicts the typical outcomes from the parametric study, which are revealed in terms of the normalized UBC with the setback distance of the strip footing. It can be noted from Figure 9(a) and Figure 9(b) that increase in c and φ of the hillslope material increases the normalized UBC, owing to the improvement in the shear strength of the slope material. Figure 9(c) indicates that an increase in β results in the decrement of the bearing strength, as the resistance of foundation soil undergoing outward lateral movement reduces due to increase in slope angle. Moreover, for steeper slopes, the progress of the passive zone towards the slope face becomes significantly curtailed due to the unavailability of the soil material. Figure 9(d) highlights that increase in footing width improves the bearing strength, owing to the fact that a large sized footing provides more contact area as well as results in larger load-spreading within the foundation soil. Figure 9(e) reflects that the enhancement in the embedment depth of the footing would lead to enhanced bearing strength, primarily due to the augmented confinement over the footing embedded at larger depths. Furthermore, from all the plots, it can be inferred that for $b/B \geq 6$, the presence of slope face has no impact on the UBC of the footing, thereby the footing performs as if it is placed on a plane ground. More details of the parametric study can be found in Acharyya and Dey (2019). The parametric study is conducted for the entire ranges and combinations of parametric magnitudes as indicated in Table 1, thereby generating a large pool of 2021 datasets to be used in the subsequent MGGP analyses.

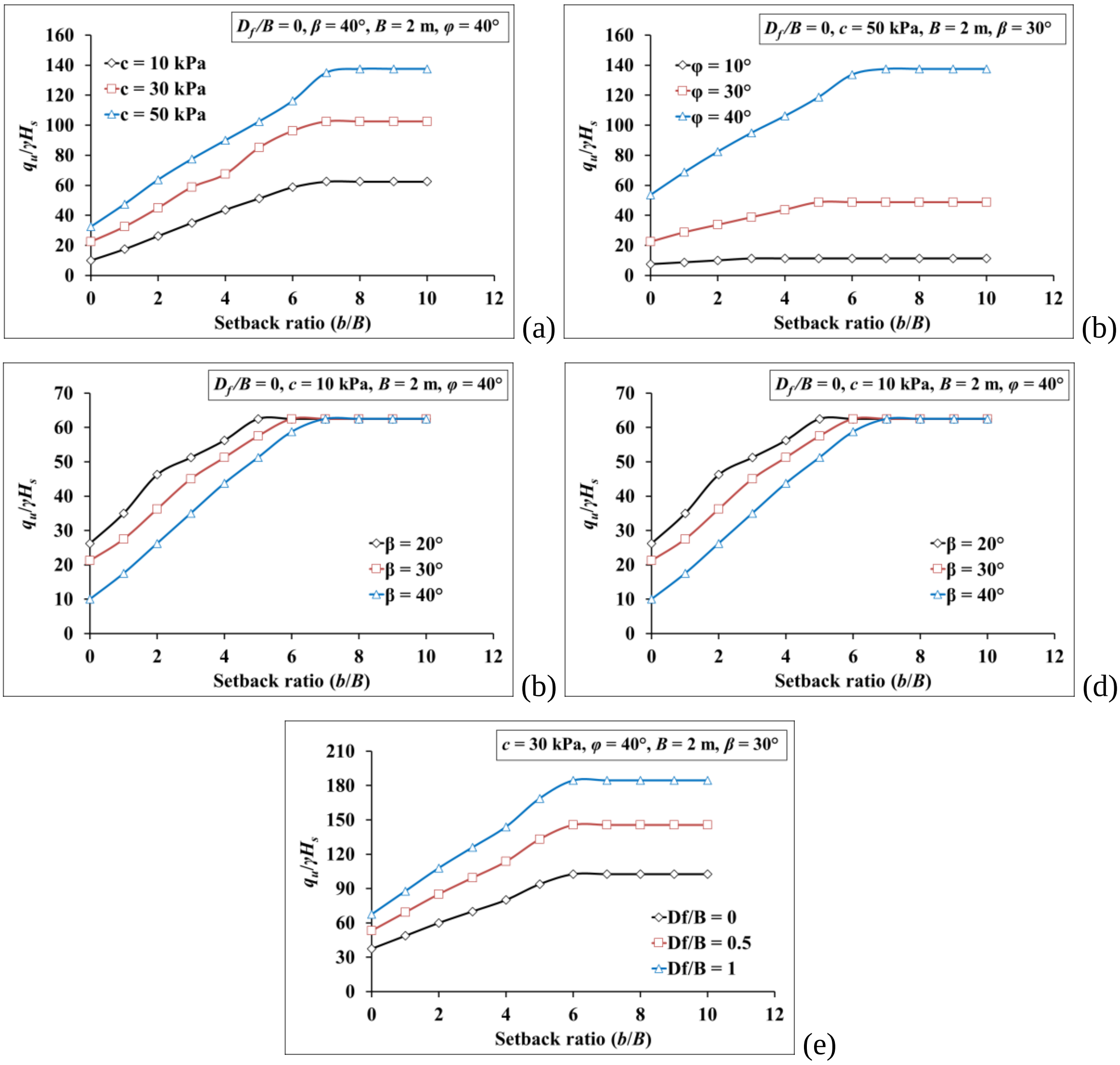
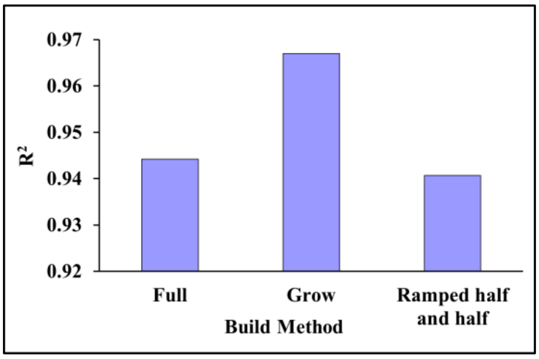


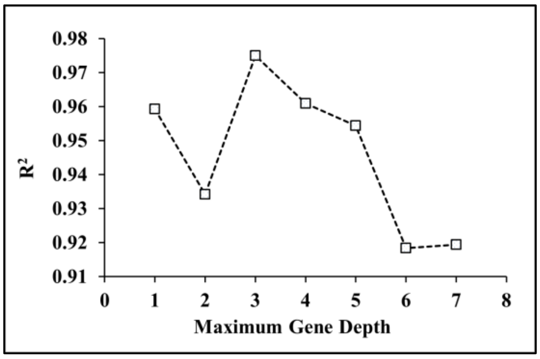
Fig. 9 Typical influence of contributing parameters on the normalized bearing strength of strip footing on a slope: (a) Cohesion of hillslope material (b) Friction angle of hillslope material (c) Inclination of hillslope (d) Width of strip footing (e) Embedment depth of footing

4.2 Outcomes and performance of developed MGGP model

The prepared dataset of 2021 combinations is used to create the MGGP model for the present study by suitably modifying the GPTIPS toolbox available in MATLAB vR2015b (Mathworks 2015). The hyper-parameters of the algorithm were substantiated over the range of their corresponding values and, finally, their optimal combination were identified. The 'build' method that signifies the technique of initializing the first-generation tree structures is explored through three techniques, namely (a) ‘Full’, (b) ‘Grow’ and (c) ‘Ramped half-and-half’, out of which the ‘Grow’ method is observed to be the best possible approach. The individual gene depth is varied in a range 1-7, while the number of genes is varied between 1 to 10. The probabilities of all the three primary operations i.e. mutation, crossover and the direct crossover (M-C-D) are explored within a range 0-1 at a uniform interval of 0.1. For the best population selection, the tournament size is varied in the range 10%-100%. The elite fraction, representing the fraction of the total population exempted from participating in the genetic operations, is varied in the range 0.05-0.3 at a uniform interval of 0.05. Lastly, the population size and number of generations are varied in the ranges 5-120 and 10-60, respectively. The optimal algorithm parameters used in modelling are finally identified as: Build method – Grow, Tournament size – 100%, Population size – 750, Number of generations – 50, M-C-D probability – 0.3, Elite fraction – 0.15, Maximum gene depth – 3 and Maximum genes – 5. For determination of the best parameter, a single parameter was altered, while the rest of the others have been maintained constant, and subsequently, the corresponding optimum value was adopted. Three repetition runs have been accomplished for individual parameter combination to remove random correlations. Figure 10 depicts the optimum magnitudes of the algorithm parameters. Keeping direct and elite fraction at their optimum magnitudes, the optimum values of population is determined. Lastly, the optimum magnitude of generation is recognized with aid of the average correlation coefficient (R^2). The average correlation coefficient of three trials has been taken as a pointer to decipher the capability of the developed MGGP model.



(a)



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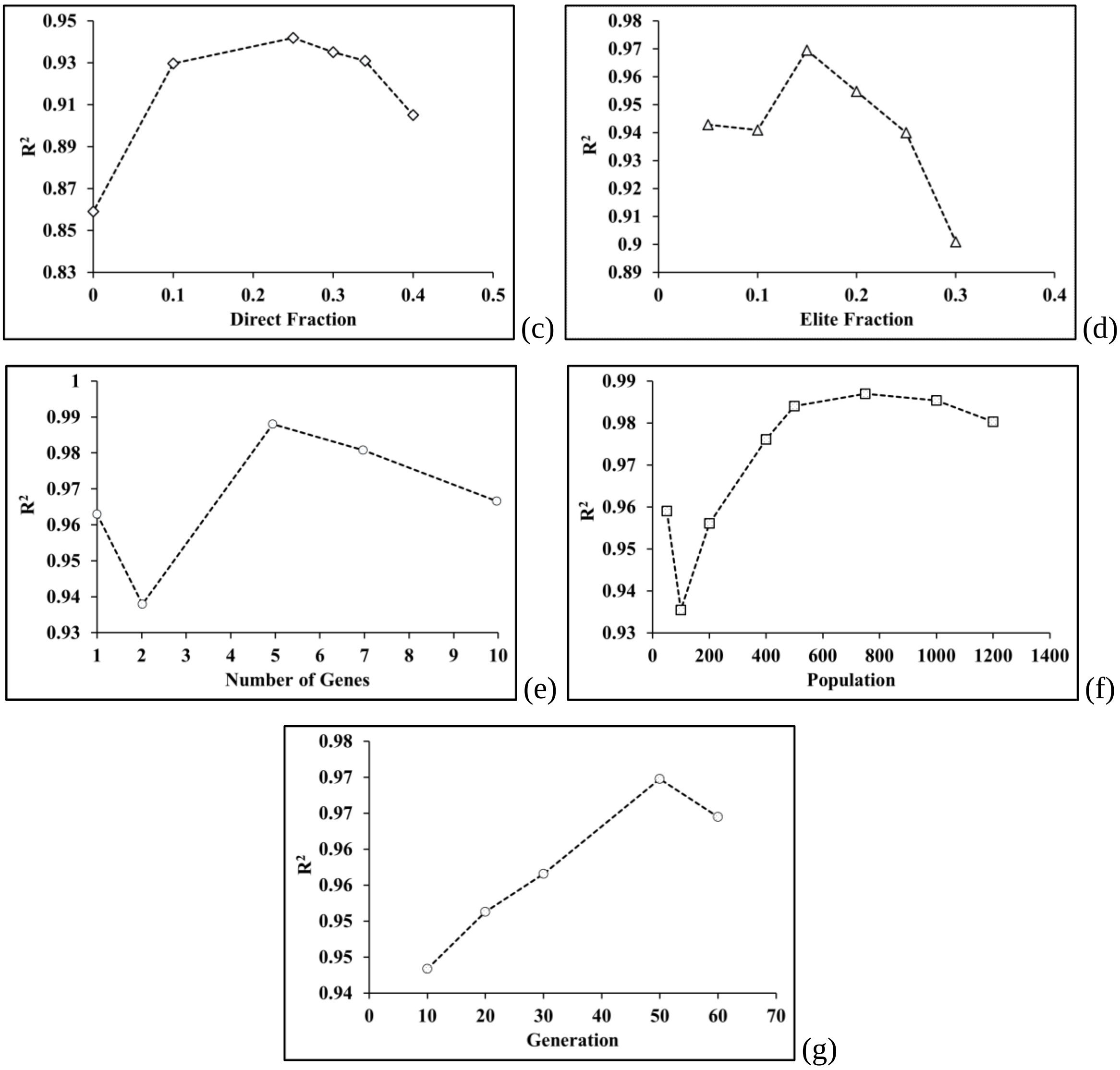


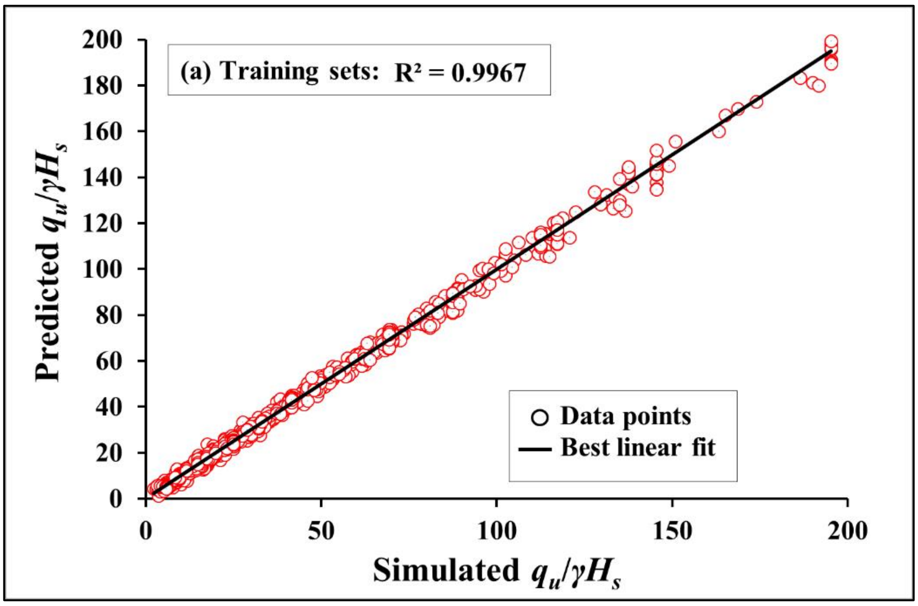
Fig. 10 Optimal magnitudes of the algorithm parameters selected for MGGP analyses

The model evaluation has been finalized on the entire data set to obtain the best model using the estimated optimum algorithm parameters. Equation 2 provides the final relationship obtained between the influencing parameters and the normalized UBC ($q_u/\gamma H_s$) using the MGGP technique. The value of R^2 has been found to be 0.997 for the present model, thereby indicating its efficiency in successfully predicting the UBC of strip footing on slopes.

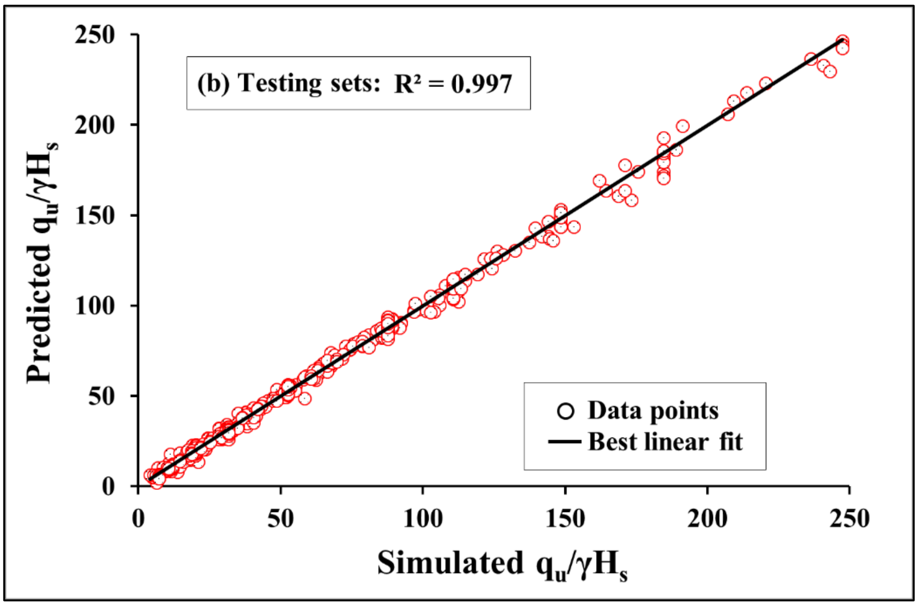
$$\frac{q_u}{\gamma H_s} = \left[\begin{array}{l} 8.01 - 41.72X_3 + \dfrac{3797X_3}{{X_2}^2} + X_1X_3e^{-1.352\times10^{-3}X_2^2X_3} \\ +0.246\dfrac{X_2^3X_3}{X_5^2} + 0.021X_2^2X_3X_6 + 9.6\times10^{-5}X_3X_4X_5 \end{array} \right] \quad (2)$$

where, $X_1 \ldots X_6$ are as per the notation given in Table 1.

Out of 2021 datasets (Acharyya and Dey, 2019), 1414 sets (i.e. 70%) and 607 (i.e. 30%) datasets are used for training and testing of the MGGP model, respectively. Figure 11 represents the capability of the generated MGGP model in terms of the similarity of simulated outcomes (from the FE analyses) to that of the predicted normalized UBC (from the MGGP model). The training performance indicates capability of the model towards generalization, while the testing performance represents predictive capability of the model. It can be noted from Figure 11 that the model exhibits superior potential in both generalizing and predicting, as observed from the high magnitudes of the correlation of regression (R^2) representing appreciable agreement between simulated and predicted outcomes. The observations suggest that the developed MGGP model can be efficiently considered in assessing the maximum bearing strength of strip footing placed on the crest of a slope comprising marginal soils.



(a)



(b)

Fig. 11 Performance of the MGGP model during training and testing phases

Furthermore, apart from the coefficient of regression (R^2), the performance of the developed MGGP model (as in Equation 2) is examined through various statistical criteria, namely, Nash–Sutcliffe efficiency (*NS*), Index of Agreement (*d*) and Modified Index of Agreement ($d_{Modified}$) (Legates and McCabe, 1999). The expressions for each of these statistical evaluators are provided in Equations 3-6. The correlation-based methods (R^2, *NS*, *d* and $d_{Modified}$) can identify the predictive capability of the developed model (Legates and McCabe 1999). It is obvious that a high magnitude for R^2, *NS*, *d* and $d_{Modified}$ (limited up to 1) designate high predictive capability of the model. The performance of Equation 2 is inspected with statistical parameters through both the training and testing datasets. The values of statistical evaluators are tabulated in Table 2, which reveals that the magnitudes of the statistical evaluators are very close to 1, thereby reconfirming that the developed MGGP model is highly efficient in predicting the UBC of strip footing positioned on crest of a marginal soil slope.

$$R^2 = \left[\frac{\sum_{i=1}^{N} (y_s - \overline{y_s}) \times (y_p - \overline{y_p})}{\sqrt{\sum_{i=1}^{N} (y_s - \overline{y_s})^2 \times (y_p - \overline{y_p})^2}} \right]^2 \tag{3}$$

$$NS = 1 - \frac{\sum_{i=1}^{N} (y_s - y_p)^2}{\sum_{i=1}^{N} (y_s - \overline{y_s})^2} \tag{4}$$

$$d = 1 - \frac{\sum_{i=1}^{N} (y_s - y_p)^2}{\sum_{i=1}^{N} \left[\left| y_p - \overline{y_s} \right| + \left| y_s - \overline{y_s} \right| \right]^2} \tag{5}$$

$$d_{Modified} = 1 - \frac{\sum_{i=1}^{N} (y_s - y_p)^2}{\sum_{i=1}^{N} \left[\left| y_p - \overline{y_s} \right| + \left| y_s - \overline{y_s} \right| \right]} \tag{6}$$

where, y_s and y_p symbolise the simulated and predicted values of normalized maximum bearing strength. $\overline{y_s}$ and $\overline{y}_p$ represent the mean values of the simulated and predicted normalized maximum bearing strength, and N represents the total number of data samples.

In the present study, the accuracy index of the MGGP model is assessed by mean relative error, root mean square error and mean absolute error, as presented in Table 3. The magnitudes of accuracy index parameters signify the high fitness of prediction.

Table 2 Statistical evaluators exhibiting the performance of developed MGGP model

Statistical evaluator	Training	Testing
R^2	0.996	0.997
NS	0.996	0.997
d	0.999	0.999
$d_{Modified}$	0.995	0.991

Table 3 Accuracy index of the developed MGGP model

Parameters	Magnitudes
Mean Relative Error = $\frac{1}{N} \times \sum_{i=1}^{N} \frac{\left| y_s - y_p \right|}{y_s}$	0.06
Mean Absolute Error = $\frac{1}{N} \times \sum_{i=1}^{N} \left| y_s - y_p \right|$	1.86
Root Mean Square Error = $\sqrt{\frac{\sum_{i=1}^{N} (y_p - y_s)^2}{N}}$	2.57

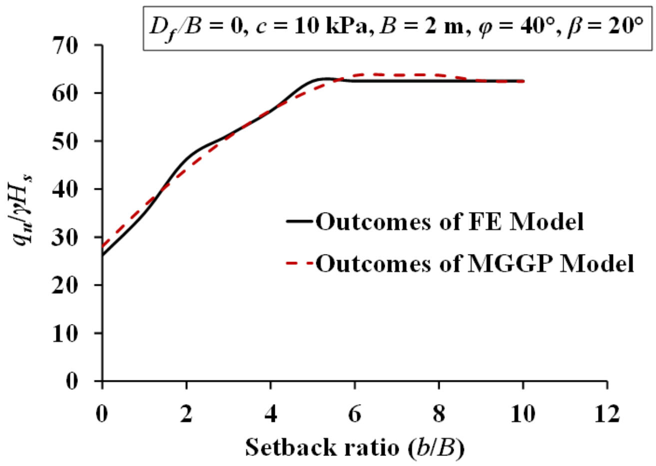


Fig. 12 Comparison of outcomes obtained from FE model and MGGP model

Figure 12 illustrates the comparison of the results found from the FE analysis and predicted from the MGGP model. It is seen that there is a substantial match between the outcomes obtained from the FE and MGGP models. It proves the fact that the proposed MGGP model has a high capability and accuracy to predict the $q_u/\gamma H_s$.

The developed MGGP-based predictive expression (Equation 2) is further compared with some experimental observations reported in literature (El-Sawwaf, 2010) and are presented in Table 4. It is to be noted herein that the experimental investigations referred herein (and mostly all others available in literature) comprise cohesionless slopes, hence only the friction angle is considered for the shear strength parameter. Hence, even with the complete absence of one contributing parameter (which played a significant role in developing the MGGP model), the closeness of the normalized bearing strength obtained from the prediction and experimental observations lead to the confidence on the efficacy of the developed model.

Table 4 A comparative of proposed MGGP model with respect to experimental observations

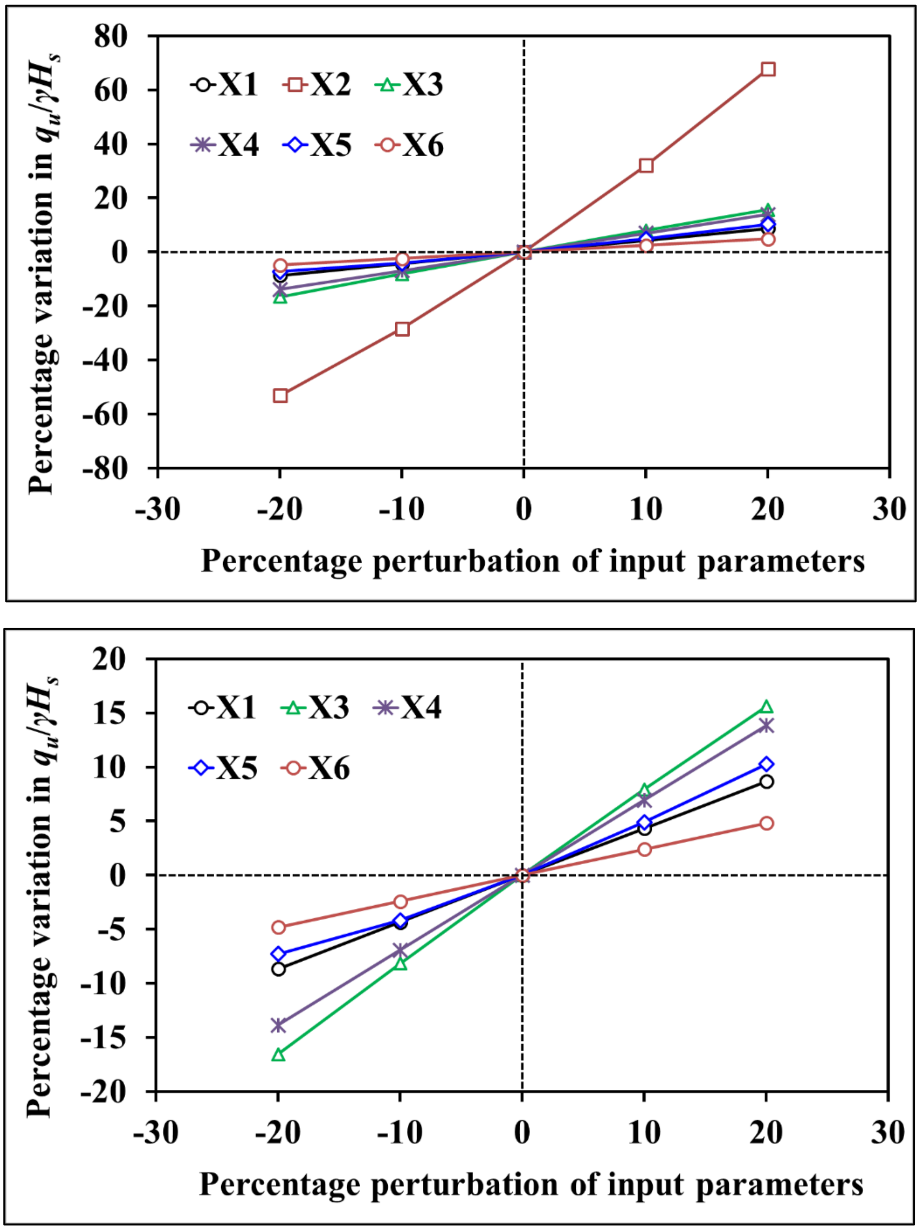
Parameters Considered	$q_u/\gamma H_s$ from the experimental investigation (El-Sawwaf 2010)	$q_u/\gamma H_s$ from the proposed MGGP model
c = 0, φ = 43°, B = 0.08 m, b/B = 0, β = 33.69°, D_f/B = 0	2.12	2.40
c = 0, φ = 37.5°, B = 0.08 m, b/B = 0, β = 33.69°, D_f/B = 0	1.25	1.55

$c = 0$, $\varphi = 34°$, $B = 0.08$ m, $b/B = 0$, $\beta = 33.69°$, $D_f/B = 0$	0.51	0.83

It is to be noted herein that the MGGP prediction equation is based on the FE simulations that comprised the effect of both the shear strength parameters (c and φ). The FE model was validated with the experimental results by Keskin and Laman (2012) so that the quality and verity of the data pool generated from FE simulations be well established. The data pool is further used to generate the MGGP prediction model whose efficacy is tested by comparing the predictions with the small-scale experimental investigations conducted by El-Sawaaf (2010). Although most of the parameters used by El-Sawaaf (2010) for various experiments (as given in Table 4) conform to the ranges of the corresponding parameters chosen to create the data pool (as given in Table 1), yet there are two major deviations. The experiments by El-Sawaaf (2010) considered cohesionless soil slopes, i.e. $c = 0$. As the MGGP prediction model is developed based on the effect of both the shear strength parameters, mismatch can originate between the experimental assessments and MGGP predictions on account of completely disregarding one of the shear strength parameters. However, it is supposed that the major reason of the deviations between the findings, as seen in Table 4, is primarily due to scale effects. It may be noted that the width of the footing considered in the small-scale experiments by El-Sawaaf (2010) is only 0.08 m, which is far smaller than the conventional real-scale footing sizes. Hence, a significant scale-effect is supposed to be induced in the small-scale experiments. The same has been also clearly mentioned by El-Sawaaf (2010) in regard to the scale effects arising from the dimensional scaling down of the footing sizes in the small-scale experiments while the grain size in the same small-scale experiments remain untouched. This leads to the mismatch in the footing width/grain size ratio between the prototype/field sized experiments and the small-scale experiments. It may be noted that the FE simulations in the present study considered the footing width in the range of 0.5-2 m, i.e. in accordance to the size of the real prototype foundations, and the same is used to develop the MGGP predictions. However, in absence of the experimental literature involving such large sized footing, the efficacy of MGGP predictions had to be sought out based on the small-sized experimental scale footings. Hence, the primary of the differences in the results can be attributed to the scale effect percolated in the actual experimental behavior. Nevertheless, the outcomes of the comparison are not found to be largely distant, and in a nutshell, can be attributed to the scale effect and extrapolation of MGGP prediction equation beyond the range for which it was originally developed. It might be interestingly noted that the difference is more pronounced for the case when the footing is resting on the cohesionless material with lowest internal friction angle (φ = 33.69º), which might be due to a phenomenal change in the failure mechanism of the small sized footing on the relatively loose sand. Each of these minor effects could not be inculcated in the MGGP prediction model.

4.3 Sensitivity assessment

Sensitivity assessment is carried out to illustrate the comparative significance of the input parameters on the prediction of normalized UBC of strip footing on the slope crest. The technique adopted herein is termed as the 'Local Perturbation Technique' or 'Local Sensitivity Analysis' and is one of the acclaimed conventional sensitivity analysis techniques owing to its easiness. In this approach, the influence of one single input parameter is studied on the variation of the output while the remaining input parameters remain fixed (Dey and Basudhar 2021; Chavan and Kumar 2018; Acharyya and Dey 2019; Li *et al*. 2023). The point-based local perturbation considering each data independent of the other is followed in this study to ascertain the sensitivity in a conventional manner. Herein, the input parameters are altered from +20% to -20% of their mean value and their effect on the variation of the model prediction is recorded. It is to be noted that when one of the input parameters were varied, the others were maintained constant at their mean values. Figure 13a describes the percentage change in prediction of $q_u/\gamma H_s$ to the percentage change in input parameters as an outcome of the sensitivity analysis; X_1 … X_6 are as per the notation given in Table 1. The negative sign, as depicted in Figure 13, indicates the decrease in the magnitude of the corresponding parameters below their mean values. It is observed that the friction angle of hillslope material, φ (i.e. X2), is by far the most significant input parameter as it has maximum effect on the predicted maximum bearing strength, followed by the effects of B and b/B (i.e. X3 and X4, respectively), β and c (i.e. X5 and X1, respectively), and D_f/B (i.e. X6). For better visualization of the remaining parameters, a local perturbation diagram is provided by Figure 13b after removing the plot for X2 from Figure 13a.



(a)

(b)

Fig. 13 Importance ranking of input parameters obtained through sensitivity studies

5. Rationality of Present Study

In the present study, it is worth mentioning that the data comes from the synthesis of software. The geo-parameters, within their individual ranges, are used as input to finite element analyses that follows a predefined constitutive behavior, thereby yielding the load-settlement responses that are further synthetized to assess the maximum bearing strength of footing on slope. The individual range of the input parameters were so chosen that their combinations (comprising specific magnitudes of each of the parameters) encompass the generic variations that can occur in the real-field soils. All the individual parameters were given equal weightage of occurrence, i.e. in other words, they follow a ‘uniform’ distribution. However, it is understandable that each of these parameters, when assessed from the field, can have different non-uniform distributions, either due to the errors and uncertainties associated with their estimation or due to the inherent spatial variability and inhomogeneity in the material present in the field. Error in field measurement is a pertinent avenue by which discrepancies creep into the scientific solutions. Field measurements need to yield the best results and such errors in measurements are necessary to be avoided at all circumstances. However, apart from measurement errors, the parameters themselves can have a range of values due to the inherent geological uncertainties. It is necessary to incorporate such uncertainties in the soft computing, and one of the best way is to include it through a probabilistic and uncertainty-based approach. Application of Monte-Carlo techniques is quite common in such cases, wherein the parameters are chosen from their statistical distributions to solve a deterministic problem in each iteration. Hence, in such cases as well, the currently developed scheme can be successfully used. Hence, as long as the range of uncertainty in field measurements is still within the range of the parameters considered herein for the development of the MGGP model, the proposed approach holds its rationality and can be suitably used.

6. Conclusions and Recommendations

Based on the numerical simulations and multi-gene genetic programming (MGGP) based prediction, the following conclusions are drawn from the present study:

- For a strip footing located at the crest of a marginal hillslope, the bearing strength is higher for slope material having higher strength parameters and for footings with higher widths and embedment depths, while the same reduces for higher slope inclinations.
- The bearing strength of strip footing on slopes is substantially influenced by setback distances lesser than six times the footing width (i.e. $S/B < 6$), beyond which the response corresponds to that of a strip footing resting on a semi-infinite horizontal ground.
- Given the high magnitudes (> 0.99) of all the statistical evaluators during the testing phase, the performance of the MGGP-based predictive expression, developed to assess the bearing strength of a strip footing resting on the crest of a hillslope, is adjudged to be excellent. Furthermore, the appreciable closeness of the bearing capacities obtained from the predictive expression to that of the experimental observations successfully validates the developed MGGP model.

- Based on local-perturbation based sensitivity analysis, the angle of internal friction of the hillslope material is found to have the highest importance ranking in terms of its sensitive influence on the bearing strength of strip footing on slopes, sequentially followed by other parameters, namely the footing width, setback ratio, cohesion of hillslope material, slope inclination and embedment depth of footing. For a perturbation of 20% about the mean value of friction angle, the normalized bearing strength varies to an extent of 60%, while the variation is in the tune of 5%-15% for the similar perturbation in other parameters.

7. Limitations of the Present Research

In the current research, M-C model is considered to represent the soil domain. The M-C soil model assumes the same elastic modulus (E) throughout the soil depth, i.e. the modulus of elasticity is stress-independent. However, it is understandable that stress-dependency is an inherent outcome of geological formations. In such case, the applicability of the Hardening soil (HS) model or any other stress-dependent models, considering variation of elastic modulus with depth of soil, can be suitably explored. It is also worth mentioning that the MGGP model expression derived herein is established by considering input parameters in certain ranges and are considered independent of each other. It is suggested that the developed expression should be used with caution if the parameters are beyond the ranges of input parameters mentioned in the manuscript. In the future, the concept of spatial variability of soil parameters could be considered to get more realistic outcomes of such geotechnical engineering problems.

Compliance with Ethical Standards

Conflict of Interest: Rana Acharyya declares that he has no conflict of interest. Arindam Dey declares that he has no conflict of interest.

Ethical approval: This article does not contain any studies with human participants or animals performed by any of the authors.

References

Acharyya R, Dey A (2017) Finite element investigation of the bearing strength of square footings resting on sloping ground. *INAE Letters* 2(3):97-105, DOI: 10.1007/s41403-017-0028-6

Acharyya R, Dey A (2021) Influence of footing typologies on the stability of a transmission tower resting on the bench of a hill slope. *Challenges and Innovations in Geomechanics Vol. 2*, Ed. M. Barla, Alice di Donna, and D. Sterpi, Springer Nature, Switzerland, Lecture Notes in Civil Engineering, Vol. 126, pp. 423-430, ISBN No. 978-3-030-64517-5.

Acharyya R, Dey A, Kumar B (2020) Finite element and ANN-based prediction of bearing strength of square footing resting on the crest of c-φ soil slope. *International Journal of Geotechnical Engineering*, 14(2):176-187, DOI: 10.1080/19386362.2018.1435022 .

Acharyya R, Dey A. (2019) Assessment of bearing strength for strip footing located near sloping surface considering ANN model. *Neural Computing and Applications* 31(11):8087-8100, DOI: 10.1007/s00521-018-3661-4 .

Armaghani DJ, Safari V, Fahimifar A, Amin MFM, Monjezi M, Mohammadi MA (2018) Uniaxial compressive strength prediction through a new technique based on gene expression programming. *Neural Computing and Applications* 30:3523–3532, DOI: 10.1007/s00521-017-2939-2.

Azzam WR, El-Wakil AZ (2015) Experimental and numerical studies of circular footing resting on confined granular subgrade adjacent to slope. *International Journal of Geomechanics, ASCE* 16(1):04015028-1-15, DOI: 10.1061/(ASCE)GM.1943-5622.0000500.

Bauer GE, Shields DH, Scott JD, Gruspier JE (1981) Bearing strength of footing in granular slope. Proceedings of 11th International Conference on Soil Mechanics and Foundation Engineering, Balkema, Rotterdam, Netherlands, 2:33-36.

Castelli F, Lentini V (2012) Evaluation of the bearing strength of footings on slopes. *International Journal of Physical Modelling in Geotechnics* 12(3):112-118, DOI: 10.1680/ijpmg.11.00015

Castelli F, Motta E (2009) Bearing strength of strip footings near slopes. *Geotechnical and Geological Engineering* 28:187-198, DOI: 10.1007/s10706-009-9277-9

Chakraborty D, Kumar J (2013) Bearing strength of foundations on slopes. *Geomechanics and Geoengineering* 8(4):274-85, DOI: 10.1080/17486025.2013.770172

Chavan R, Kumar B (2018) Prediction of scour depth and dune morphology around circular bridge piers in seepage affected alluvial channels. *Environmental Fluid Mechanics* 18:923-45. DOI: 10.1007/s10652-018-9574-z

Chen J, Zeng Z, Jiang P, Tang H (2016) Application of multi-gene genetic programming based on separable functional network for landslide displacement prediction. *Neural Computing and Applications* 27: 1771–1784, DOI: 10.1007/s00521-015-1976-y

Dey A, Basudhar PK (2021) Bearing strength of surface footings on reinforced sandy bed: A revised regression model. *Indian Geotechnical Journal* 52(2): 448-462, DOI: 10.1007/s40098-021-00572-1 .

Drescher A, Detournay E (1993) Limit load in translational failure mechanisms for associative and non-associative materials. *Geotechnique* 43(3):443-456, DOI: 10.1680/geot.1993.43.3.443.

El Sawwaf M (2010) Experimental and numerical study of strip footing supported on stabilized sand slope. *Geotechnical and Geological Engineering* 28:311-323. DOI: 10.1007/s10706-009-9293-9

Gandomi AH, Alavi AH (2012) A new multi-gene genetic programming approach to non-linear system modelling: Part II: geotechnical and earthquake engineering problems. *Neural Computing and Applications* 21: 189–201, DOI: 10.1007/s00521-011-0735-y .

Guven A, Gunal M (2008) Genetic programming approach for prediction of local scour downstream of hydraulic structures. *Journal of Irrigation and Drainage Engineering, ASCE* 134(2):241-249, DOI: 10.1061/(ASCE)0733-9437(2008)134:2(241)

Hansen JB (1970) A revised and extended formula for bearing strength. *Bulletin of the Danish Geotechnical Institute* 28:5-11.

Hii C, Searson DP, Willis MJ (2011) Evolving toxicity models using multigene symbolic regression and multiple objectives. *International Journal of Machine Learning and Computing* 1(1):30-35. DOI: 10.7763/IJMLC.2011.V1.5

Johari A, Habibagahi G, Ghahramani A (2006) Prediction of soil–water characteristic curve using genetic programming. *Journal of Geotechnical and Geoenvironmental Engineering, ASCE* 132(5):661-665, DOI: 10.1061/(ASCE)1090-0241(2006)132:5(661).

Keskin MS, Laman M (2013) Model studies of bearing strength of strip footing on sand slope. *KSCE Journal of Civil Engineering* 17(4):699-711, DOI: 10.1007/s12205-013-0406-x .

Legates DR, McCabe Jr GJ (1999) Evaluating the use of “goodness-of-fit” measures in hydrologic and hydroclimatic model validation. *Water Resources Research* 35(1):233-241, DOI: 10.1029/1998WR900018

Leshchinsky B (2015) Bearing strength of footings placed adjacent to c'-φ' slopes. *Journal of Geotechnical and Geoenvironmental Engineering, ASCE* 141(6): 04015022-1-13, DOI: 10.1061/(ASCE)GT.1943-5606.0001306.

Li D, Jiang P, Hu C, Yan T (2023) Comparison of local and global sensitivity analysis methods and application to thermal hydraulic phenomena. *Progress in Nuclear Energy* 158:104612-1-11. DOI: 10.1016/j.pnucene.2023.104612

MathWorks (2015) Matlab user’s manual. Version 2015b, The MathWorks, Inc., Natick.

Meyerhof GG (1951) The maximum bearing strength of foundations. *Geotechnique* 2:301-332, DOI: 10.1680/geot.1951.2.4.301

Meyerhof GG (1951) The maximum bearing strength of foundations. *Geotechnique* 2:301-332.

Mishra AK, Kumar B, Vadlamudi S (2017) Prediction of hydraulic conductivity for soil–bentonite mixture. *International Journal of Environmental Science and Technology* 14(8):1625-1634, DOI: 10.1007/s13762-017-1247-9 .

Murthy VNS (2008) Principles and Practices of Soil Mechanics and Foundation Engineering, Marcel Dekker, New York, USA.

Naderi E, Hataf H (2014) Model testing and numerical investigation of interference effect of closely spaced ring and circular footings on reinforced sand. *Geotextiles and Geomembranes* 42(3):191-200, DOI: 10.1016/j.geotexmem.2013.12.010

Narendra BS, Sivapullaiah PV, Suresh S, Omkar SN (2006) Prediction of unconfined compressive strength of soft grounds using computational intelligence techniques: A comparative study. *Computers and Geotechnics* 33(3):196-208, DOI: 10.1016/j.compgeo.2006.03.006.

Nasr AM (2014) Behaviour of strip footing on fiber-reinforced cemented sand adjacent to sheet pile wall. *Geotextiles and Geomembranes* 42(6):599-610, DOI: 10.1016/j.geotexmem.2014.10.004

Noh H, Kwon S, Seo IW, Baek D, Jung SH (2021). Multi-gene genetic programming regression model for prediction of transient storage model parameters in natural rivers. *Water*, 13(1): 76-1-29. https://doi.org/10.3390/w13010076.

Pattanaik ML, Choudhary R, Kumar B (2017) Evaluation of frictional pavement resistance as a function of aggregate physical properties. *Journal of Transportation Engineering, ASCE, Part B: Pavements* 143(2): 04017003-1-8, DOI: 10.1061/JPEODX.0000005 .

PLAXIS (2015) Reference Manual, 2D-Version 2015.02, Plaxis, Delft, The Netherlands.

Rezania M, Javadi AA (2007) A new genetic programming model for prediction settlement of shallow foundations. *Canadian Geotechnical Journal* 44(12):1462–1473, DOI: 10.1139/T07-063 .

Sarma CP, Dey A, Murali Krishna A (2020b) Investigation of rainfall induced landslides at the hillslopes of Guwahati region, Assam. *Geotechnics for Natural Disaster Mitigation and Management*, Ed. A. Murali Krishna and T. Katsumi, Springer, Singapore, pp. 75-87: ISBN No. 978-981-13-8827-9.

Sarma CP, Dey A, Murali Krishna A (2020a) Influence of digital elevation models on the simulation of rainfall-induced landslides in the hillslopes of Guwahati, India. *Engineering Geology*, 268:105523-1-13, DOI: 10.1016/j.enggeo.2020.105523

Sarma CP, Murali Krishna A, Dey A (2014) Probabilistic slope stability analysis considering spatial variability of soil properties: Influence of correlation length. *Proceedings of the 14th International Conference of the International Association for Computational and Numerical Methods in Geomechanics (IACMAG 2014)* (Eds. F. Oka, A. Murakami, R. Uzuoka and S. Kimoto), Kyoto, Japan, pp. 1125-1130.

Sarma CP, Murali Krishna A, Dey A (2018) Geotechnical characterization of hillslope soils of Guwahati region" *Geotechnical Characterisation and Geoenvironmental Engineering*, *Lecture Notes in Civil Engineering Vol. 16*, Ed. V. K. Stalin, M. Muttharam, Springer, Singapore, pp. 103-110: ISBN No. 978-981-13-0899-4.

Searson D (2009) GPTIPS: Genetic programming & symbolic regression for MATLAB. http://gptips.sourceforge.net.

Searson DP, Leahy DE, Willis MJ (2010). GPTIPS: An open source genetic programming toolbox for multigene symbolic regression. *Proceedings of the International Multiconfernece of Engineers and Computer Scientists* (IMECS 2010), 1-4. https://www.iaeng.org/publication/IMECS2010/IMECS2010_pp77-80.pdf.

Shahnazari H, Dehnavi Y, Alavi AH (2010) Numerical modeling of stress-strain behavior of sand under cyclic loading. *Engineering Geology* 116(1–2):53–72, DOI: 10.1016/j.enggeo.2010.07.007.

Shahnazari H, Tutunchian MA, Mashayekhi M, Amini AA (2012) Application of soft computing for prediction of pavement condition index. *Journal of Transportation Engineering*, *ASCE* 138(12): 1495-1506, DOI: 10.1061/(ASCE)TE.1943-5436.0000454.

Shields DH, Scott JD, Bauer GE, Deschenes JH, Barsvary AK (1977) Bearing strength of foundation near slopes. Proceedings of the 10th International Conference on Soil Mechanics and Foundation Engineering, Tokyo, Japan, 2:715-720.

Skempton AW (1951) The bearing strength of clay. Building Research Congress, England.

Terzaghi K (1943) Theoretical Soil Mechanics. John Wiley and Sons Inc., New York, USA.

Vesic AS (1973) Analysis of ultimate loads of shallow foundation. *Journal of Soil Mechanics and Foundation Division*, ASCE 99(SM1):45-73. DOI: 10.1061/JSFEAQ.0001846

Wang W, Zhao R, Shang Y, Liu Y (2016) Test data generation efficiency prediction model for EFSM based on MGGP. In: Sarro, F., Deb, K. (eds) Search Based Software Engineering. SSBSE 2016. *Lecture Notes in Computer Science*, 9962: 1-15. Springer, Cham. https://doi.org/10.1007/978-3-319-47106-8_12.

Xiao-Li Y, Nai-Zheng G, Lian-Heng Z, Jin-Feng Z (2007) Influences of non-associated flow rules on seismic bearing strength factors of strip footing on soil slope by energy dissipation method. *Journal of Central South University of Technology* 14(6):842-847, DOI: 10.1007/s11771-007-0160-7.