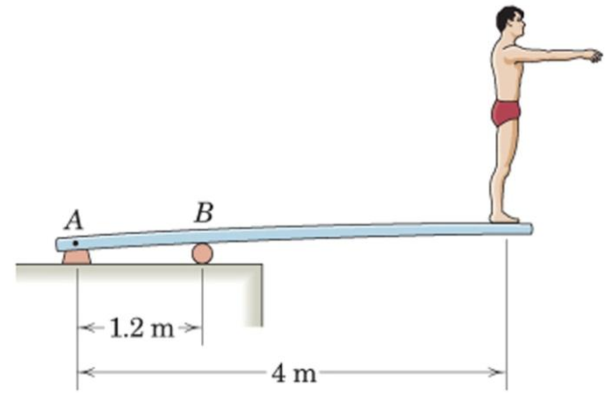
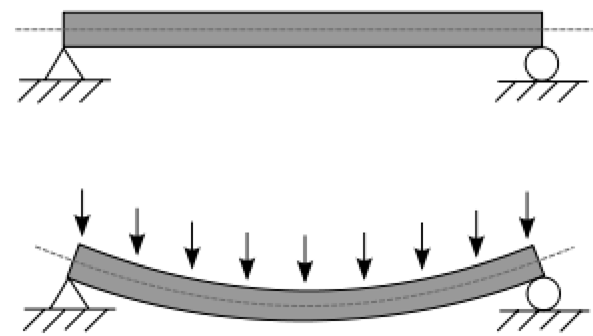
Beams



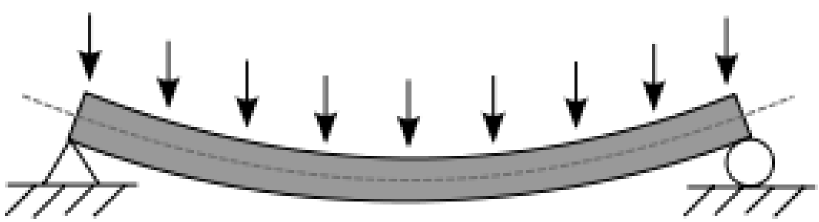


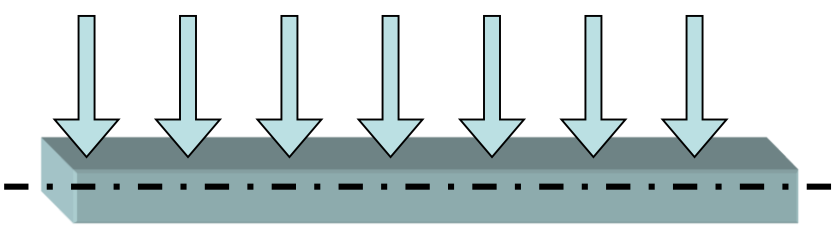
Beams are structural members that offer resistance to bending due to applied load

Beams

- Long prismatic members
  - Non-prismatic sections also possible
- Each cross-section dimension << Length of member

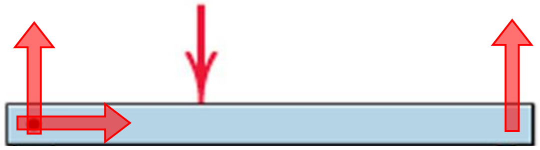
- Loading $\perp^r$ to the member axis





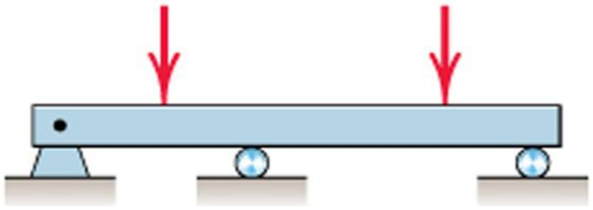
Beams

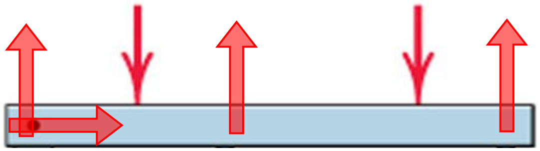
- Determinacy
  - Statically **determinate** beam
    - Only equilibrium equations required to obtain support reactions



3 equilibrium eqs. (**1 redundant**)

  - Statically **indeterminate** beam
    - Deformability required to obtain support reactions

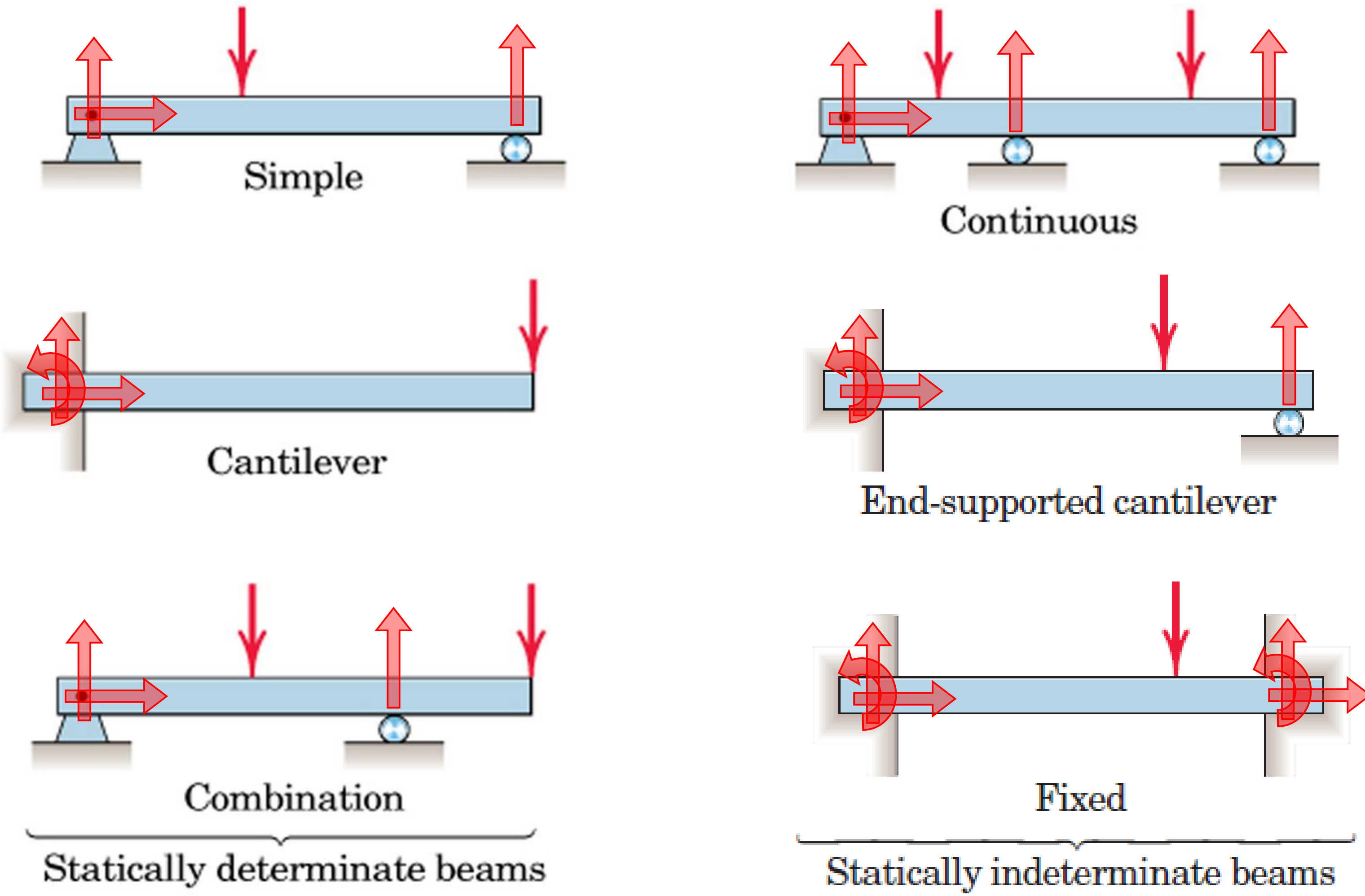




3 equilibrium eqs. (**insufficient**)

Types of Beams

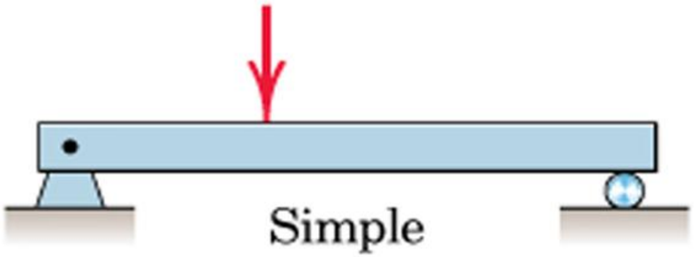
- Based on support conditions



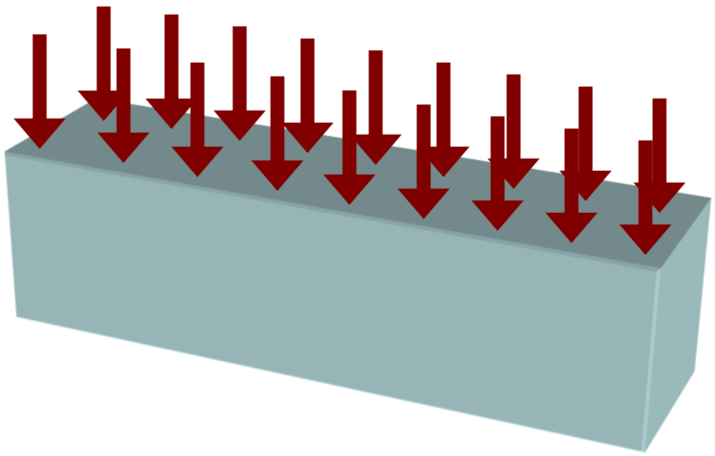
Propped Cantilever

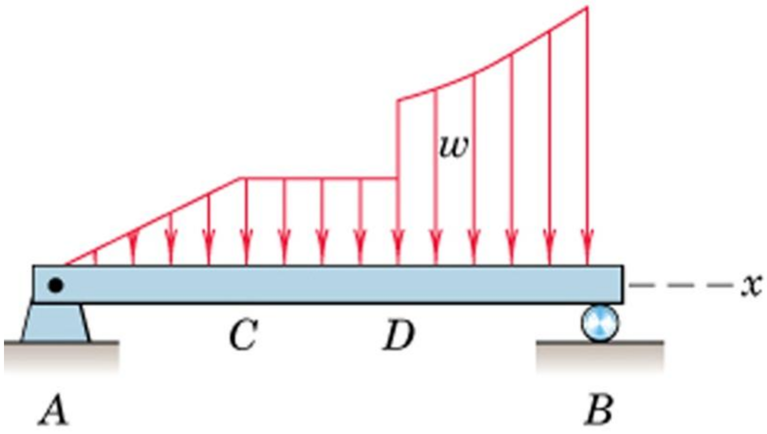
Types of Beams

- Based on pattern of external loading
  - **Concentrated** load



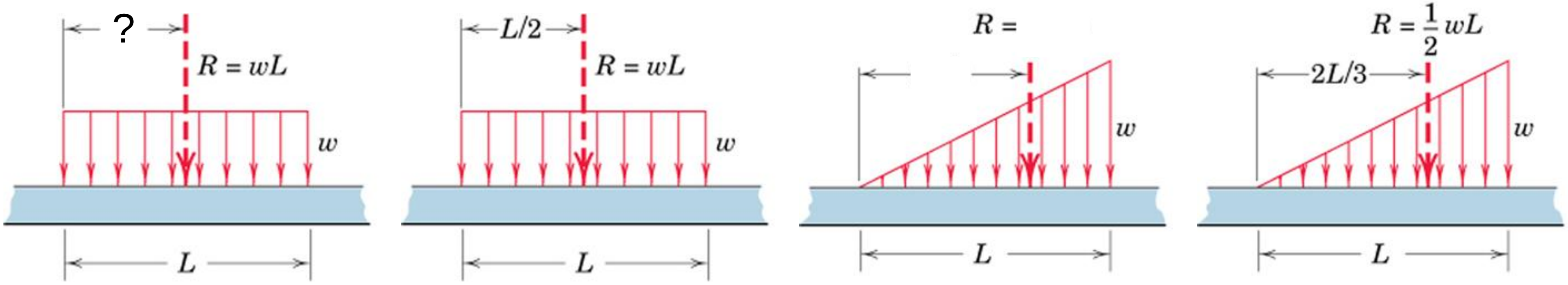
  - **Distributed** load
    - **Intensity** (w) expressed **force** per **unit length** of beam

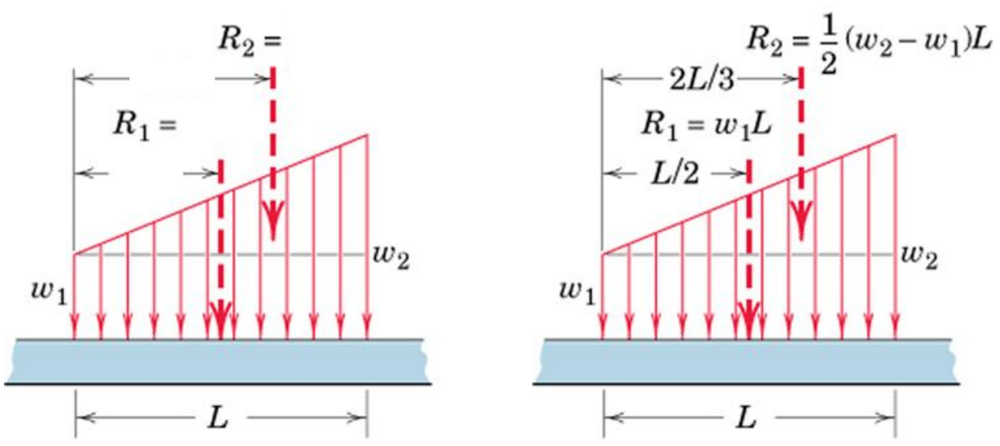




Beams

Resultant force (R) on beams





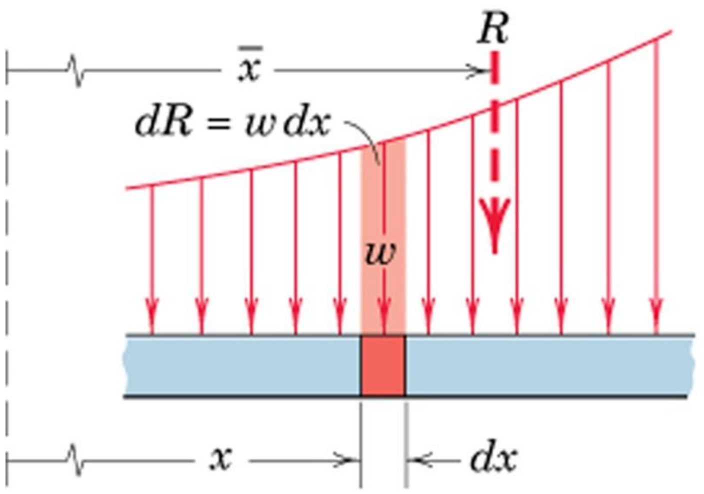
R :: area formed by w and length L over which the load is distributed

: *R* passes through centroid of this area

Beams

Distributed Loads on beams

: General Load Distribution



Differential increment of force is

dR = w dx

Total load *R* is sum of all the differential forces

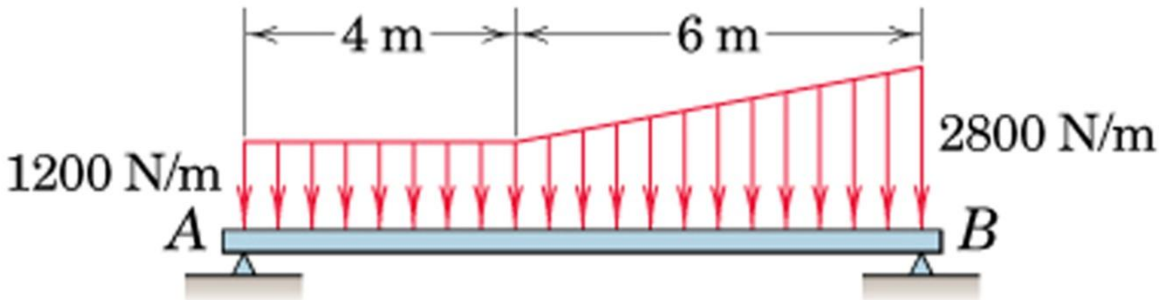
$R = \int w\,dx$ acting at centroid of the area under consideration

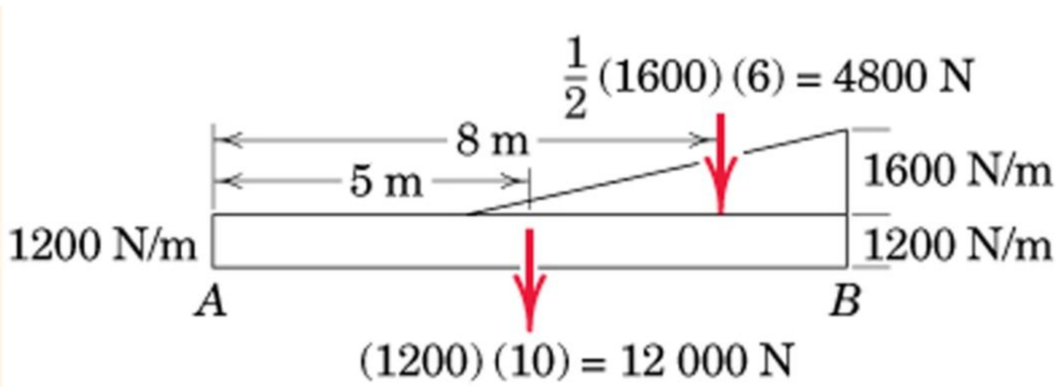
$$\bar{x} = \frac{\int xw\,dx}{R}$$

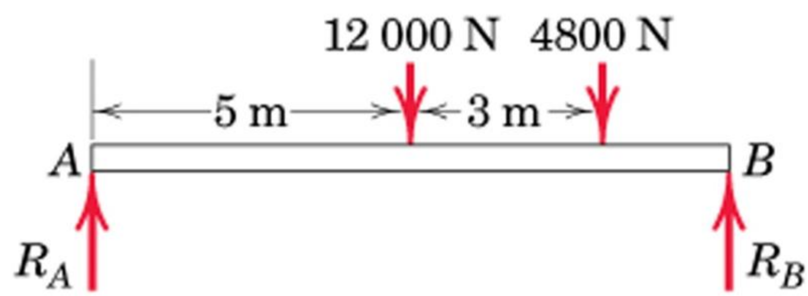
Once *R* is known reactions can be found out from Statics

Beams: Example

Determine the external reactions for the beam



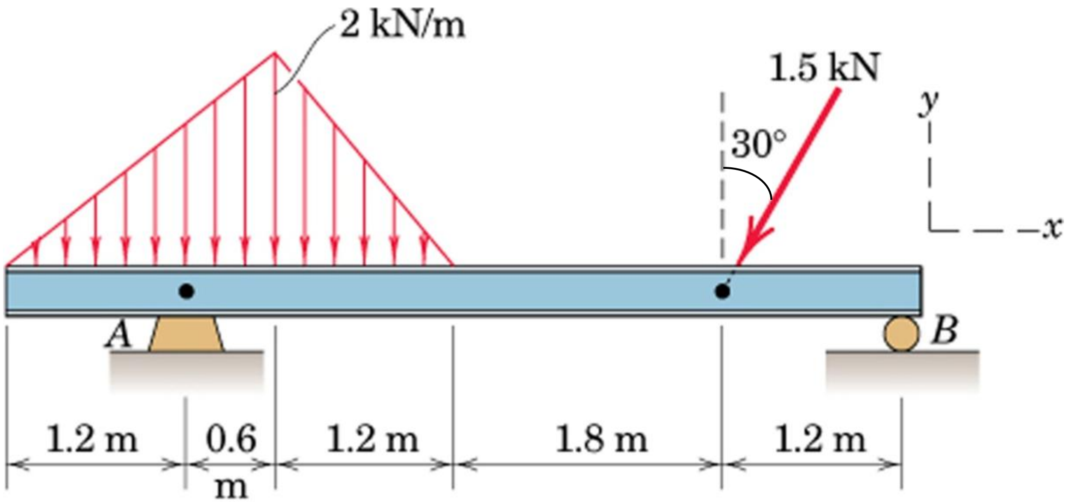




R_A = 6.96 kN, R_B = 9.84 kN

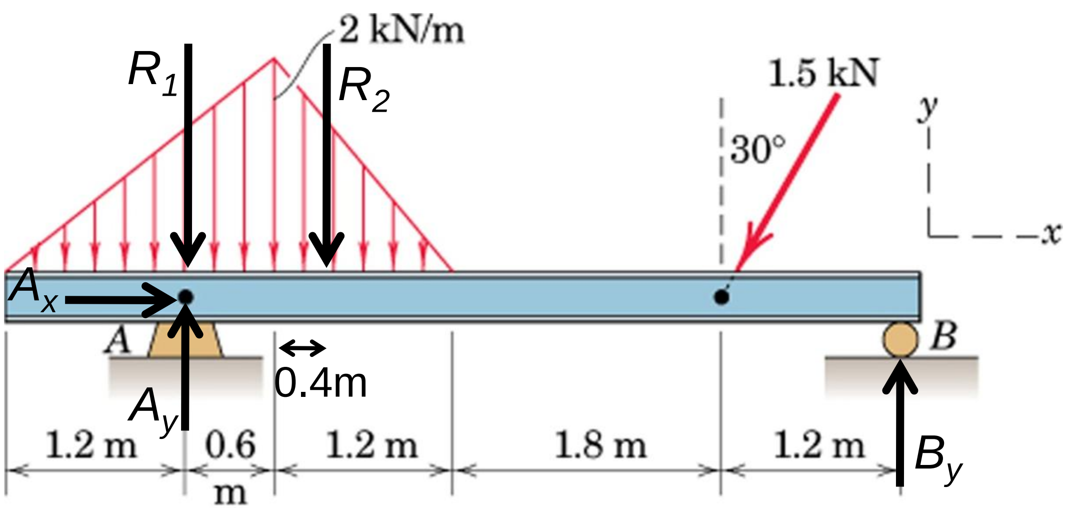
Beams: Example

Determine the external reactions for the beam



Dividing the un-symmetric triangular load into two parts with resultants R_1 and R_2 acting at point A and 1m away from point A, respectively.

$R_1 = 0.5\text{x}1.8\text{x}2 = 1.8$ kN
$R_2 = 0.5\text{x}1.2\text{x}2 = 1.2$ kN



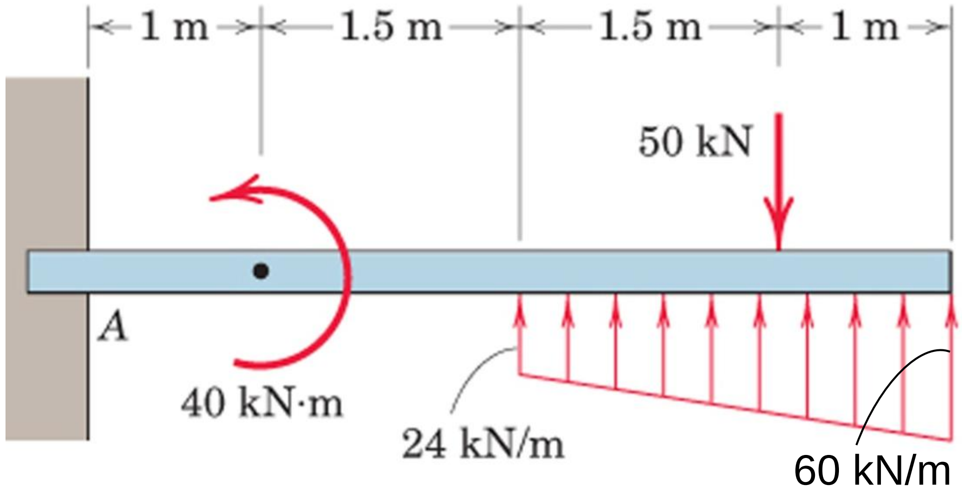
$\sum F_x=0 \rightarrow$ $\boldsymbol{A_x} = 1.5\sin30 =$ **0.75 kN**

$\sum M_A=0 \rightarrow$
$4.8\text{x}B_y = 1.5\cos30\text{x}3.6 + 1.2\text{x}1.0$
$\boldsymbol{B_y}$ **= 1.224 kN**

$\sum F_y=0 \rightarrow$
$A_y = 1.8+1.2+1.5\cos30-1.224$
$\boldsymbol{A_y}$ **= 3.075 kN**

Beams: Example

Determine the external reactions for the beam

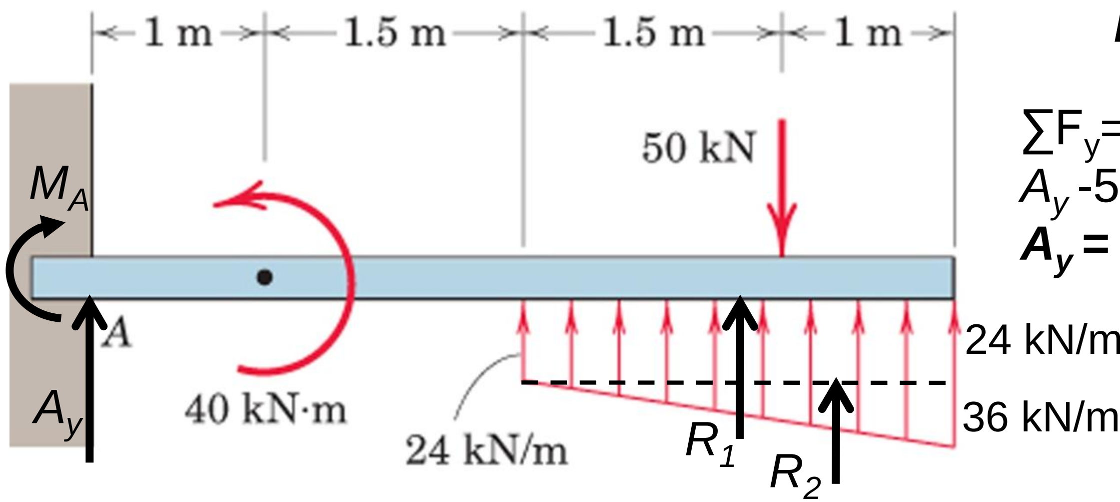


Dividing the trapezoidal load into two parts with resultants R_1 and R_2

R_1 = 24x2.5 = 60 kN @ 3.75m → A
R_2 = 0.5x2.5x36=45 kN@4.17m → A
(distances from A)

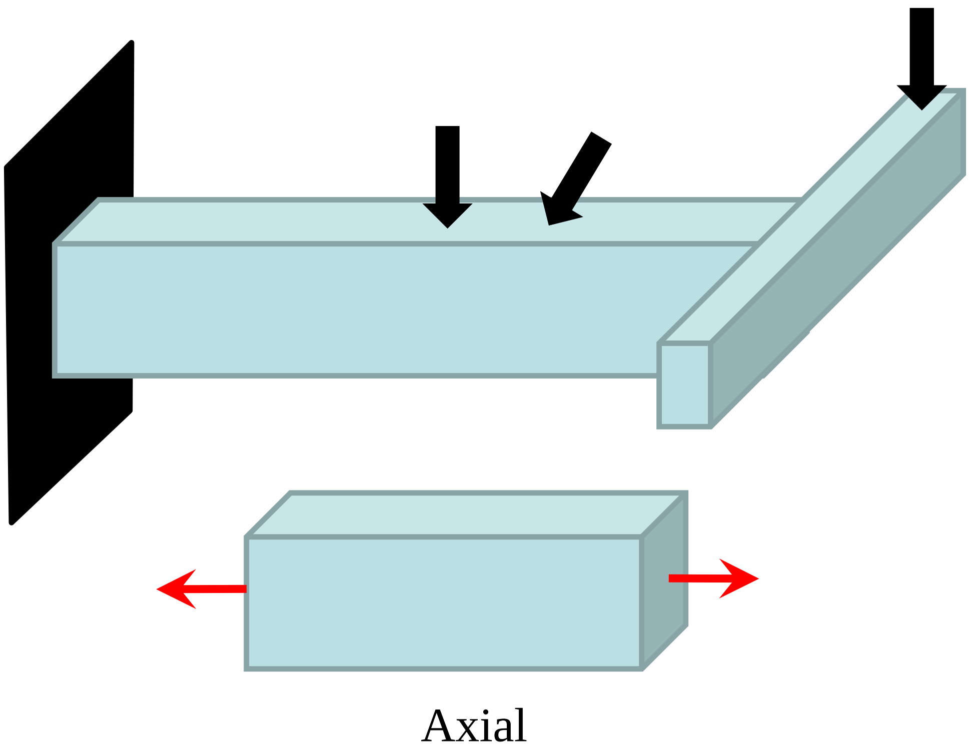
$\sum M_A=0$ →
M_A-40+50x4.0-60x3.75-45x4.17=0
M_A = 253 kNm

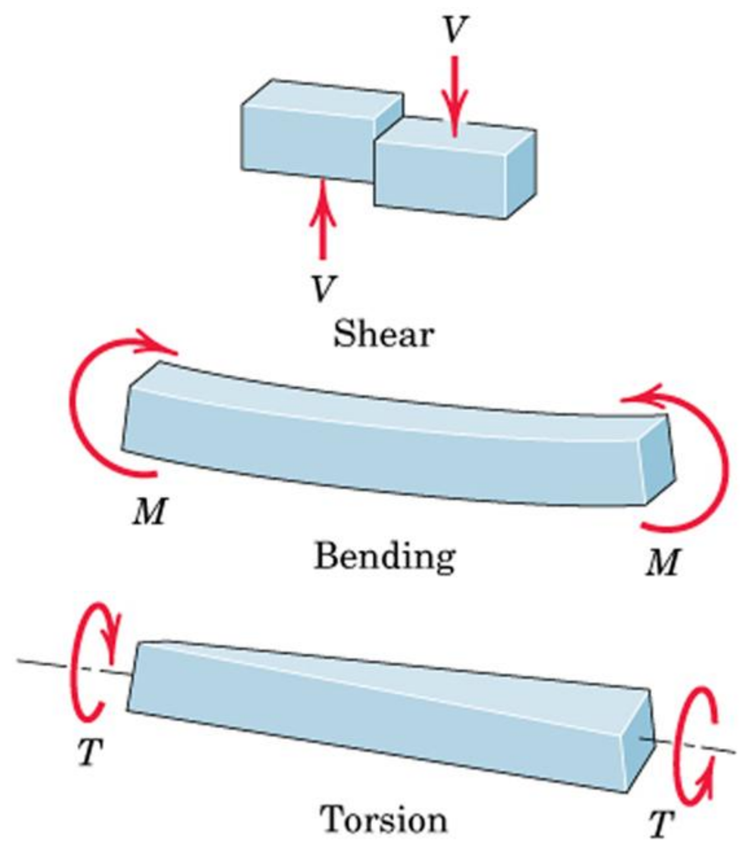
$\sum F_y=0$ →
A_y -50+60+45 = 0
A_y = -55 kN → Downwards



Beams – Internal Effects

- Internal Force Resultants
- Axial Force (*N*), Shear Force (*V*), Bending Moment (*M*), Torsional Moment (*T*) in Beam
  - Method of Sections is used

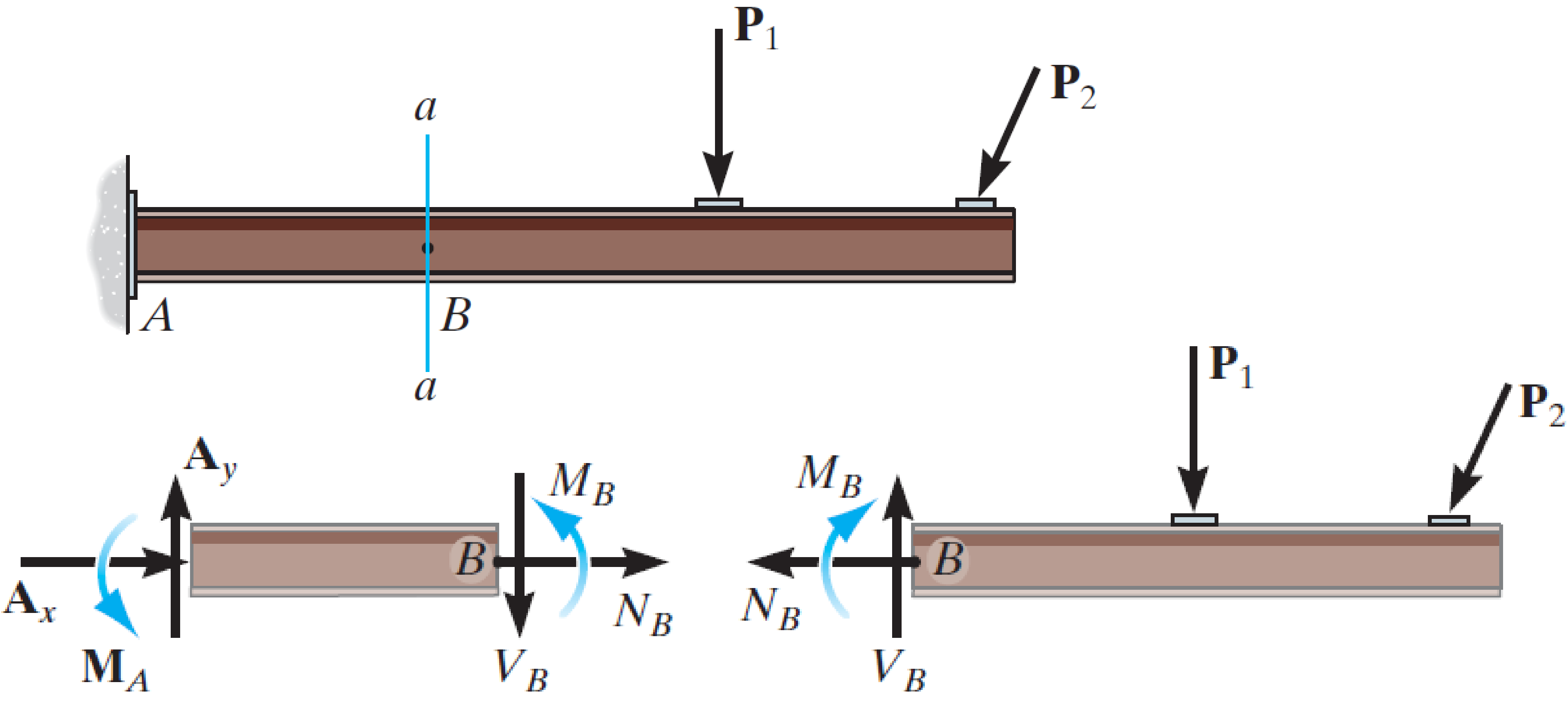




Beams – Internal Effects

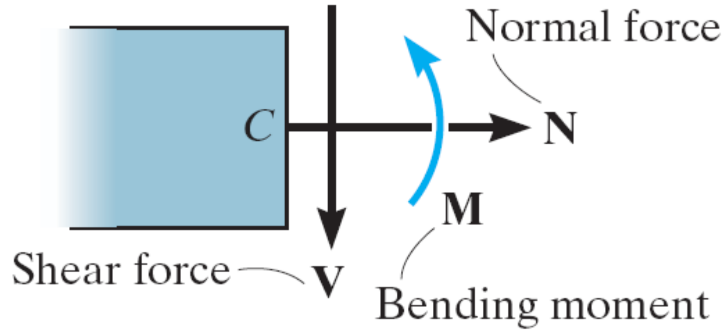
- Method of Section:

  Internal Force Resultants at B → Section a-a at B and use equilibrium equations in both cut parts

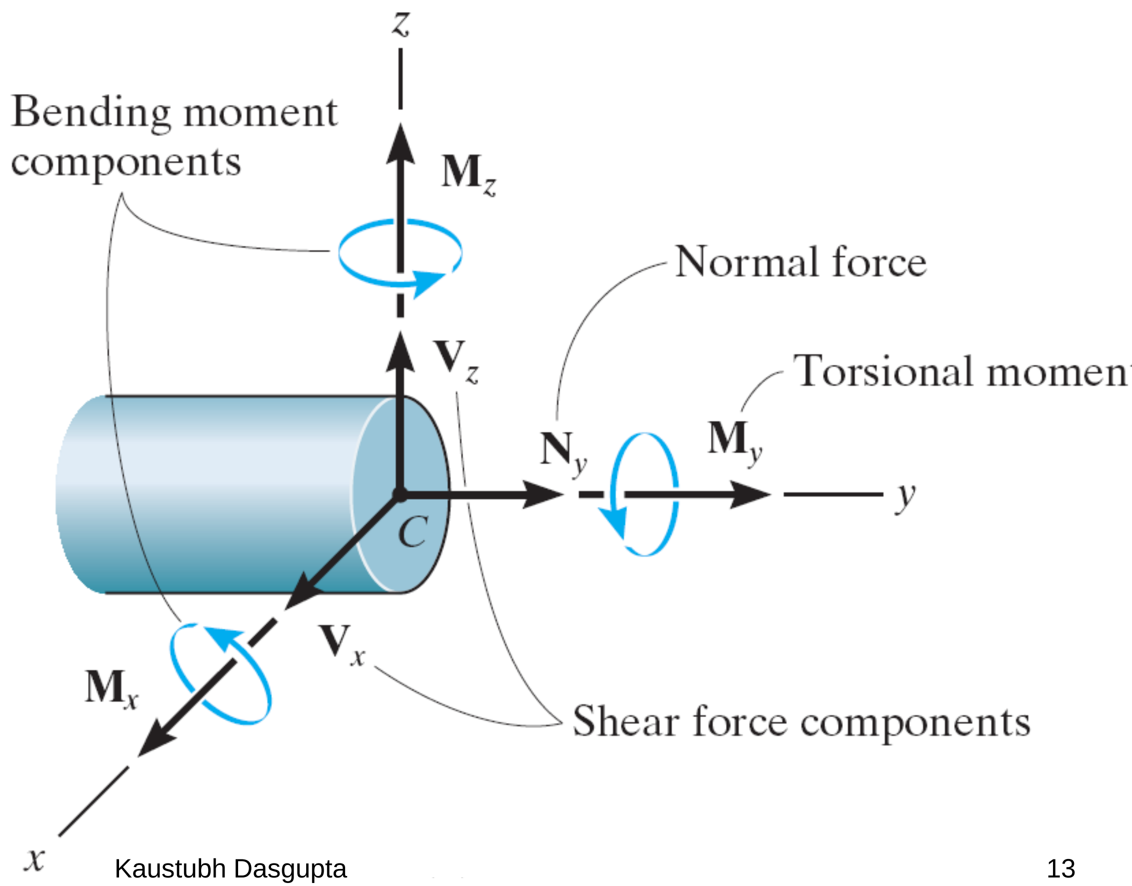


Beams – Internal Effects

- 2D Beam



- 3D Beam

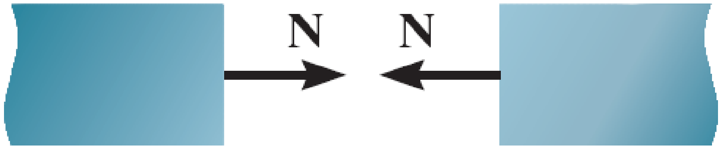


The **Force Resultants** act at the centroid of the section

Beams – Internal Effects

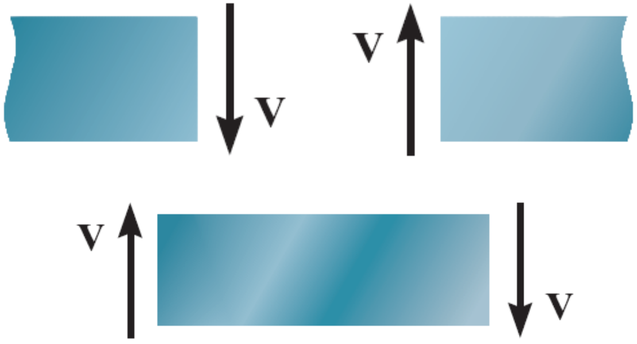
Sign Convention

Positive Axial Force creates Tension



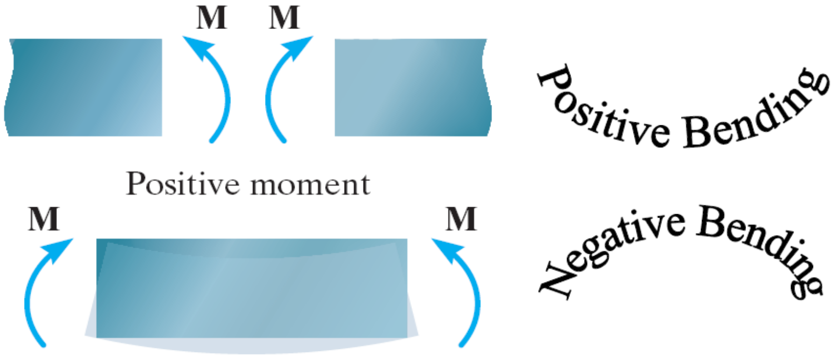
Positive normal force

Positive shear force will cause the Beam segment on which it acts to rotate clockwise



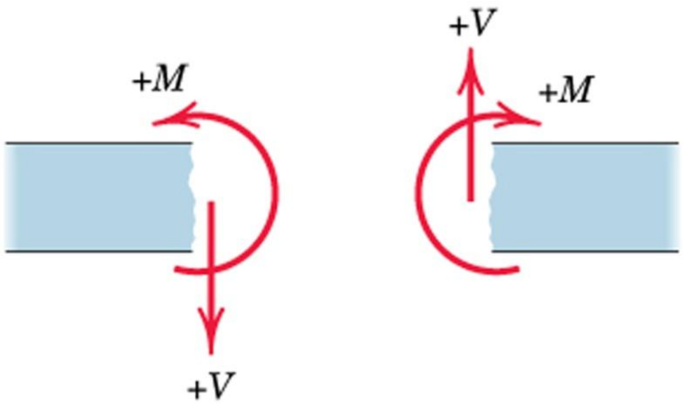
Positive shear

Positive bending moment will tend to bend the segment on which it acts in a concave upward manner (compression on top of section).

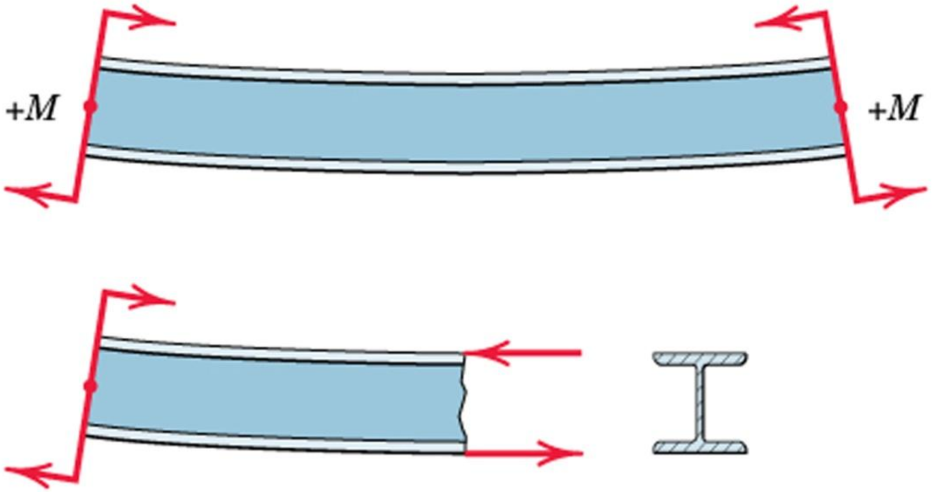


Beams – Internal Effects

Sign convention in a single plane



Interpretation of Bending Couple



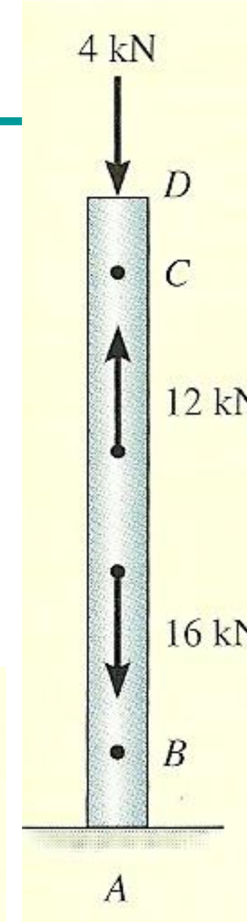
H-section Beam bent by two equal and opposite positive moments applied at ends

Neglecting resistance offered by web

Compression at top; Tension at bottom

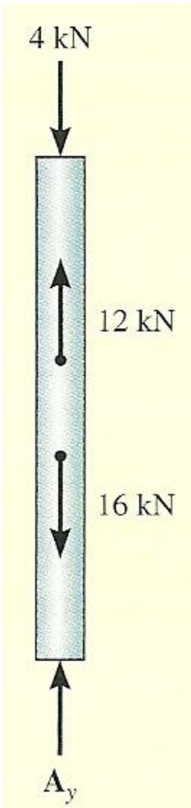
Resultant of these two forces (one tensile and other compressive) acting on any section is a Couple and has the value of the Bending Moment acting on the section.

Beams – Internal Effects



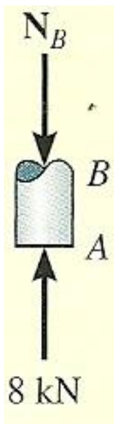
Example: Find the axial force in the fixed bar at points B and C

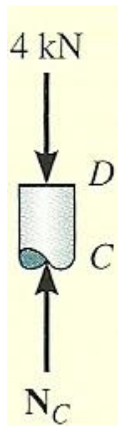
Solution: Draw the FBD of the entire bar



$$+\uparrow \Sigma F_y = 0; \quad A_y - 16\text{ kN} + 12\text{ kN} - 4\text{ kN} = 0 \quad A_y = 8\text{ kN}$$

Draw sections at B and C to get AF in the bar at B and C





Segment AB

$$+\uparrow \Sigma F_y = 0; \quad 8\text{ kN} - N_B = 0 \quad N_B = 8\text{ kN}$$

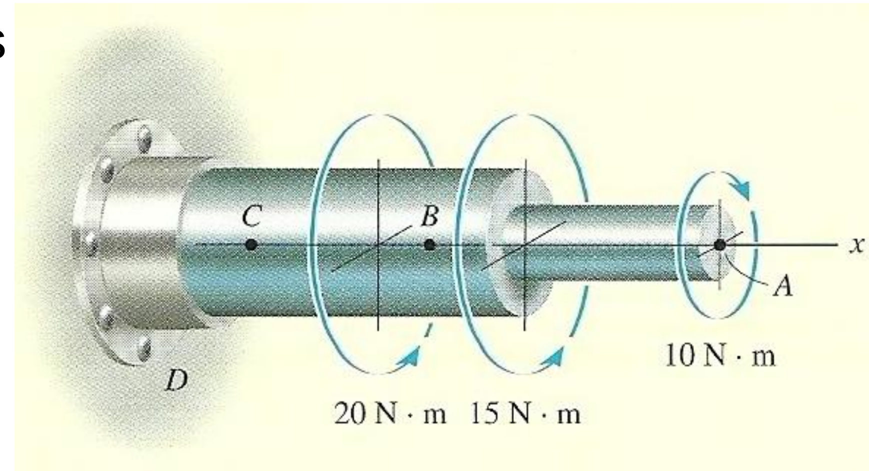
Segment DC

$$+\uparrow \Sigma F_y = 0; \quad N_C - 4\text{ kN} = 0 \quad N_C = 4\text{ kN}$$

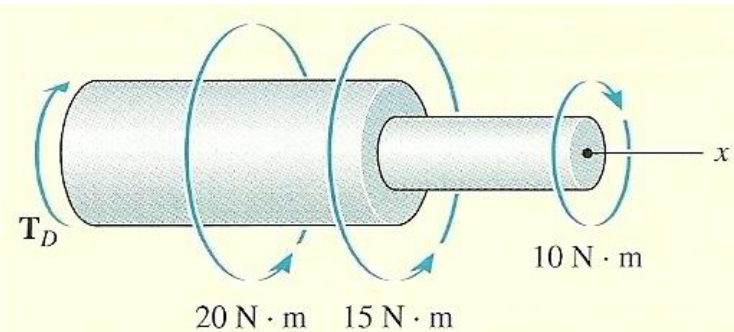
Alternatively, take a section at C and consider only CD portion of the bar
Then take a section at B and consider only BD portion of the bar
→ no need to calculate reactions

Beams – Internal Effects

Example: Find the internal torques at points B and C of the circular shaft subjected to three concentrated torques



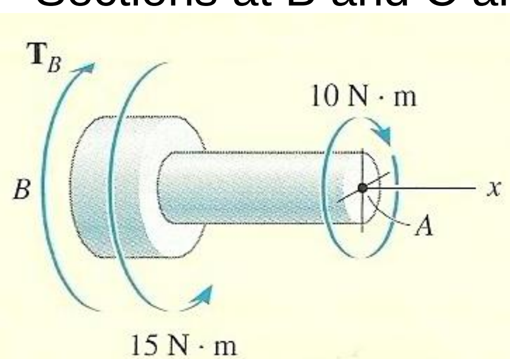
Solution: FBD of entire shaft

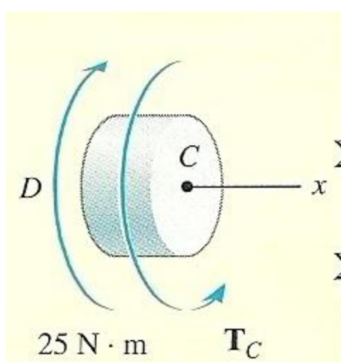


$$\Sigma M_x = 0; \quad -10\ \text{N}\cdot\text{m} + 15\ \text{N}\cdot\text{m} + 20\ \text{N}\cdot\text{m} - T_D = 0$$

$$T_D = 25\ \text{N}\cdot\text{m}$$

Sections at B and C and FBDs of shaft segments AB and CD





Segment AB

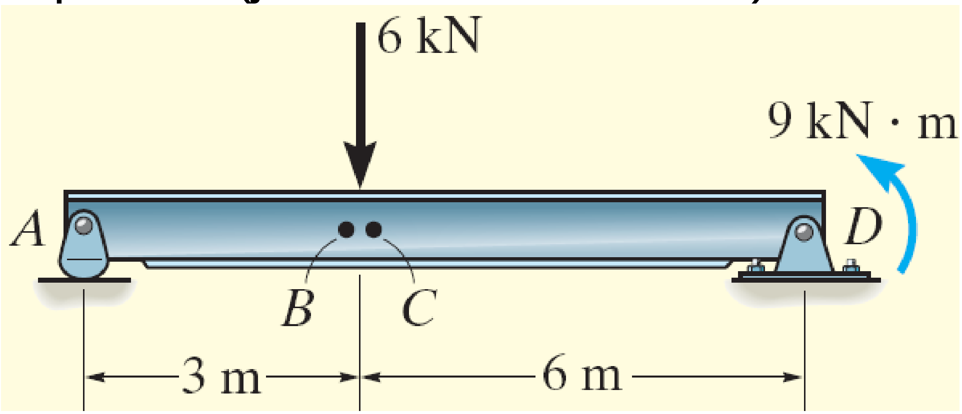
$$\Sigma M_x = 0; \quad -10\ \text{N}\cdot\text{m} + 15\ \text{N}\cdot\text{m} - T_B = 0 \qquad T_B = 5\ \text{N}\cdot\text{m}$$

Segment CD

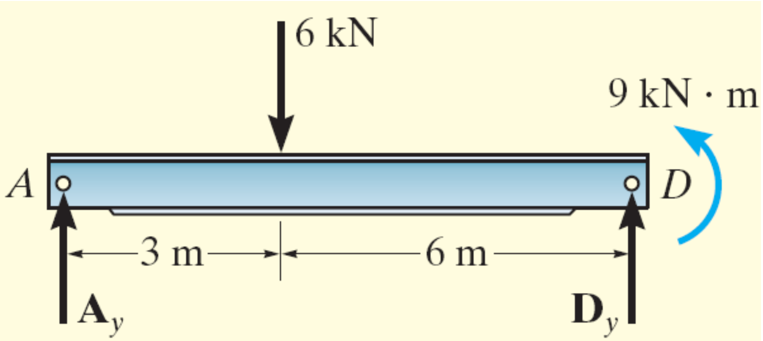
$$\Sigma M_x = 0; \quad T_C - 25\ \text{N}\cdot\text{m} = 0 \qquad T_C = 25\ \text{N}\cdot\text{m}$$

Beams – Internal Effects

Example: Find the AF, SF, and BM at point B (just to the left of 6 kN) and at point C (just to the right of 6 kN)



Solution: Draw FBD of entire beam

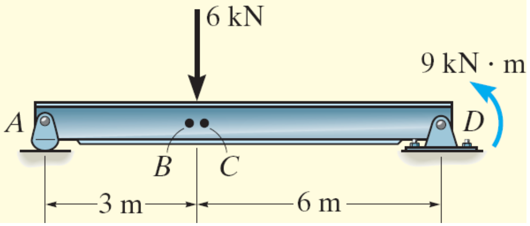


$$\circlearrowleft + \Sigma M_D = 0;\ 9\ \text{kN}\cdot\text{m} + (6\ \text{kN})(6\ \text{m}) - A_y(9\ \text{m}) = 0$$

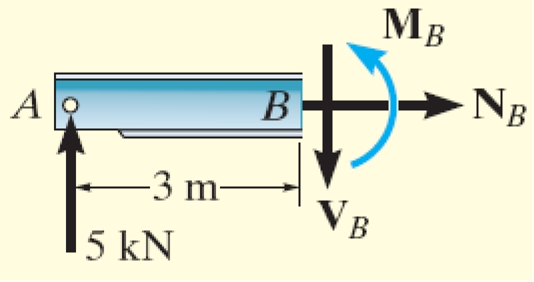
$$A_y = 5\ \text{kN}$$

D_y need not be determined if only left part of the beam is analysed

Beams – Internal Effects



Example Solution: Draw FBD of segments AB and AC and use equilibrium equations

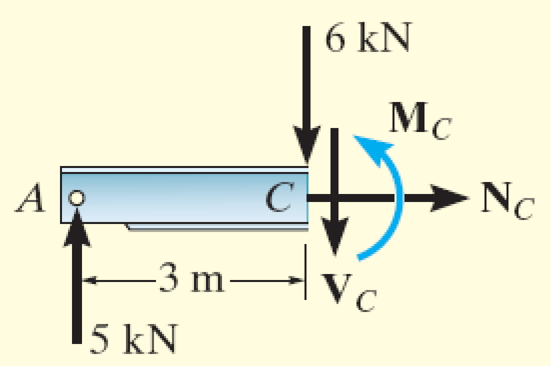


Segment AB

$\xrightarrow{+} \Sigma F_x = 0; \quad N_B = 0$

$+\uparrow \Sigma F_y = 0; \quad 5\text{ kN} - V_B = 0 \quad V_B = 5\text{ kN}$

$\circlearrowleft + \Sigma M_B = 0; \quad -(5\text{ kN})(3\text{ m}) + M_B = 0 \quad M_B = 15\text{ kN}\cdot\text{m}$



Segment AC

$\xrightarrow{+} \Sigma F_x = 0; \quad N_C = 0$

$+\uparrow \Sigma F_y = 0; \quad 5\text{ kN} - 6\text{ kN} - V_C = 0 \quad V_C = -1\text{ kN}$

$\circlearrowleft + \Sigma M_C = 0; \quad -(5\text{ kN})(3\text{ m}) + M_C = 0 \quad M_C = 15\text{ kN}\cdot\text{m}$

Beams – SFD and BMD

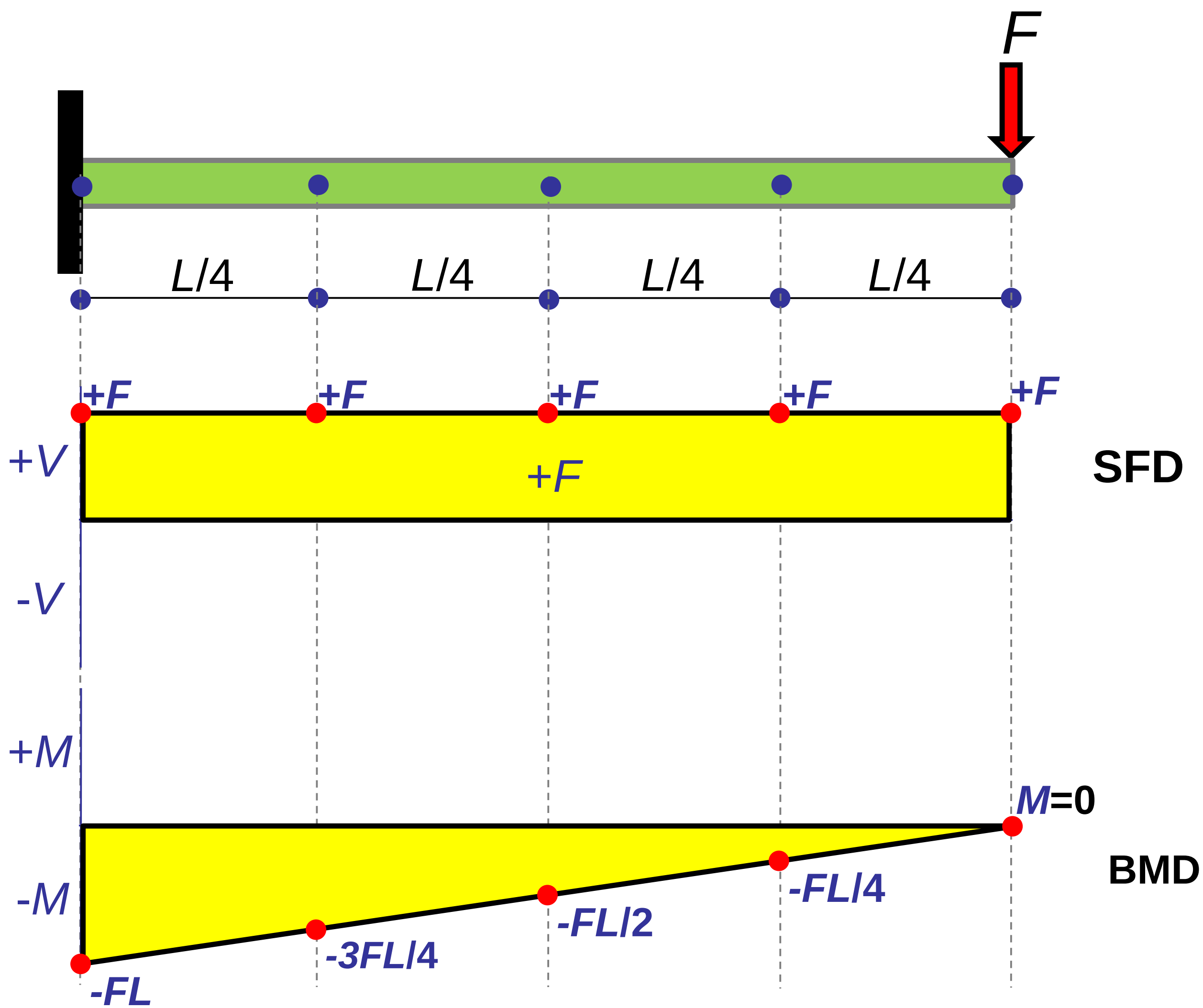
Shear Force Diagram and bending Moment Diagram

- Variation of SF and BM over the length of the beam → SFD and BMD
- Maximum magnitude of BM and SF and their locations is prime consideration in beam design
- SFDs and BMDs are plotted using method of section
  – Equilibrium of FBD of entire Beam
    → External Reactions
  – Equilibrium of a cut part of beam
    → Expressions for SF and BM at the cut section

Use the positive sign convention consistently

Beams – SFD and BMD

Draw SFD and BMD for a cantilever beam supporting a point load at the free end



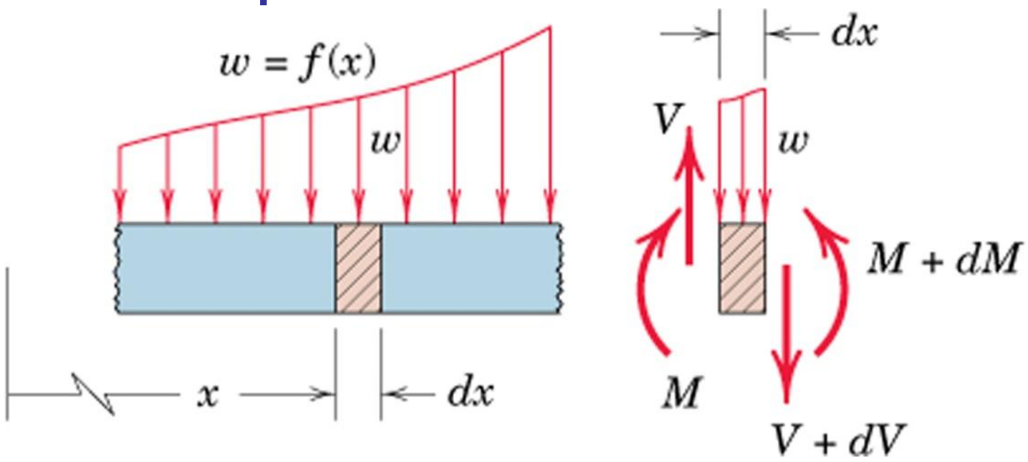
Beams – SFD and BMD

Shear and Moment Relationships

Consider a portion of a beam

Isolate an element *dx*

Draw FBD of the element



Vertical equilibrium in *dx* →

$$V - w\,dx - (V + dV) = 0 \qquad w = -\frac{dV}{dx}$$

Moment equilibrium in *dx* (∑*M*@ the left side of the element)→

$$M + wdx\frac{dx}{2} + (V + dV)dx - (M + dM) = 0 \qquad V = \frac{dM}{dx}$$

Terms $w(dx)^2/2$ and $dVdx$ may be dropped since they are differentials of higher order than those which remains.

Beams – SFD and BMD

Shear and Moment Relationships

$$w = -\frac{dV}{dx}$$

Slope of the shear diagram = - Value of applied loading

$$V = \frac{dM}{dx}$$

Slope of the moment curve = Shear Force

Both equations not applicable at the point of loading because of discontinuity produced by the abrupt change in shear.

Beams – SFD and BMD

Shear and Moment Relationships

Expressing *V* in terms of *w* by integrating $w = -\frac{dV}{dx}$

$$\int_{V_0}^{V} dV = -\int_{x_0}^{x} w\,dx$$

OR **$V = V_0$ + (the negative of the area under the loading curve from x_0 to x)**

V_0 is the shear force at x_0 and V is the shear force at x

Expressing *M* in terms of *V* by integrating $V = \frac{dM}{dx}$

$$\int_{M_0}^{M} dM = \int_{x_0}^{x} V\,dx$$

OR **$M = M_0$ + (area under the shear diagram from x_0 to x)**

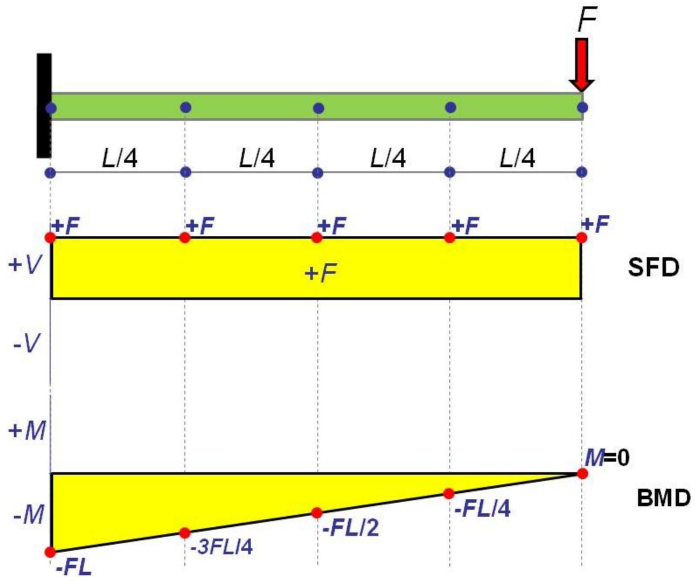
M_0 is the BM at x_0 and M is the BM at x

Beams – SFD and BMD

$V = V_0$ **+ (negative of area under the loading curve from** x_0 **to** x**)**

$M = M_0$ **+ (area under the shear diagram from** x_0 **to** x**)**

If there is no externally applied moment M_0 at $x_0 = 0$, total moment at any section equals the area under the shear diagram up to that section



When V passes through zero and is a continuous function of x with $dV/dx \neq 0$ (i.e., nonzero loading)

→ $$\frac{dM}{dx} = 0$$

→ BM will be a maximum or minimum at this point

Critical values of BM also occur when SF crosses the zero axis discontinuously (Example: Beams under concentrated loads)

Beams – SFD and BMD

$$w = -\frac{dV}{dx}$$ Degree of *V* in *x* is one higher than that of *w*

$$V = \frac{dM}{dx}$$ Degree of *M* in *x* is one higher than that of *V*

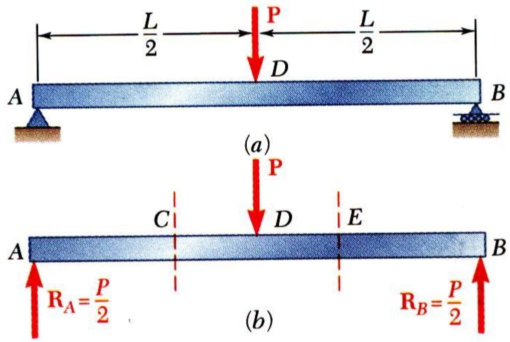
→ Degree of *M* in *x* is two higher than that of *w*

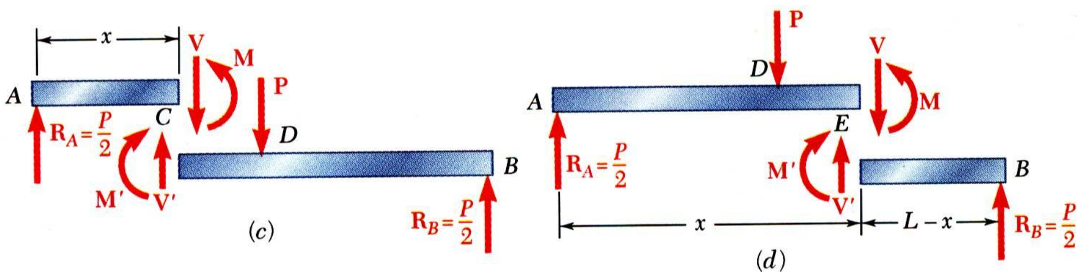
Combining the two equations → $$\frac{d^2M}{dx^2} = -w$$

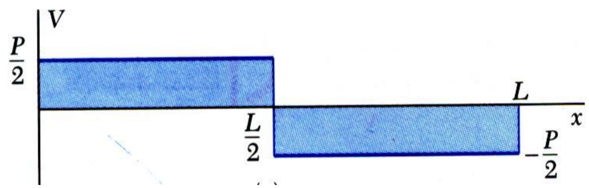
→ If *w* is a known function of *x*, BM can be obtained by integrating this equation twice with proper limits of integration.

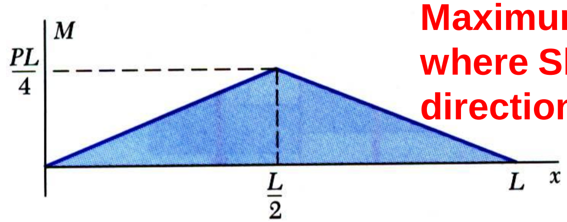
- → Method is usable only if *w* is a continuous function of *x* (other cases not part of this course)

Beams – SFD and BMD: Example









Maximum BM occurs where Shear changes the direction

- Draw the SFD and BMD.
- Determine reactions at supports.
- Cut beam at *C* and consider member *AC*,

$$V = +P/2 \quad M = +Px/2$$

- Cut beam at *E* and consider member *EB*,

$$V = -P/2 \quad M = +P(L-x)/2$$

- For a beam subjected to <u>concentrated loads</u>, shear is constant between loading points and moment varies linearly.