24/01/2017

Lecture 7

Hydrostatic Forces on Surface

In the last class we discussed about the

- Hydrostatic Pressure
- Hydrostatic condition, etc.

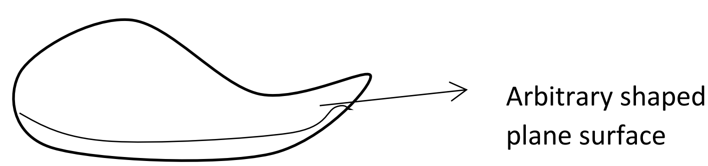
To retain water or other liquids, you need appropriate solid container or retaining structure like

- Water tanks
- Dams
- Or even vessels, bottles, etc.

In static condition, forces due to hydrostatic pressure from water will act on the walls of the retaining structure. You have to design the walls appropriately so that it can withstand the hydrostatic forces.

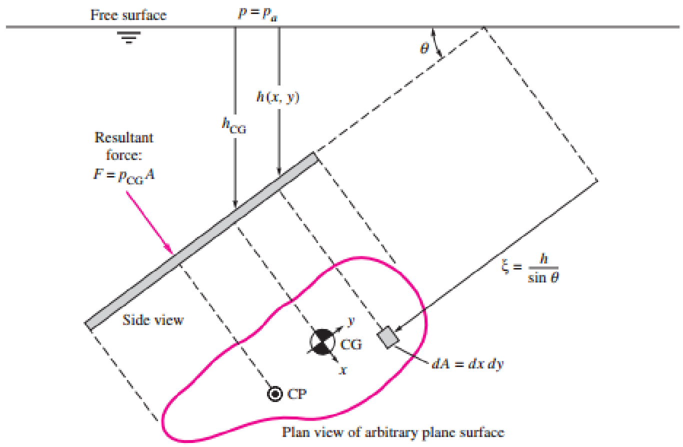
To derive hydrostatic force on one side of a plane

Consider a purely arbitrary body shaped plane surface submerged in water.



This plane surface is normal to the plain of this paper and is kept inclined at an angle of θ from the horizontal water surface.

Our objective is to find the hydrostatic force on one side of the plane surface that is submerged.



(Source: Fluid Mechanics by F.M. White)

Let ‘h’ be the depth from the free surface to any arbitrary element area ‘dA’ on the plane.

Pressure at h(x,y) will be

$p = p_a + \rho g h$ where, p_a = atmospheric pressure.

For our convenience to make various points on the plane, we have taken the x-y coordinates accordingly.

We also introduce a dummy variable ξ that show the inclined distance of the arbitrary element area dA from the free surface.

The total hydrostatic force on one side of the plane is

$F = \int_A p\,dA$ where, ‘A’ is the total area of the plane surface.

This hydrostatic force is similar to application of continuously varying load on the plane surface (recall solid mechanics).

We can find the resultant of hydrostatic force on the one side of the plane as such:

$$F = \int (p_a + \rho g h)\,dA$$
$$= p_a A + \rho g \int h\,dA$$

Now from the fig, we can see that $h=\xi \sin\theta$. From the theory of first moment of inertia

$\int hdA = \sin\theta \int \xi\, dA$; $\frac{1}{A}\int \xi dA = \xi_{CG}$

where ξ_{CG} is centroid slant distance.

As the plate is inclined and not moving, the angle θ will be contact:

$F = p_a A + \rho g^*\sin\theta \int \xi dA$

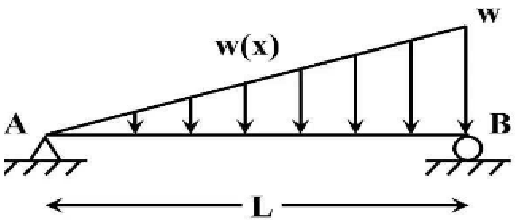
$= p_a A + (\rho g^* \sin\theta)^* \xi_{CG} {}^*A$

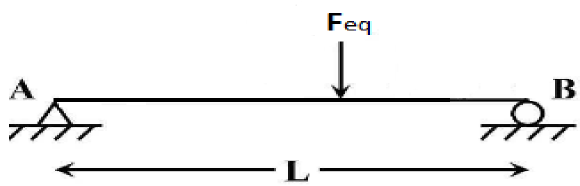
Again $\xi_{CG} \sin\theta = h_{CG}$ = the depth straight down from the surface to the plate centroid.

$F = (p_a + \rho g\, h_{CG})A$

$F = p_{CG} A$

The resultant force will be hydrostatic pressure at the area's centroid multiplied by the area.
As studied in the solid mechanics the equivalent force of the varying load will be applied at a distance 'l' from joint B





In a similar way the equivalent hydrostatic force on the one side of the plane will be acting at a point is called center of pressure.
The magnitude of equivalent force, however, will be

$F = p_{CG} A$

(Note that area centroid h_{CG} and the center of pressure are not the same.)

To find the location of center of pressure we can find them through the plane coordinate (x,y) and assuming the origin of the coordinate at the centroid of the area. Let (xcp , ycp) be the point of center of pressure,

Taking moment about centroid

$Fy_{cp} = \int ypdA = \int y(p_a + \rho g\times \xi\sin\theta)dA$

$= \rho g \sin\theta \int y\times\xi dA + \int y\times p_a dA$

As p_a is constant, $p_a \int ydA = p_a \times 0 = 0$

(since, $\int ydA = 0$; the first moment about centroid)

$Fy_{cp} = \rho g \sin\theta \int y \times \xi dA$

Also, $\xi = \xi_{CG} - y$ and

$\xi \sin\theta = h$

$Fy_{cp} = \rho g \sin\theta\, [\xi_{CG} \int ydA - \int y^2 dA]$

$Fy_{cp} = -\rho g \sin\theta \int y^2 dA$

$Fy_{cp} = -\rho g \sin\theta\ I_{xx}$

where, I_{xx} = second moment of inertia of the plane area about its centroid x-axis.

$y_{cp} = -\rho g \sin\theta \frac{Ixx}{p_{CG}A}$

The negative sign indicate that the center of pressure usually lies below the centroid of the area. Similarly, to determine the x-coordinate of the center of pressure.

$Fx_{cp} = \int xpdA = \int x(p_a + \rho g^*(\xi_{CG} - y)\sin\theta)dA$

(since $\int xdA = 0$, $\int ydA = 0$; the first moment about centroid)

$Fx_{cp} = -\rho g \sin\theta \int xydA$

$Fx_{cp} = -\rho g \sin\theta\ I_{xy}$

where, I_{xy} = product of inertia of the plane.

$x_{cp} = -\rho g \sin\theta \frac{Ixy}{p_{CG}A}$