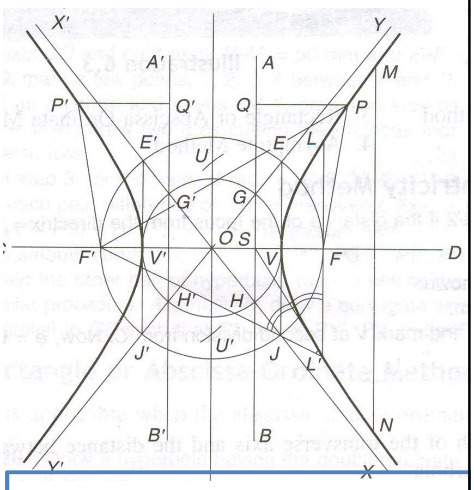
Hyperbola

- **A hyperbola is a conic whose eccentricity is greater than 1.**
- The hyperbolas exist in a pair. It has two foci(F and F′), two directrices(AB and A′B′), an axis and two vertices (V and V′)
- **Transverse axis or Major axis (V and V′)** : The difference of the distances of a point on the curve from the two foci
- **Double ordinate(MN):** Any chord, Say MN , perpendicular to the axis
- **Abscissa:** Distance of the nearest vertex from the given ordinate.
- **Asymptotes:** The two lines X-X′ and Y-Y′ intersection at O
- **Directrix (GH and G′H′):** O as centre and OV and OV′ as radius, draw arcs to cut the asymptotes.

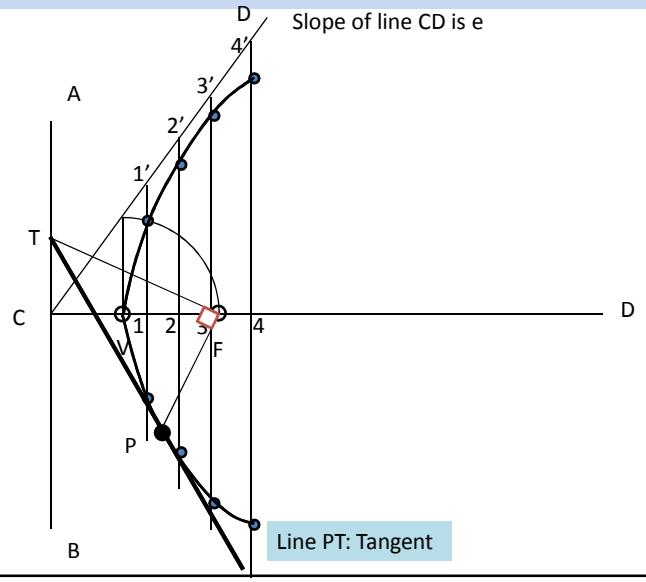


Draw a circle with O as centre and radius OF. Draw perpendicular to (V V′) at V and V′ cutting the circle at E′, J. The diagonals E, J′ are the asymptotes

33

Focus Directrix or Eccentricity Method

- Distance between foci and directrix and ‘e’ are given.



AB: Directrix

CD: Axis

Center: V
Radius: VF

Center: F
Radius: 1-1′

Center: F
Radius: 2-2′

Tangent at point P

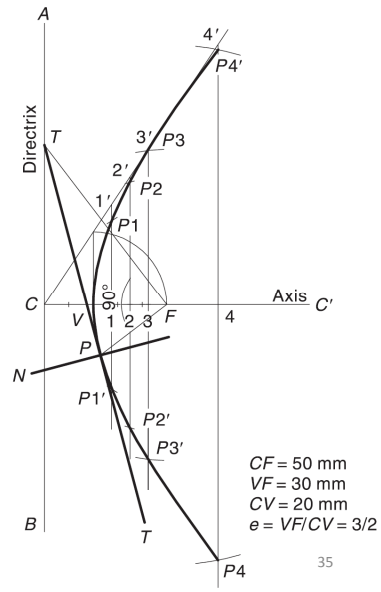
Line PT: Tangent

34

Steps for Focus Directrix or Eccentricity Method

Draw a hyperbola of e = 3/2 if the distance of the focus from the directrix = 50 mm.

1. **Draw directrix AB and axis CC' as shown.**
2. **Mark F on CC' such that CF = 50 mm.**
3. **Divide CF in to 3 + 2 = 5 equal parts and mark V at second division from C. Now, e = VF/ VC = 3/2.**
4. **Follow steps as in ellipse and parabola**



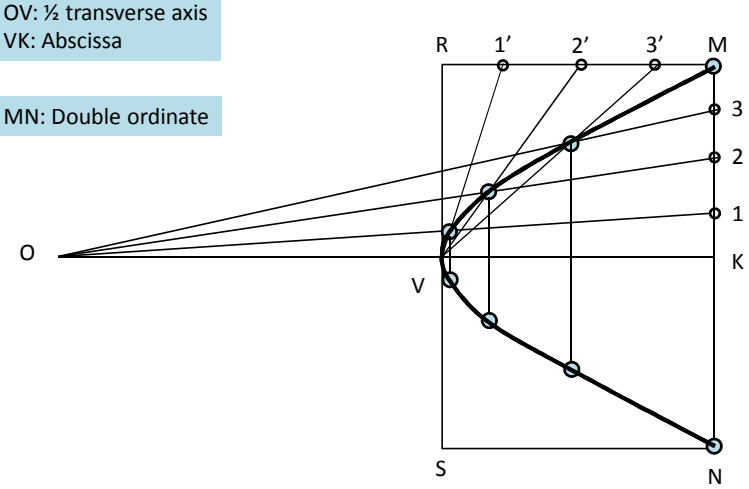
35

Rectangle or Ordinate-Abscissa Method

- Abscissa, double ordinate and transverse axis are given.

OV: ½ transverse axis
VK: Abscissa

MN: Double ordinate



36

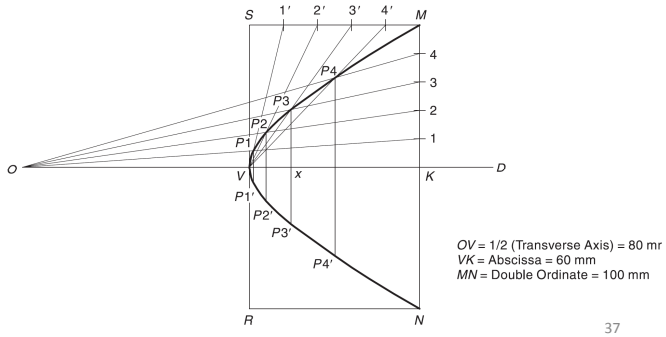
Steps for Rectangle or Abscissa-Ordinate Method

Draw a hyperbola having the double ordinate of 100 mm, the abscissa of 60 mm and the transverse axis of 160 mm.

1. Draw axis OD and mark V and K on it such that OV = 1/2(Transverse Axis) = 80 mm and VK =Abscissa = 60 mm.
2. Through K, draw double ordinate MN = 100 mm.
3. Construct rectangle MNRS such that NR = VK.
4. Divide MK and MS into the same number of equal parts, say 5. Number the divisions as shown.

5. Join O–1, O–2, O–3, etc. Also join V–1', V–2', V–3', etc. Mark P1, P2, P3, etc., at the intersections of O–1 and V–1', O–2 and V–2', O–3 and V–3', etc., respectively.

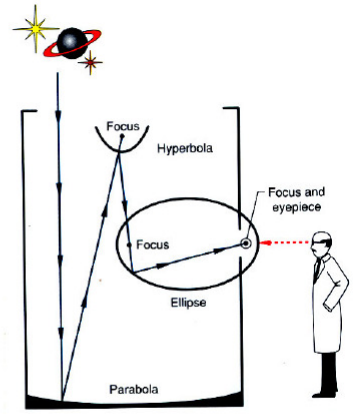
6. Obtain P1', P2', P3', etc., in other half in a similar way. Alternatively, draw P1/ P1', P2/P2', P3/P3', etc., such that P3 –x = x–P3' and likewise.



37

Few Applications of Hyperbola

The cooling tower is of hyperbolic shape



Hyperbolic mirror is used in Telescopes

38

Involutes

- An *involute* is a curve traced by the free end of a thread unwound from a circle or a polygon, in such a way that the thread is always tight and tangential to the circle or the sides of the polygon.
- Depending on whether the involute is traced over a circle or a polygon, the involute is called an *involute of circle* or *involute of polygon*

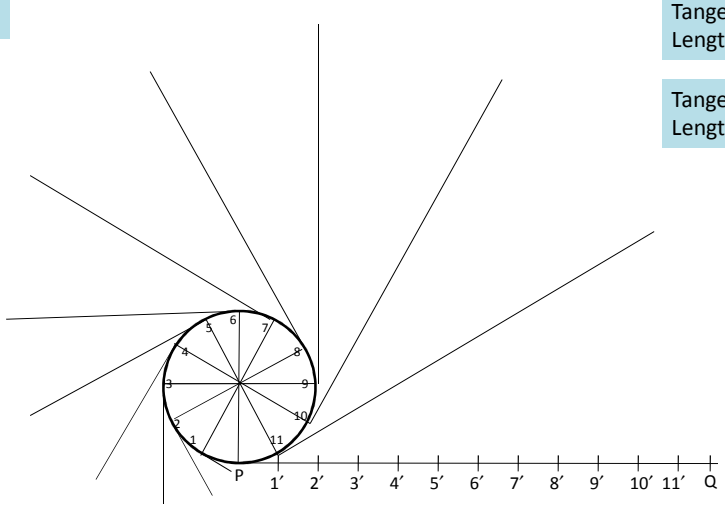
39

Draw the involute of a circle, 40 mm in diameter.

PQ = πD

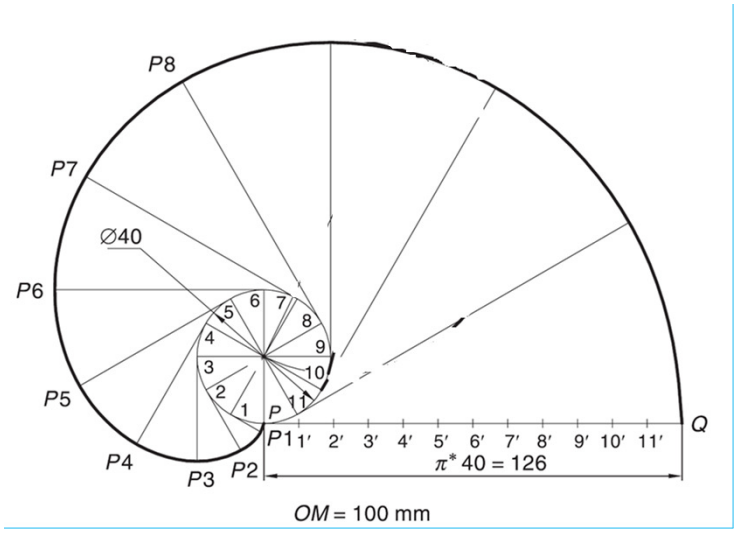
Tangent at 1
Length = P-1′

Tangent at 2
Length = P-2′



40

Involute



41

STEPS:
DRAW INVOLUTE AS USUAL.

MARK POINT **Q** ON IT AS DIRECTED.

JOIN **Q** TO THE CENTER OF CIRCLE **C**. CONSIDERING **CQ** DIAMETER, DRAW A SEMICIRCLE AS SHOWN.

MARK POINT OF INTERSECTION OF THIS SEMICIRCLE AND POLE CIRCLE AND JOIN IT TO **Q.**

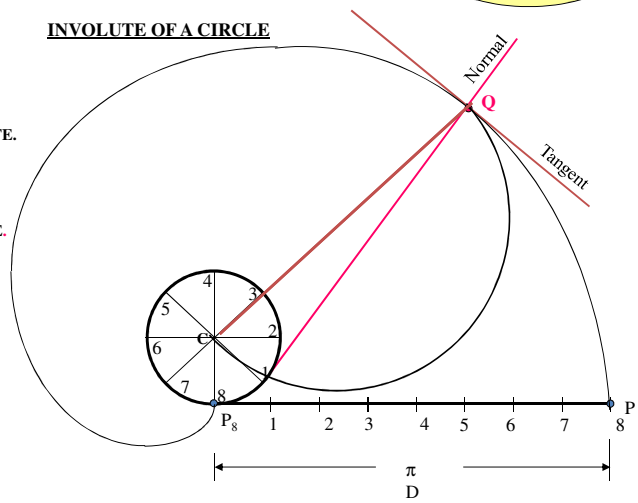
THIS WILL BE **NORMAL TO INVOLUTE.**

DRAW A LINE AT RIGHT ANGLE TO THIS LINE FROM **Q.**

IT WILL BE TANGENT TO INVOLUTE.

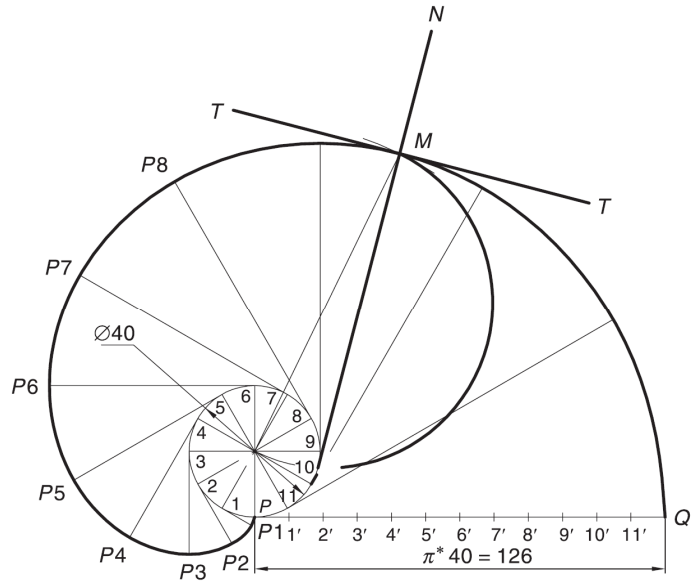
Involute Method of Drawing Tangent & Normal

INVOLUTE OF A CIRCLE



42

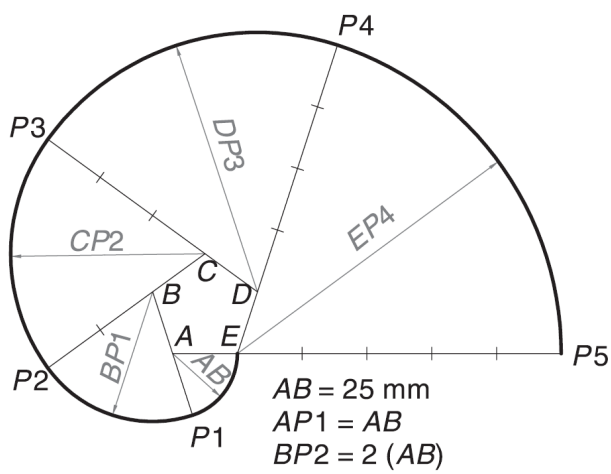
Tangent and Normal to Involute



OM = 100 mm

43

<u>Involute of Polygon:</u> Draw the involute of a pentagon of 25 mm side.



AB = 25 mm
*AP*1 = *AB*
*BP*2 = 2 (*AB*)
*CP*3 = 3 (*AB*)
*DP*4 = 4 (*AB*)
*EP*5 = 5 (*AB*)

44

Application of Involute

Involute gear

45

CYCLOIDAL CURVES: A **cycloid** is a curve generated by a point on the circumference of a circle rolling along a straight line without slipping. The rolling circle is called a generating circle and the straight line is called a directing line or base line. The point on the generating circle which traces the curve is called the generating point.

The cycloid is called the **epicycloid** when the generating circle rolls along another circle outside it.

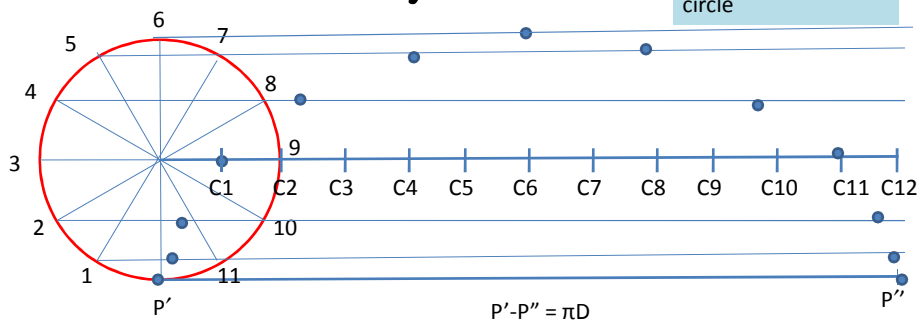
Hypocycloid, opposite to the epicycloid, is obtained when the generating circle rolls along another circle inside it.

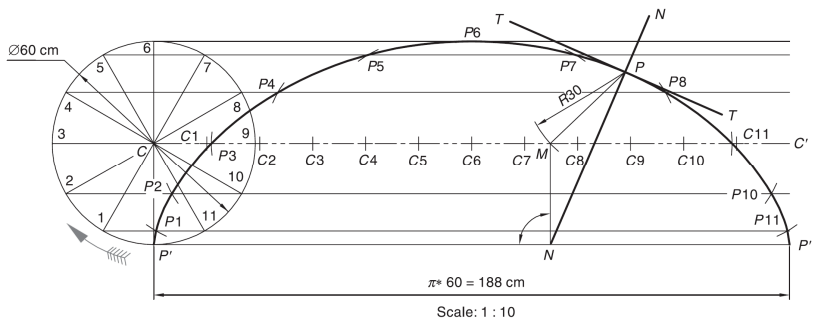
The other circle along which the generating circle rolls is called the directing circle or the base circle.

46

Cycloid

Center: C1, Radius of circle





47

Cycloid: A wheel of diameter 60 cm rolls on a straight horizontal road. Draw the locus of a point P on the periphery of the wheel, for one revolution of the wheel, if P is initially on the road.

Draw the base line P'–P" equal to the circumference of generating circle, i.e., πx 60 cm = 188 cm.

Draw the generating circle with C as a centre and radius = 30 cm, tangent to P' –P" at P'. Point P is initially at P'.

Draw C–C" parallel and equal to P' –P" to represent the locus of the centre of the generating circle.

Obtain 12 equal divisions on the circle. Number the divisions as 1, 2, 3, etc., starting from P' as shown. Through 1, 2, 3, etc., draw lines parallel to P' –P".

Obtain 12 equal divisions on C–C" and name them as C1, C2, C3, etc.

With C1, C2, C3, etc. as the centres and radius = CP' = 30 mm, cut the arcs on the lines through 1, 2, 3, etc., to locate respectively P1, P2, P3, etc.

Join P', P1, P2, P3, etc. by a smooth curve.

48

CYCLOID
Method of Drawing Tangent & Normal

STEPS:
DRAW CYCLOID AS USUAL.
MARK POINT **Q** ON IT AS DIRECTED.

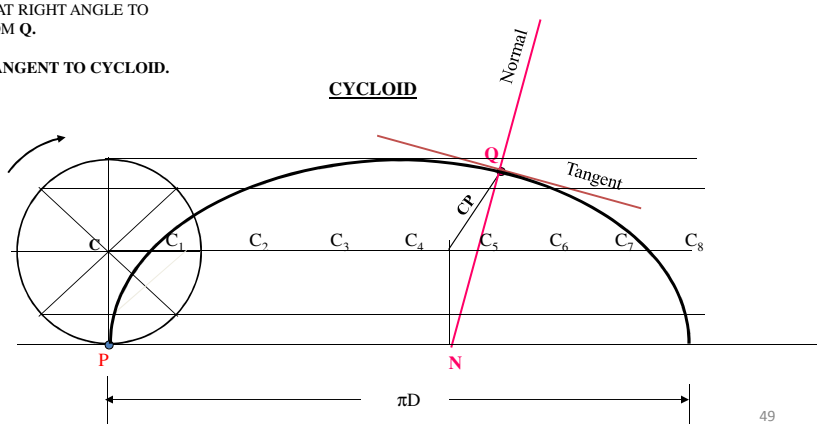
WITH CP DISTANCE, FROM **Q.** CUT THE POINT ON LOCUS OF **C** AND JOIN IT TO **Q.**

FROM THIS POINT DROP A PERPENDICULAR ON GROUND LINE AND NAME IT N

JOIN N WITH Q.THIS WILL BE **NORMAL TO CYCLOID.**

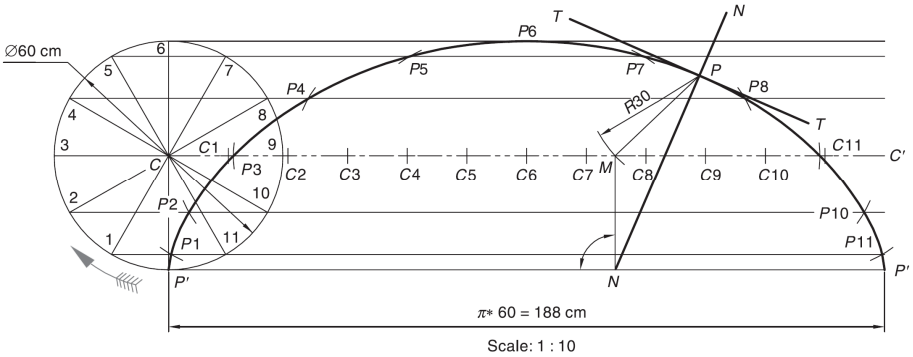
DRAW A LINE AT RIGHT ANGLE TO THIS LINE FROM **Q.**

IT WILL BE TANGENT TO CYCLOID.



49

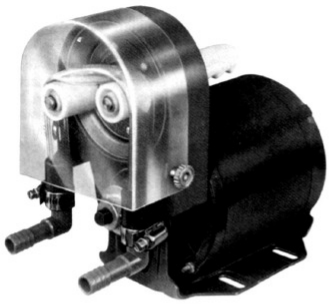
Tangent and Normal to Cycloid



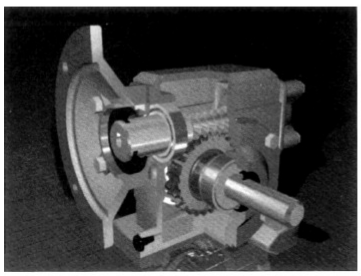
1. **With P as a centre and radius = CP' (i.e., radius of generating circle), cut an arc on C–C" at M.**
2. **From M, draw a normal MN to P' –P". In case of epicycloid and hypocycloid, this can be done by joining MO and then locating N at the intersection of P' –P" and MO (produced it necessary).**
3. **Join NP for the required normal. Draw tangent T–T perpendicular to NP at P.**

50

Applications



Cycloids find application in gears for rotary pumps, watches, etc



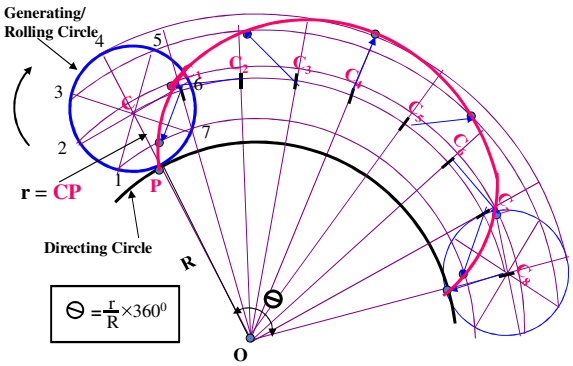
High power transmission gear teeth profiles are involutes

51

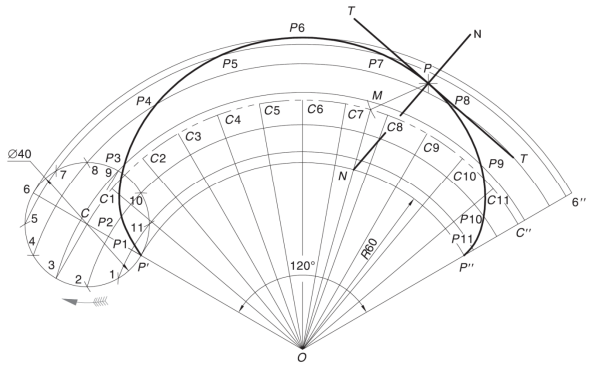
DRAW LOCUS OF A POINT ON THE PERIPHERY OF A CIRCLE WHICH ROLLS ON A CURVED PATH. **Take diameter of rolling Circle 50 mm And radius of directing circle i.e. curved path, 75 mm.**

EPI CYCLOID

Length of the arc of directing circle = $\pi D = R\theta$



52



Draw an epicylcoid if a circle of 40 mm diameter rolls outside another circle of 120 mm diameter for one revolution.

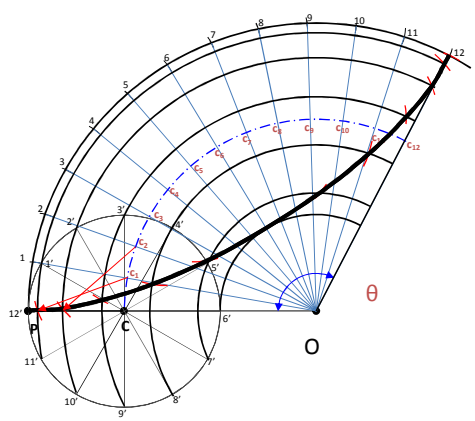
Length of the arc of directing circle = $\pi D = R\theta$

∴ Included angle of the arc,
D/ R × 180) = 40/60 × 180 = 120°

1. With O as a centre and radius = 60 mm, draw the directing arc P' –P'' of the included angle 120°.
2. Produce OP' and locate C on it such that CP' = radius of generating circle = 20 mm. With C as centre and radius = CP', draw a circle.
3. With O as a centre and radius = OC, draw an arc C–C'' such that ∠COC'' = 120°. Arc C–C'' represents the locus of centre of generating circle.
4. Divide the circle into 12 equal parts. With O as a centre and radii = O–1, O–2, O–3, etc., draw the arcs through 1, 2, 3, etc., parallel to arc P' –P''.

53

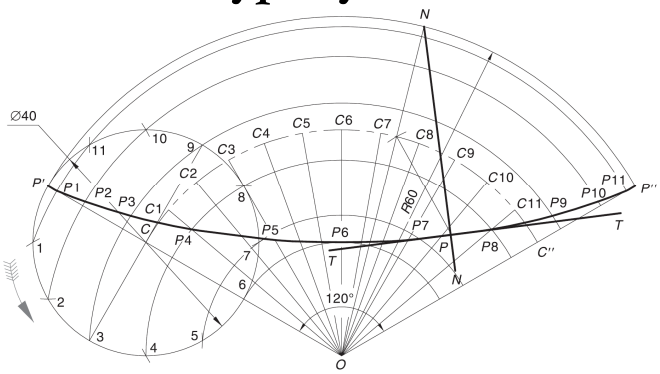
HYPOCYCLOID



OP=Radius of directing circle=75mm
PC=Radius of generating circle=25mm
θ=r/R X360º= 25/75 X360º=120º

54

Hypocycloid



A circle of diameter 40 mm rolls inside another circle of radius 60 mm. Draw the hypocycloid traced by a point on the rolling circle initially in contact with the directing circle for one revolution.

Included angle of the arc, θ = (D/ R × 180) = 40/60 × 180 = 120°.
With O as a centre and radius = 60 mm, draw the directing arc P' –P" of included angle 120°.
On OP', locate C such that CP' = 20 mm. With C as a centre and radius = CP', draw a circle.
Follow steps as cycloid and epicycloids

55

9th Century Chand Baori well in the Rajasthan is the world's deepest, extending 100 feet below the surface of the earth.

56