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Unit cell consolidation of a PVD incorporated soft soil: Comparative of 2D axisymmetric and plane strain analysis considering ideal and actual drain

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Abstract. The paper presents two-dimensional (2D) finite element-based study of a surcharge-induced consolidating unit cell model comprising fully saturated soft soil. Radial drainage in the unit cell is ensured through a single prefabricated vertical drain (PVD) along with vertical drainage through the top surface. Finite element based 2D axisymmetric and plane strain modelling has been conducted by considering the smearing of cohesive medium. The PVD is represented by both 1D drain element (ideal drain) and through geometric modelling considering well resistance. The parameters for the 'Soft Soil' model for the cohesive medium are adopted from the subsurface data at Krishnapatnam Ultra Mega Power Project (KUMPP), India. Comparative assessment of unit-cell consolidation is carried out between the 2D axisymmetric and equivalent plane strain condition based on Hird's permeability compatibility approach. The two approaches yielded noticeable disparity in the time rates of consolidation for time factor ≤ 0.4, beyond which the disparity decreases, and the two methods lead to identical results beyond time factor = 4. Further, the influence of permeability of a geometrically modelled PVD and the height of unit cell on the time-rate of consolidation is elucidated, thereby manifesting the importance of well resistance and drainage length in such analyses.

1. Introduction

Implementation of prefabricated vertical drains (PVDs) with surcharge preloading has been one of the effective ground improvement technique for treating low permeable and high compressible soft cohesive soil. This leads to the enhanced settlement rate as well as increased bearing capacity of the soft cohesive layer due to its accelerated consolidation in presence of PVDs subjected to a preload. Analytical solutions were proposed for radial consolidation of soft soil through a vertical sand drain with equal or free strain assumptions [1-4]. Such analyses are based on the radial consolidation of an axisymmetric unit cell comprising a cylindrical vertical drain within adjacent soil column. The consolidation is governed by the influence area of the surrounding soft soil depending upon the spacing between the drains and its pattern of installation within the soft soil. Moreover, with the invention of cardboard wicks and PVDs, radial consolidation through equivalent circular diameter to the band shaped drains are suggested through various studies [5-7]. Subsequently, the development in the finite element analysis has facilitated in performance prediction of soft soil improved with drains under complex boundary conditions [8-10]. In order to save the computational effort through finite element analysis of an actual three-dimensional case, the stability of embankment supported on a layered subsoil with vertical drains is often carried out in a two-dimensional scenario as a plane strain analysis [11-14]. Numerous studies in the past have indicated in establishing an equivalency between an axisymmetric unit cell of soil incorporated with a vertical drain to a plane strain unit cell with a rectangular drain wall [15-18].

In the present study, finite element analyses in PLAXIS 2D is carried out for the consolidation analysis of a unit soil element comprising uniform soft cohesive soil incorporated with a PVD in an axisymmetric condition. The vertical drain acts as an ideal drain, allowing infinite discharge of the developed pore pressure during the radial drainage. The material property of the cohesive soil is adopted from the soft silty clay of Krishnapatnam Ultra Mega Power project (KUMPP), Andhra Pradesh, India. Further, a comparative analysis between the axisymmetric soil column involving the vertical drain as PVD and its equivalent plane strain analysis is made. The equivalency of the plane strain soil column is based on permeability compatibility approach of Hird [15]. Smear zone characteristics, such as, smear zone diameter and reduced smear zone permeability coefficient are also included in analyses. Moreover, in order to appraise controlled discharge of developed excess pore pressure through the PVDs as, in practice, a geometrically modelled vertical drain is considered for an axisymmetric analysis of a unit cell soil column. The behaviour of the geometrically modelled vertical drain is assumed to be linearly elastic and coefficient of permeability of the drain indicates its limited discharge capacity relative to that of the adjacent soft soil. The limited discharge capacity is a manifestation of well resistance that can originate due to various reasons such as clogging or bending of PVDs. A relative assessment of the finite permeability coefficients of drain on the time rate of consolidation of the soft soil with smear effect is carried out with that in case of an ideal drain that enforces infinite permeability.

2. Finite element analysis of unit cell

2.1 Axisymmetric and plane strain analyses with an ideal line drain element

The finite element analysis of an axisymmetric unit cell is carried out in a finite element software PLAXIS 2D v2018. PLAXIS allows automatic generation of triangular mesh elements with facilities for global and local mesh refinements. For modelling the soil column, 15 -noded triangular elements are chosen and 5-noded drainage line element is employed as a vertical drain representing a single PVD. The drainage line element acts as ideal drain by allowing infinite discharge and thus imposing zero resistance to the dissipation of generated excess pore water pressure (EPWP) of the adjacent soft cohesive soil (undergoing radial consolidation) in contact with the drain element. Mesh sensitivity analysis is conducted and 'medium coarse mesh' with an average element size of 0.06 m is considered in the present study.

2.1.1. Geometry and material properties. The diameter of the influencing zone of the soil column surrounding a PVD, is taken as $D_e = 1.13$ S, for square pattern of installation [19] and spacing (S) between the drains is assumed 2 m centre to centre. The height of the soil column is considered as $H = 5$ m over which a uniformly distributed surcharge $q = 50$kPa is applied. During the consolidation, the soil column is allowed to deform vertically under the applied load, while the base of the soil column is completely restrained. The consolidation of the soil column by dissipation of generated EPWP occurring radially (horizontally) along the vertical drain (PVD) as well as vertical drainage allowed only at the top boundary of the soil column. All other boundaries are impermeable. Being an axisymmetric model, only one radial axis of a realistic 3D cylindrical model of a unit cell, having a radius of influence, $r_e = 1.13$ m, is adopted in the analysis. A typical finite element (FE) meshed unit cell of a soil column containing a vertical drain at the axis of symmetry with surrounding smear zone is shown in Figure 1.

For depicting the constitutive behavior of a soft cohesive soil, 'Soft Soil' (SS) model from the PLAXIS2D material menu, has been adopted for the current analysis. The SS is a Cam Clay type model that simulates the behavior of normally consolidated clays and peat under primary compression. The material parameters describing the SS model is based on the soft clay characteristics of Krishnapatnam Ultra Mega Power Project (KUMPP), Nellore, Andhra Pradesh, India [20]. Table 1 shows the material parameters employed in the present axisymmetric analysis.

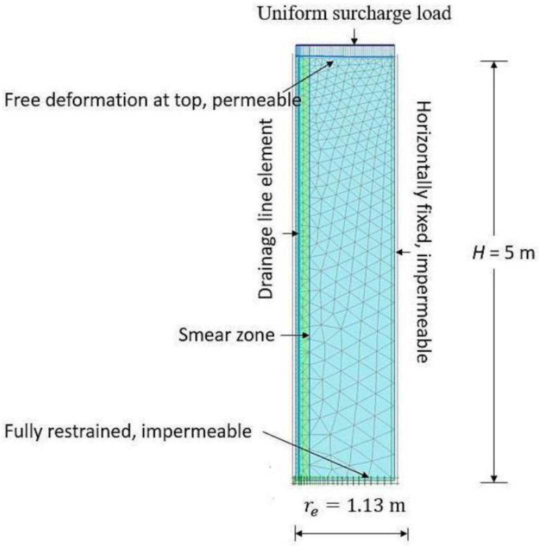

Figure 1. Typical FE axisymmetric unit cell model of clay column with a vertical drain and surrounding smear zone.

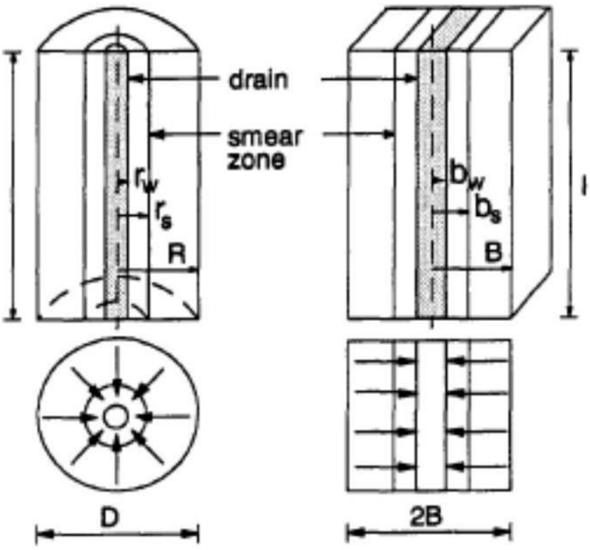

Figure 2. An axisymmetric unit cell of a clay column with a single drain into its equivalent plane strain unit cell [17]

Smear zone is considered in the present analysis by adopting the ratio smear zone diameter (d_s) to the equivalent diameter of the PVD (d_w), i.e., $s = 3.5$. Within the smear zone, reduced horizontal permeability coefficient of the remolded soil is considered to be $1/4^{\text{th}}$ of that in undisturbed soil surrounding the smear zone [21]. Although the vertical drain represented by the PLAXIS line element has negligible cross-sectional area, yet the equivalent diameter of the line drain is assumed as $d_w = 70$ mm for determining the extent of smear zone around the vertical line drain. The equivalent diameter of the circular vertical drain cross-section can be obtained from Hansbo's expression of perimeter equivalence with rectangular cross-section of the band drains [5], given by Eqn. (1) as

$$d_w = \frac{2(a+b)}{\pi} \tag{1}$$

where a and b are the width and thickness of a rectangular band shaped PVD.

2.1.2 Equivalent plane strain analysis from the axisymmetric analysis of unit cell. Radial consolidation of a surrounding soil through a vertical drain under a uniform surcharge load is appraised through an axisymmetric analysis. On the other hand, two-dimensional analysis for radial consolidation of soft cohesive soil under a prescribed surcharge preload, occurring through a series of PVDs arranged at a designated spacings and pattern (square or triangular), requires a procedure of expressing an axisymmetric condition into its plane strain equivalence at any cross-section. In this regard, plane strain equivalence can be achieved by three approaches: (a) Geometry matching of unit cell - the spacings between the drains are matched keeping permeability coefficients same in axisymmetric and plane strain analysis (b) Permeability matching of unit cell - adjusting the permeability coefficients while keeping the spacings between the drains equal in axisymmetric and plane strain analysis (c) Combination of both geometry and permeability matching [17]. Figure 2 shows an equivalent plane strain model obtained from an axisymmetric unit cell model through permeability matching.

The present study employs Hird's permeability compatibility approach [15] for establishing an equivalence between plane strain and axisymmetric analysis of a unit cell. As such, the width of the plane strain unit cell is assumed equal to the diameter of influence zone of soft soil in axisymmetric analysis, i.e., $2B = D_e$. The equivalent plane-strain horizontal hydraulic conductivity coefficient is then determined by the following expression, derived on the basis of achieving equal rate of consolidation from both axisymmetric and the plane strain condition at any time instant and at a given stress level:

$$\frac{k_{pl}}{k_{ax}} = \frac{2}{3\left[\ln(n) + \left(\frac{k_{ax}}{k_s}\right)\ln(s) - \frac{3}{4}\right]} \tag{2}$$

where, k_{pl} is the horizontal permeability of undisturbed soil in plane strain unit cell, k_{ax} is the horizontal permeability of undisturbed zone in axisymmetric unit cell, k_s is the horizontal permeability of the smear zone in axisymmetric unit cell, n is the influence ratio given by $r_e/r_w, s$ is the smear ratio given by $r_s/r_w, r_e$ is the radius of the influence zone of the axisymmetric unit cell, r_s is the radius of the smear zone of the axisymmetric unit cell and r_w is the radius of the vertical drain.

Table 1. Parameters of the SS model used in the axisymmetric unit cell analysis.

Parameter	Clay
Material model	Soft Soil (SS)
Drainage Type	Undrained
Soil unit weight above phreatic level $[\gamma_{\text{unsat}}](\mathrm{kN/m^2})$	15
Soil unit weight below the phreatic level $[\gamma_{sat}]$	17
$(\mathrm{kN/m^2})$	1.1
Initial void ratio (e_0)	1.6×10^{-5}
Horizontal permeability (without smear) k_{ax}(m/ day)	4×10^{-6}
Vertical permeability (without smear) k_y(m/ day)	2
Effective cohesion $[c'](\mathrm{kPa})$	26.55
Effective friction angle $[\varphi'](\,^\circ)$	0
Dilatancy angle $[\psi](\,^\circ)$	0.1035
Modified compression index (λ^*)	0.01294
Modified swelling index (κ^*)	0.55
Change of coefficient of permeability (c_k)	

The plane strain geometry and the horizontal hydraulic conductivity coefficient (with smear properties $s = 3.5$ and $\frac{k_{ax}}{k_s} = 4$), obtained from the Eqn. (2) is presented in Table 2. It is to be noted that the vertical permeability coefficients for the axisymmetric and plane strain cases are kept equal. The remaining material model parameters for the plane strain analysis is similar to that listed in table 1 for axisymmetric analysis.

2.2 Axisymmetric analysis with a geometrically modelled vertical drain

The conventional radial consolidation theory assumes a circular vertical drain. This study geometrically models a PVD using a 15-noded triangular element with an equivalent diameter of $d_w = 70$ mm using a 'medium coarse' mesh with an average element size of 0.06 m. The drain is assumed to behave as a linearly elastic (LE) material, for simplicity, in the analysis, with the soft soil undergoing radial drainage and top vertical drainage under a uniform surcharge load of $q = 50\mathrm{kPa}$. The Young's modulus of elasticity $E = 150\mathrm{kPa}$ and Poisson's ratio $v = 0.3$ is employed for the material modelling of the drain. The coefficient of hydraulic conductivity is assumed to be isotropic and correspondingly the limited discharge capacity being a manifestation of the well resistance experienced by the PVD during radial drainage. The study considers the impact of finite permeability coefficient of the PVD model on the time rate of consolidation of the soil column. In the present study, the smear zone has also been taken into consideration by assuming the smear zone properties to be same as that considered in axisymmetric analysis with the PLAXIS line drain.

Table 2. Geometric properties and horizontal coefficient of permeability for plane strain unit-cell.

Parameters	Magnitudes and Units
Width of plane strain unit cell $(2B)$	$2.26\ \mathrm{m}$
Spacing of drains [square pattern of arrangement]	$2\ \mathrm{mc/c}$
Drain spacing factor, $n = \frac{r_e}{r_w}$	32.286
Coefficient of horizontal equivalent plane strain permeability $\left(k_{pl}\right)$ [with smear zone]	$1.385 \times 10^{-6}\ \mathrm{m/\ day}$

3. Results and discussions

3.1 Degree of consolidation-time plot for 2D axisymmetric and equivalent plane strain unit cell

The soil column under the prescribed surcharge load is allowed to undergo 90% consolidation in presence of PLAXIS line drain element as a PVD. Figure 3 shows the degree of consolidation plots for different time factors (T_h) at the mid-depth of the soil column for axisymmetric and plane strain analyses. The rate of consolidation in axisymmetric condition is higher than the equivalent plane strain condition especially at the initial phases of consolidation, i.e. approximately for $T_h \leq 0.1$ At the time factor when 10% consolidation for the axisymmetric condition is achieved, i.e. $T_h = 0.4$, the consolidation in plane strain condition is yet to commence. This is attributed to the fact that in the axisymmetric case, more water is fetched from the surrounding soil leading to higher rate of dissipation in the earlier phases of consolidation. However, in case of plane strain, the dissipation of the generated EPWP takes place horizontally from the surrounding soil through a permeable drain wall. Beyond 15% consolidation, the time rate of consolidation remains uniform for either of the plane strain and axisymmetric cases. It is to be noted that within a consolidation range of $15\% - 50\%$, for the same time factor, the degree of consolidation of the plane strain condition marginally supersedes than that achieved for axisymmetric condition. Beyond 50% (i.e., at $T_h = 4$), the time rate of consolidation for both axisymmetric and plane strain coincides with each other.

3.2 Time-rate of consolidation for a unit cell of a soil column with a geometrically modelled PVD in 2D axisymmetric condition

Figure 4 shows the time rate of consolidation plots between a geometrically modelled PVD and an ideal drain (PLAXIS line element) of a soil column of height $H = 5$ m with smear zone; the measurements are made at mid-depth in each case. The geometrically modelled PVD is characterised by its finite permeability coefficient (k_w), denoted in terms of ratios $\frac{k_w}{k_{ax}}$. It can be noticed that owing to the finite coefficient of permeability, the time rate of consolidation in case of geometrically modelled PVD is much delayed compared to ideal drain that offers no resistance in dissipation of the generated excess pore water pressure.

However, the rate of consolidation for geometrically modelled vertical drain having different ranges of its permeability are not distinctly different. On the other hand, from Figure 5, the time-rate and degree of consolidation for a soil column with a geometrically modelled PVD of $H = 30$ m under identical material properties and boundary condition are different with the chosen ranges of drain's permeability coefficient. In this regard, a threshold permeability ratio of drain equal to 1.3×10^4 can be observed, below which the effect of well resistance (limiting discharge through the drain) on the rate of consolidation is prominent. Besides radial permeability of the soil and discharge length of the drain, previous studies suggested that the maximum discharge capacity of the vertical drain is also affected by probable geometric defects (bending, folding, kinking etc.) that can be considered as governing factors influencing well resistance [22-25]. Moreover, it can be noticed that the actual degree of consolidation attained by the soil column is a function of coefficient of permeability of the geometrically modelled PVD incorporated in it. Figure 5 shows the degree of consolidation attained by the 30 m high soil column varying between $80 - 99\%$ corresponding to the range of permeability coefficients of the PVD considered in this study.

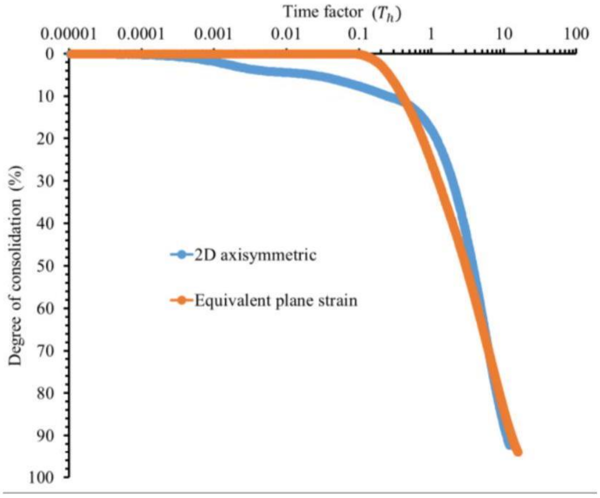

Figure 3. Comparison of time rate of consolidation between 2D axisymmetric and plane strain unit cell with smear zone properties $s = 3.5$ and $\frac{k_{ax}}{k_s} = 4$.

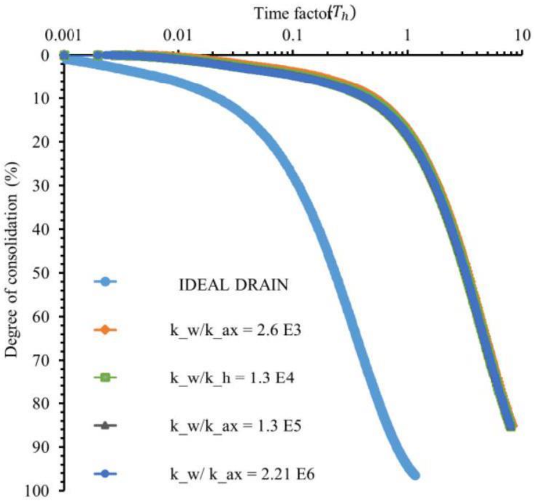

Figure 4. Comparison of time-rate of consolidation for a 2D axisymmetric soil column of $H = 5$ m with varying smear zone scenarios of geometrically modeled PVD with that of an ideal drain.

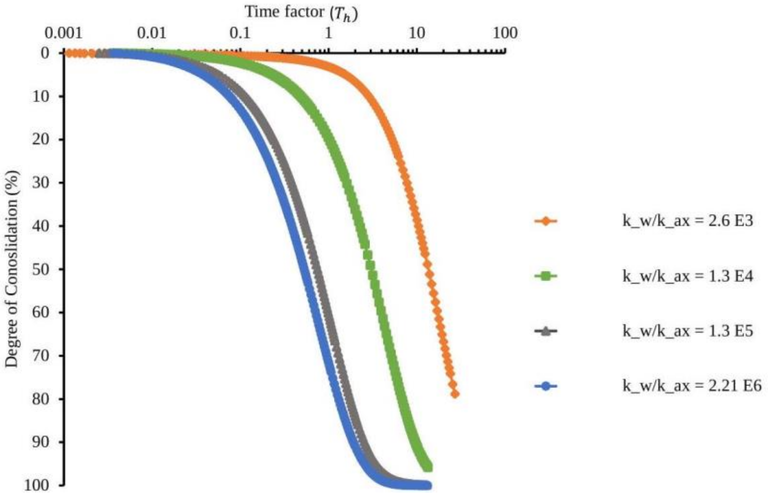

Figure 5. Time -rate of consolidation of an axisymmetric soil column of $H = 30$ m with smear zone for a geometrically modelled PVD with varying $\frac{k_w}{k_{ax}}$.

4. Summary and conclusions

The present study deals with two-dimensional finite element analysis of an axisymmetric unit cell subjected to a uniform surcharge load. The unit cell comprises a homogenous cohesive soil column with a PVD. Initially, the PVD is modelled by a PLAXIS drain element along with surrounding smear zone. A comparative assessment is carried out between 2D axisymmetric and equivalent plane strain consolidation to understand the agreements and differences in the time-rate of consolidation between the two approaches. Besides, the rate of consolidation of an axisymmetric soil column is also studied by considering smear zone around a geometrically modelled PVD, and comparison is established with the results obtained by considering ideal drain condition. Finally, the influence of drainage length of the PVD with its finite permeability coefficient on magnitude and time-rate of consolidation of a soil column of larger depth is also examined.

The following conclusions can be drawn out of the present study:

The rate of consolidation in 2D axisymmetric unit cell is faster up to 10% consolidation, i.e., up to a time factor $T_h \leq 0.4$, beyond which the time-rate of consolidation of the cohesive soil including smear around PVD line drain are in close agreements to the equivalent plane strain condition. Beyond a consolidation of 50% (i.e., at $T_h = 4$), the resulting time-rate of consolidation plots by the two approaches coincides with each other.

The degree of consolidation-time plots for the 2D axisymmetric condition of a soil column with a geometrically modelled PVD revealed a significant delay in the time rate of consolidation compared to that with an ideal vertical drain represented by PLAXIS drain element.

The differences in time-rates of consolidation with varying finite permeability coefficient of the PVD are not distinct for a lower drainage path and coincides with each other. The influence of variability of permeability of PVD on the time-rate of consolidation is more prominent for samples with larger drainage paths.

For the chosen magnitudes of PVD's permeability coefficients, a limiting value is noticed, below which the time rate of consolidation is observed to be visibly delayed. Hence, the effect of well resistance due to increased drainage length on the rate of consolidation is evident.

The present study reinforces the notion that equivalency of axisymmetric and plane-strain unit cell modeling of PVDs through a permeability compatibility approach [15] leads to an equal degree to primary consolidation. Yet, the novelty of present study lies in the succinct elucidation of the differences in the time-rate of consolidation to achieve the final consolidation. This observation would aid the practitioners to have a better insight into the actual magnitude of the degree of consolidation achieved at any given time intermittent to the total completion time of primary consolidation. Such prior information would help manage the mid-stage constructions on such PVD-treated soft soil. Further, conventional numerical modeling uses an ideal drain behavior and models the PVDs as infinite discharge elements, while it is not true for field situations where PVDs rationally exhibit finite discharge capacity. In contrast, the novelty in the present study lies in successful elucidation of the effect of considering finite permeability and discharge capacity of the PVD, along with highlighting the smear effect, on the time-rate of consolidation. This consideration is very important for embankments resting on PVDs as the realistic functionality of the PVDs is only reflected when it is geometrically modeled with finite permeability and smear scenarios.

References

[1] Barrron R A 1948 Transactions of the American Society of Civil Engineers **113** 718-24.
[2] Hansbo S 1979 J. *Ground Engineering* **12** 16-25.
[3] Yoshikuni H and Nakanodo H 1974 J. *Soils and Foundations* **14** 35-46.
[4] Onoue A 1988 J. *Soils and Foundations* **28** 165-74.
[5] Hansbo S 1981 Proc. 10th *Intl. Conf. on Soil Mechanics* (Stockholm) vol **3** pp 677-82.
[6] Rixner J J, Kraemer S R and Smith A D 1986 Prefabricated Vertical Drains (Vol **I**, II and III) Summary of Research Report: Final report Federal Highway Administration (Washington DC) FHWA-RD-86 p 169.
[7] Pradhan T B S, Imai G, Murata T, Kamon M and Suwa S 1993 *Proc.* 11th *Southeast Asian Geotech. Conf.* vol **1** pp 391-96.
[8] Zeng G X, Xie K H and Shi Z Y 1987 Proc. 8th *Asian Regional Conf. on Soil Mechanics* (Kyoto) vol **1** pp 139-42.
[9] Duncan J M and Schaefer V R 1988 J. *Comput. Geotech.* **6** 301-24.

[10] Borges J L 2004 J. *Comput. Geotech*. **31** 665-76.
[11] Indraratna B and Redana I W 2000 *Can. Geotech*. J. **37** 132-45.
[12] Indraratna B, Bamunawita C, Redana I W and Balasubramanium A S 2001 Proc. 3rd *Intl. Conf. on Soft Soil Engineering* (Hong Kong) pp 329-38.
[13] Indraratna B, Rujikiatkamjorn C and Sathananthan I 2005 *Can. Geotech. J*. **42** 994-1015.
[14] Rujikiatkamjorn C, Indraratna B and Chu J 2008 *Intl. J. Geomech. ASCE* **8** 144-56.
[15] Hird C C, Pyrah I C and Russell D 1992 J. *Ge*otechnique **42** 499-511.
[16] Hird C C, Pyrah I C, Russell D and Cinicioglu F 1995 *Can. Geotech*. J **32** 795-807.
[17] Indraratna B and Redana I W 1997 J. Geotech. En*g. ASCE* **123** 474-78.
[18] Indraratna B and Redana I W 1999 J. *Geotech. Geoenvironmental Eng. ASCE* **125** 96-99.
[19] Bergado D T Improving Techniques of Soft Ground in Subsiding and Lowland Environment ed 2nd (Rotterdam Netherlands: A A Balkema).
[20] Giridhar Rajesh B, Chukka S and Dey A 2018 *Geotech. Eng. J*. SEAGS & AGSSEA **49** 63-72.
[21] Indraratna B and Redana I W 1998 J. *Geotech. Eng. ASCE* **124** 180-84.
[22] Mesri G and Lo D O K 1991 Proc. *Intl. Conf. on Geotech. Eng*. for Coastal Development- Theory to Practice (Yokohama, Japan) vol **1** pp 231-36.
[23] Jamiolkowski M, Lancelotta R and Wolski W 1983 Proc. 8th Europe on Conf. Soil Mech. *Found. Eng. J*. (Helsinki, Finland) vol **3** pp 1201-26.
[24] Lin D G, Kim H K and Balasubramaniam A S 2000 *Geotech. Eng. J*. **31** 109-25.
[25] Kim R, Hong S J, Lee M J and Lee W 2011 Mar. *Georesources Geotechnol*. **29** 131-44.