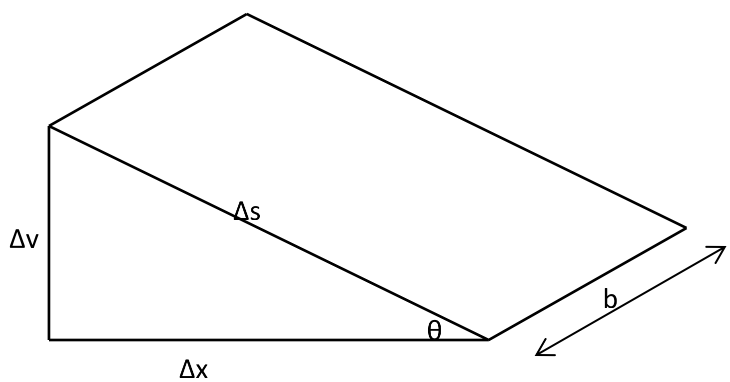
18/01/2016

LECTURE – 5

Pressure distribution in a fluid:

- ✓ There are many instances where the fluid is in stationary condition. That is the movement of liquid (or gas) is not involved.
- ✓ Yet, we have to solve some engineering problems involving stationary liquids (similar to what you have studied in statics of the course Engineering Mechanics).
- ✓ The condition of fluid, for such case is termed as hydrostatic.
- ✓ Recall, a fluid at rest cannot resist shear.
- ✓ Therefore, the normal stress on any plane through a fluid element at rest is called fluid pressure, p. (Recall you have studied normal stress & shear stress in Solid Mechanics course).
- ✓ This normal stress, when fluid is at rest, is taken positive for compression (conventionally taken through & across various course).
- ✓ To describe about this pressure, let us describe through equilibrium of forced in a small wedge of fluid at rest. (again recall principles of statics)



- ✓ For a body in static condition you should satisfy

$$\sum F_x = 0$$

$$\sum F_y = 0$$

$$\sum F_z = 0$$

- ✓ The above fluid element is also in static equilibrium.

- ✓ As this element is isolated from its surroundings, we need to incorporate corresponding forces (Free body diagram).

- ✓ By convention the compressive forces may be acting on the wall of the fluid element, i.e., forces act into the plane in respective directions.
- ✓ Let p_x be the force per unit area acting on the plane bΔz.
- ✓ Let p_z be the force per unit area acting on the plane bΔx.
- ✓ Let p_y be the force per unit area acting on the plane ΔxΔz.
- ✓ Let p_s be the force per unit area acting on the plane bΔs.

For $\sum F_x = 0$;

$$p_x b\Delta z - p_s b\Delta s \sin\theta = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

For $\sum F_z = 0$;

$$p_z b\Delta x - p_s b\Delta s \cos\theta - \frac{1}{2}\rho g \Delta x \Delta z b = 0 \ldots\ldots\ldots (2)$$

For $\sum F_y = 0$;

$$-\frac{1}{2} p_y \Delta x \Delta z + \frac{1}{2} p_y \Delta x \Delta z = 0 \ldots\ldots\ldots\ldots\ldots\ldots.. (3)$$

From (1),

$$p_x b\Delta z - p_s b\Delta s \sin\theta = 0$$
$$p_x b\Delta z = p_s b\Delta z$$
$$p_x = p_s$$

From (2),

$$p_z b\Delta x - p_s b\Delta x - \frac{1}{2}\rho g \Delta x \Delta z b = 0$$
$$p_z = p_s + \frac{1}{2}\rho g \Delta z$$

For limit ($\Delta z \to 0$; $\Delta x \to 0$) $p_z \to p_s$

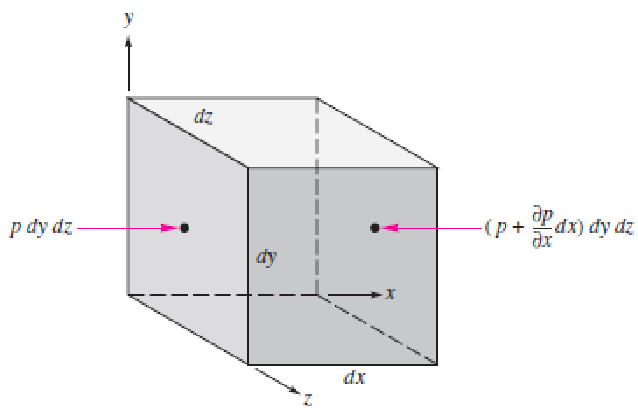
So, $p_z = p_s = p_x = p_y = p$ = pressure

At a static point, pressure is a scalar property without any orientation. P(x,y,z,t)

<u>Pressure force on fluid element</u>:

Due to pressure, forces act on the respective plane of interest.

Let us consider a rectangular elemental prism of volume $\Delta x \Delta y \Delta z$



Net *x* force on an element due to pressure variation

(Source: Fluid Mechanics by F.M. White)

- ✓ Force due to pressure is called <u>pressure force</u>.
- ✓ At any mathematical point pressure is described as p(x,y,z,t) as it is a scalar quantity.
- ✓ Therefore, let us suggest that in x direction there are two planes normal.
- ✓ On the left plane, let p be the pressure.
- ✓ On the right plane, pressure will be $p + \frac{\partial p}{\partial x} \Delta x$

So, <u>net force</u> due to pressure acting in the x direction will be:

$$\Delta F_{px} = p \Delta y \Delta z - (p + \frac{\partial p}{\partial x} \Delta x) \Delta y \Delta z$$

$$= - \frac{\partial p}{\partial x} \Delta x \Delta y \Delta z$$

Similarly, net force in y direction:

$$\Delta F_{py} = -\frac{\partial p}{\partial y} \Delta x \Delta y \Delta z$$

Similarly, net force in z direction due to pressure

$$\Delta F_{pz} = -\frac{\partial p}{\partial z} \Delta x \Delta \text{ y} \Delta \text{ z}$$

Force is a vector & the net pressure force is given by,

$$\overrightarrow{\Delta F}_p = \Delta F_{px}\hat{i} + \Delta F_{py}\hat{j} + \Delta F_{pz}\hat{k}$$

$$= [-\frac{\partial p}{\partial x}\ \hat{\text{i}}\ \text{-}\ -\frac{\partial p}{\partial y}\ \hat{\text{j}}\ -\frac{\partial p}{\partial z}\ \hat{\text{k}}]\Delta x\ \Delta y\ \Delta z$$

As the volume ΔxΔyΔz is arbitrary & choosen by us,

the net pressure force per unit volume

$$\vec{f}_p = [-\frac{\partial p}{\partial x}\ \hat{\text{i}}\ \text{-}\ -\frac{\partial p}{\partial y}\ \hat{\text{j}}\ -\frac{\partial p}{\partial z}\ \hat{\text{k}}]$$

$$= \nabla p$$

= gradient of pressure

i.e., it is the gradient of pressure that cause force in the fluid.

<u>Gage Pressure & Vacuum Pressure</u>:

In the physics classes, you might have already seen how pressure is expressed.

- ✓ The units are N/m^2 or Pa or kPa etc.
- ✓ Might have heard about 1. Absolute pressure,
  2. Gage pressure,
  3. Vacuum pressure

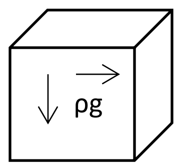
<u>Hydrostatic Pressure Condition</u>:

- If a fluid is in rest, it will not have any shear force.
- Similarly, in a static liquid, there will not be any <u>viscous force</u>.
- For the same rectangular prism element of the fluid in rest (or static):

We should apply principles of statics

$\sum F = 0$

- As the fluid is in static, the forces acting on it will be
  - pressure force
  - gravity force



$$\vec{g} = 0\hat{i} + 0\hat{j} + g\hat{k}$$

i.e. $\sum F_x = 0$ means $-\frac{\partial p}{\partial x}\Delta x \Delta y \Delta z = 0$

Or, $\frac{\partial p}{\partial x} = 0$

Similarly, $\sum F_y = 0$ implies $\frac{\partial p}{\partial y} = 0$

Similarly, $\sum F_z = 0$ means $-\frac{\partial p}{\partial x}\Delta x \Delta y \Delta z - \rho g_z \Delta x \Delta y \Delta z = 0$

Or, $\frac{\partial p}{\partial z} = -\rho g_z$

It can be also expressed as,

$\overrightarrow{\sum F} = 0$, $-\nabla \vec{p}\Delta x \Delta y \Delta z + \rho \vec{g} \Delta x \Delta y \Delta z = 0$

$\vec{\nabla} p = \rho \vec{g}$

i.e., $\frac{\partial p}{\partial z} = -\rho g$

or $\vec{\nabla} p = \rho \vec{g}$

In hydrostatic condition, it is now clear that,

$\frac{\partial p}{\partial x} = 0$ and $\frac{\partial p}{\partial y} = 0$

That is, there won’t be variation of pressure in horizontal direction for the same fluid.

So, we can hence write $\frac{\partial p}{\partial z}$ as $\frac{dp}{dz}$ in static conditions.

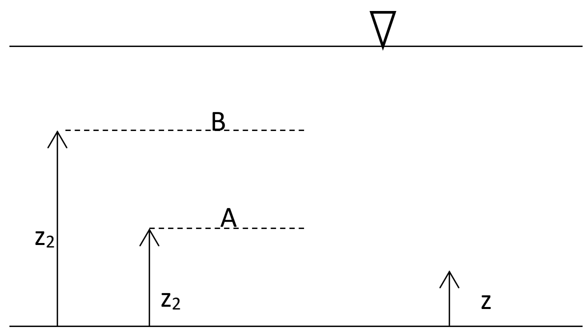
$$\frac{dp}{dz} = -\rho g_z$$

$$dp = -\rho g_z dz$$

$$\int_A^B dp = -\int_A^B \rho g_z dz$$

$$p_B - p_A = -\rho g_z (z_2 - z_1)$$

$$p_A = p_B + \rho g_z (z_2 - z_1)$$



If B is water surface, then P_B = atmospheric pressure = 0 (gage pressure)

So, $p_A = \rho g_z(z_2 - z_1) = \rho g h$, where h = height of water surface level from surface.