

ME 532 Finite Element Methods in Engineering

3 0 0 6

Introduction: Historical background, basic concept of the finite element method, comparison with finite difference method; Variational methods: calculus of variation, the Rayleigh-Ritz and Galerkin methods; Finite element analysis of 1-D problems: formulation by different approaches (direct, potential energy and Galerkin); Derivation of elemental equations and their assembly, solution and its post processing. Applications in heat transfer, fluid mechanics and solid mechanics. Bending of beams, analysis of truss and frame. Finite element analysis of 2-D problems: finite element modelling of single variable problems, triangular and rectangular elements; Applications in heat transfer, fluid mechanics and solid mechanics; Numerical considerations: numerical integration, error analysis, mesh refinement. Plane stress and plane strain problems; Bending of plates; Eigen value and time dependent problems; Discussion about preprocessors, postprocessors and finite element packages.

Texts:

1. J N Reddy, **An introduction to the Finite Element Method**, McGraw-Hill, New York, 1993.
2. K H Huebner, D L Dewhirst, D E Smith and T G Byrom, **The Finite Element Method for Engineers**, Fourth Edition, Wiley-Interscience Publication, John Wiley & Sons, Inc., 2001.
3. R D Cook, D S Malkus and M E Plesha, **Concepts and Applications of Finite Element Analysis**, 3rd ed., John Wiley, New York, 1989.
4. K J Bathe, **Finite Element Procedures in Engineering Analysis**, Prentice-Hall, Englewood Cliffs, NJ, 1982.
5. T J T Hughes, **The Finite Element Method**, Prentice-Hall, Englewood Cliffs, NJ, 1986.
6. O C Zienkiewicz and R L Taylor, **The Finite Element Method**, 3rd ed. McGraw-Hill, 1989.

GRADING SYSTEM

1. End Semester Examination (Open note book)	40% (Three hours)
2. Mid Semester Examination (Open note book)	20% (Two hours)
3. Quizzes (Two) (Open note book)	20% (10% Each; One hour each)
4. Assignments	05%
5. Term project (Computational)	15%

ME 532 Finite Element Methods in Engineering**3 0 0 6****Topic-wise Lectures Details**

S. N.	Topic	No. of Lectures	Cumulative no. of lectures	
1.	Introduction: Historical background, basic concept of the finite element method, comparison with finite difference method	3 (2)	3	
2.	Direct method: Derivation of elemental equations and their assembly, solution and its post processing.	5 (4)	8	
3.	Variational methods: calculus of variation, the Rayleigh-Ritz method;	4 (5)	12	
4.	Quiz # 1	1	13	
5.	Galerkin method	4	17	
6.	Interpolation functions	5	22	
Mid-Semester Examination				
7.	Numerical Integration & Error analysis	4	26	
8.	Applications in Solid Mechanics & Transient Analysis (2-Dimensional problems)	5	31	
9.	Quiz # 2	1	32	
10.	Applications in Heat Transfer & Fluid Mechanics (2-Dimensional problems)	5	37	
11.	Project Presentation	3	40	
End Semester Examination				
Total		40	40	

ME 532 Finite Element Methods in Engineering

Term Projects Topics-2004

I Software development for the following elements (individually or in combination):

S N	Title	Group I		Group II		Group III		Group IV	
		Name	Roll No. & Email Id	Name	Roll No. & Email Id	Name	Roll No. & Email Id	Name	Roll No. & Email Id
1	Springs (linear, nonlinear)	1.K Satish narayana.	04410335 snk@iitg.ernet.in	1.K.V.V.N. S K.Mohan	04410314 k.mohan@iitg.ernet.in				
		2.S.Saibabu	04410333 -----	2.Venkata Srikanth Manda	04410346 v.manda@iitg.ernet.in				
		3.R.Vijayaraj Chakravarthy	04410326 -----						
2	Electric circuits (linear, nonlinear)								
3	Trusses (2-D,3-D)	1. Raja Ramesh.J	04410328 r.ramesh@iitg.ernet.in						
		2. Mohammed Mastan	04410323 m.mastan@iitg.ernet.in						
		3. Radha Krishna Y	04410348 yerra@iitg.ernet.in						

S N	Title	Group I		Group II		Group III		Group IV	
		Name	Roll No. & Email Id	Name	Roll No. & Email Id	Name	Roll No. & Email Id	Name	Roll No. & Email Id
4	Rods: axial and torsional deflections (1-D,2-D)								
5	Strings (1-D,2-D)								
6	Beams: transverse deflections (1-D,2-D)	1.Mahadevan P	04410320 mahadevan@iitg.ernet.in	1.Rahul Jain	01010320 rahulj@iitg.ernet.in				
		2. G.Balaji	04410310 g.balaji@iitg.ernet.in	2.Neeraj Kuar	01010314 -----				
		3. Sanjeev Kumar	04410332 yerra@iitg.ernet.in	3 Writuparna Sutradhar	01010328 writu@iitg.ernet.in				

S N	Title	Group II		Group II		Group III		Group IV	
		Name	Roll No. & Email Id	Name	Roll No. & Email Id	Name	Roll No. & Email Id	Name	Roll No. & Email Id
7.	Frames (1-D,2-D)								
8	Plates (2-D)	1.Yuvaraj N	01010329 yuvraj@iitg.ernet.in						
		2.Abhishek Kumar	010101301 -----						
9	Heat transfer (1-D,2-D)	1. K.Karthik	01010311 katra@iitg.ernet.in	1. -----	04410301 a.koushik@iitg.ernet.in	1.Siddhartha Shankar Choudhury	01010321 ssc@iitg.ernet.in	1.Amit Kushwaha	----- kushwaha@iitg.ernet.in
		2. Ashutosh Aggarwal	01010308 -----	2. -----	04410325 r.reddy@iitg.ernet.in	2. Pranav Kumar	01010330 kumarp@iitg.ernet.in	2. Deep Singh	----- -----
		3. Pranshu Chug	01010318 -----						

S . N	Title	Group I		Group II		Group III		Group IV	
		Name	Roll No. & Email Id	Name	Roll No. & Email Id	Name	Roll No. & Email Id	Name	Roll No. & Email Id
1 0	Flow through Pipe	1.Elaprolu Vishnu Vardhana Rao	----- evrao@iitg.ernet.in	1.Om Prakash	01010315 -----				
	GroupI(1-D)			2.Somesh Khandelwal	01010322 -----				
	GroupII - (2D)			3. Vijay Kumar Gupta	01010327 -----				
1 1	Reynolds's equations etc.	1. Praveen Sharma	01010319 -----						
		2. Piyush Grover	01010320 -----						
		3. Vishesh	01010327 vishesh@iitg.ernet.in						
1 2	Problems related composite materials, smart materials, etc.								
1 3	Any other problem in design, fluids, thermal, manufacturing, structures etc.								

II. Using ANSYS/Pro-engineer Software

S N	Title	Group II		Group II		Group III		Group IV	
		Name	Roll No. & Email Id	Name	Roll No. & Email Id	Name	Roll No. & Email Id	Name	Roll No. & Email Id
1.	Contact stress analysis (ball and roller on a flat/curved surfaces)	1.Souptick Chanda,	04410338 souptick@iitg.ernet.in	1.Tarapada RayBagdi	04610306 tarapada@iitg.ernet.in				
		2.Prasenjit Mallik	04410324 Prasenjit@iitg.ernet.in						
		3.Sushen Kirtania	04410342 sushen@iitg.ernet.in						
2	Gear tooth stress analysis	1.Satya narayan houksey	04410336 satya@iitg.ernet.in	1.Srinivasa Rao Thalapala	04410340 srt@iitg.ernet.in				
		2. chandrakant maheswari	04410309 m.chandra@iitg.ernet.in	2.Ramesh Meena Ghashiram	04410322 meena@iitg.ernet.in				
		3. Tanuja s. vaidya	04410343 tanuja@iitg.ernet.in	3. Rajendra Kishore Oliganti	04410329 r.kishore@iitg.ernet.in				

S N	Title	Group I		Group II		Group III		Group IV	
		Name	Roll No. & Email Id	Name	Roll No. & Email Id	Name	Roll No. & Email Id	Name	Roll No. & Email Id
3.	Stress analysis of Gas Turbine Blade	1. P.Kashyap	01010316 pkashyap@iitg.ernet.in						
		2. T.S.H.V. Reddy	01010325 -----						
4	Cam face stress analysis								
5	Rolling bearing load distributions analysis (Indeterminant problem)								
6	Any other real life machine elements, structures etc								

Note:

1. Last date of giving options is 08-08-2004 (Otherwise I will allot in random fashion).
2. Two to three students in a group.
3. Please submit title of the problem and brief outline of the problem with names, roll numbers and email of the students in the group
through email to rtiwari@iitg.ernet.in.

Website for the course:

<http://www.iitg.ernet.in/engfac/rtiwari/resume/course%20to%20be%20offered.html>

ME-532 Assignment Sheet No.1 Submission date: On or before 17-08-2004

Q1. Consider the spring-mounted bar shown in Figure 1. Solve for displacements of points P and Q using bar elements (assume $AE = \text{constant}$).

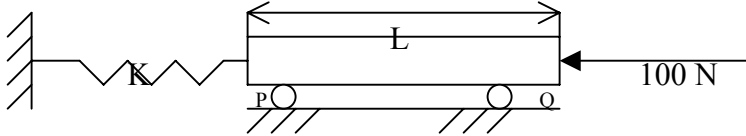


Fig.1. Spring mounted bar.

Q2. For a uniform circular cross section bar (diameter d , length L , modulus of rigidity G) under twist, determine the stiffness equation i.e.,

$$\begin{Bmatrix} T_1 \\ T_2 \end{Bmatrix} = \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix} \begin{Bmatrix} \phi_1 \\ \phi_2 \end{Bmatrix}$$

Use this to solve for the angle of twist ϕ at points A and B in Figure 2.

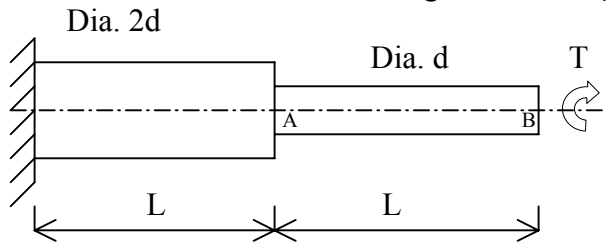


Fig.2. Torsion of a stepped shaft.

Q3. The plane wall shown in Figure 3 is 0.5 m thick. The left surface of the wall is maintained at a constant temperature of 200°C , and the right surface is insulated. The thermal conductivity $K = 25 \text{ W/m}^\circ\text{C}$ and there is a uniform heat generation inside the wall of $Q = 400 \text{ W/m}^3$. Determine the temperature distribution through the wall thickness using linear elements.

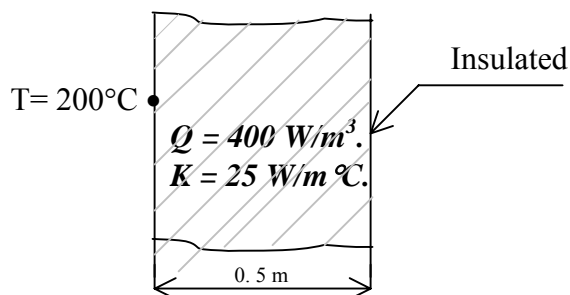


Fig.3. Heat conduction in a slab

ME-532 Assignment No. 2 (Variational approach)
(Due date of submission: 31 August 2004)

Q1. Displacement degree of freedom (d.o.f) d_i at ends A and B of a uniform bar element have the directions as shown in the Figure 1. Write an expression for the potential energy functional (Π_p) in terms of these d.o.f. From the stationary condition $\delta\Pi_p = 0$, obtain the element stiffness matrix that operates on these four d.o.f

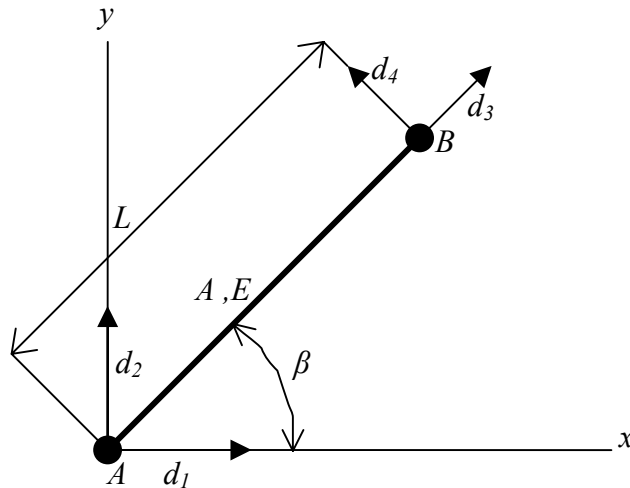


Fig.1. Uniform bar element.

Q2. A certain physical problem has the functional $\Pi_p = \int_0^L \left(\frac{1}{2} \Phi_{,x}^2 - 50\Phi \right) dx$. Determine the Euler equation for this functional. Then solve the Euler equation with the essential boundary condition $\Phi = 0$ at $x = 0$ and $\Phi = 20$ at $x = L$ to determine Φ as a function of x and L .

Q3. The functional, for a one-dimensional problem in structural mechanics is given by

$$\Pi_p = \frac{EI}{2} \int_0^L \left(\frac{d^2 y}{dx^2} \right)^2 dx - Py(L), \quad y(0) = y'(0) = 0$$

By performing the variation $\delta\Pi_p = 0$, derive the governing differential equation of the problem and the boundary conditions at $x = L$. What is the physical situation represented by this functional?

Assignment No.3 (ME-532 Galerkin approach)

(Last date of submission: 10-09-2004; Please submit your assignment in the old CAD lab, now it is called Project Staff room, to Mr M. Karthikeyan)

Q1. Solve the ordinary differential equation,

$$\frac{d^2\Phi}{dx^2} + 10x^2 = 0, \quad 0 \leq x \leq 1,$$

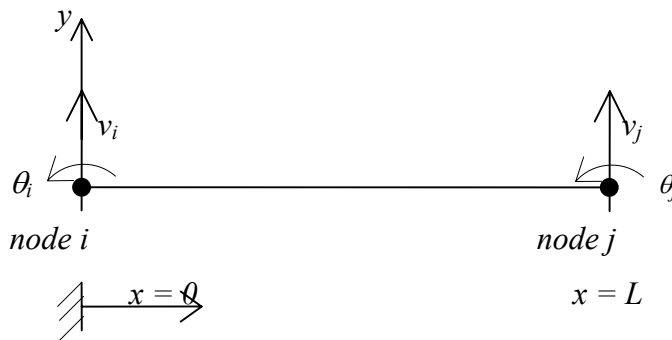
Subject to boundary conditions, $\Phi(0) = 0, \Phi(1) = 0$ using the Galerkin method with the trial functions

a) $N_0(x) = 0, \quad N_1(x) = x(1-x), \quad N_2(x) = x(1-x^2)$

b) $N_0(x) = 0, \quad N_1(x) = \sin \pi x, \quad N_2(x) = \sin 3\pi x, \quad N_3(x) = \sin 5\pi x$

Compare each with exact solution by plotting $\Phi(x)$ and $\tilde{\Phi}(x)$

Q2. Derive beam element matrices (stiffness and nodal forces) for the Euler-Bernoulli beam element, shown in Figure 1, using Galerkin method.



where, v_i, v_j ---- displacements in y direction at node i and node j ,

θ_i, θ_j --- slopes in x - y plane at node i and node j ,

L ---- Length of the element

Figure 1. The Euler-Bernoulli beam element

Q3. Develop the weak form and the element model of the following differential equation over an element:

$$-\frac{d}{dx} \left(a \frac{du}{dx} \right) + \frac{d^2}{dx^2} \left(b \frac{d^2u}{dx^2} \right) + cu = f \quad \text{for } x_A < x < x_B$$

where a, b, c and f are known functions of position x . Ensure that the element coefficient matrix $[\mathbf{K}]^{(e)}$ is symmetric. What is the nature of the interpolation functions for the problem?

Assignment No. 4 (ME-532 Elements and Interpolation functions)

(Last date of submission: 22-10-2004)

Q.1. For the three-nodded triangular element shown in Fig. 1, calculate the temperature at the point P , give the nodal temperatures as $T_1 = 100^\circ\text{C}$, $T_2 = 200^\circ\text{C}$, and $T_3 = 300^\circ\text{C}$.

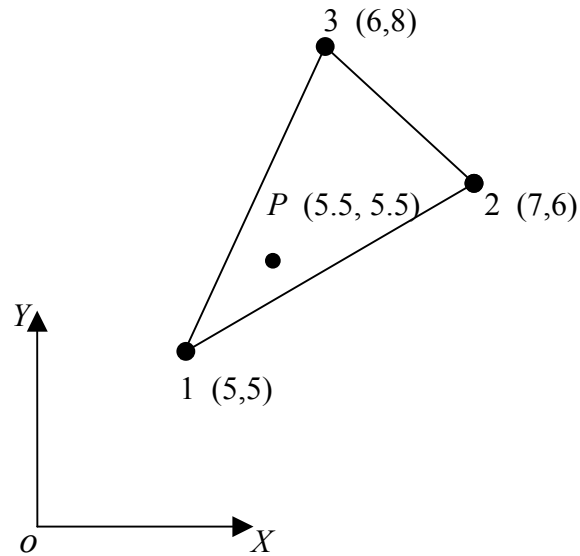


Figure 1. Temperature at an interior point

Q.2. Derive and plot the geometric variation, of the Lagrange interpolation functions at nodes 1, 2 and 5 of the nine-node quadratic element shown in Fig. 2.

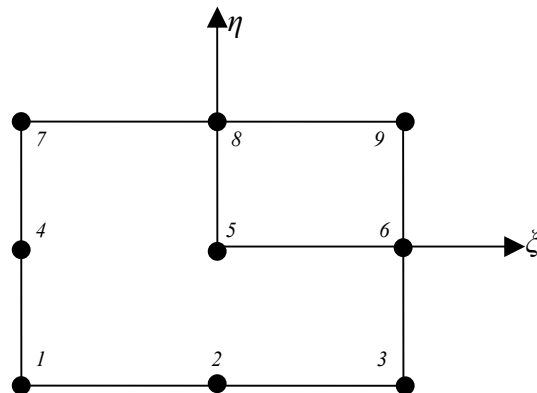


Figure 2. Nine-node quadratic element.

Q.3. Two different isoparametric elements are used together as shown in Fig.3. There is inter element boundary between $(x, y) = (1,1)$ and $(x, y) = (2,2)$. Show that the variable is continuous and compatible across the interface boundary. The interpolation function for the triangular element is $u = a_0 + a_1x + a_2y$, and for the quadrilateral element is $u = b_0 + b_1x + b_2y + b_3xy$.

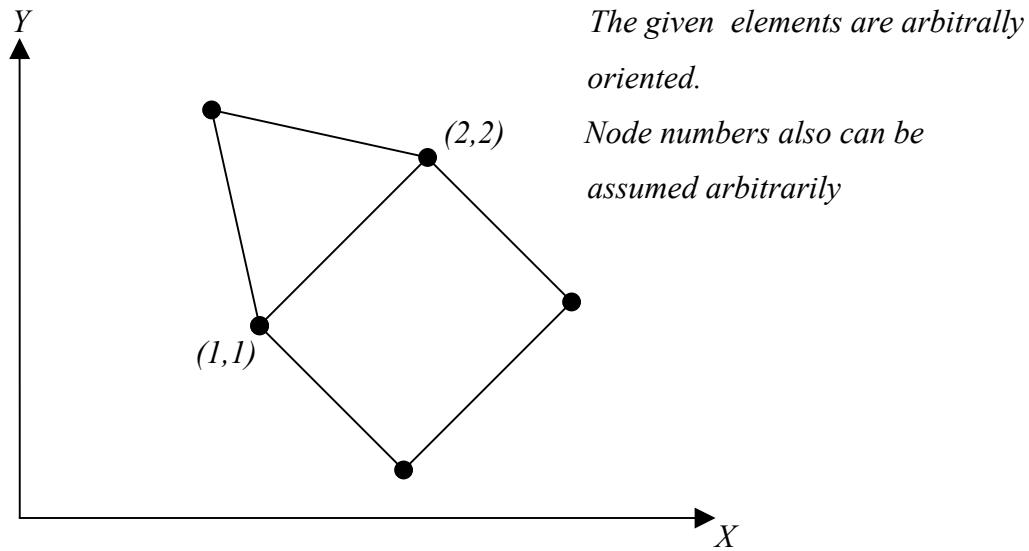


Figure 3. Isoparametric elements

Q.4. Consider the isoparametric quadrilateral with nodes 1-4 at $(15,0)$, $(17,12)$, $(7,10)$, and $(6,2)$, respectively. Compute the Jacobian matrix and its determinant at the element centroid for the given element. Also calculate the area of the element and its parent and compare the ratio the two to your calculated $|J|$ for each case. Sketch the element and its parent to scale.