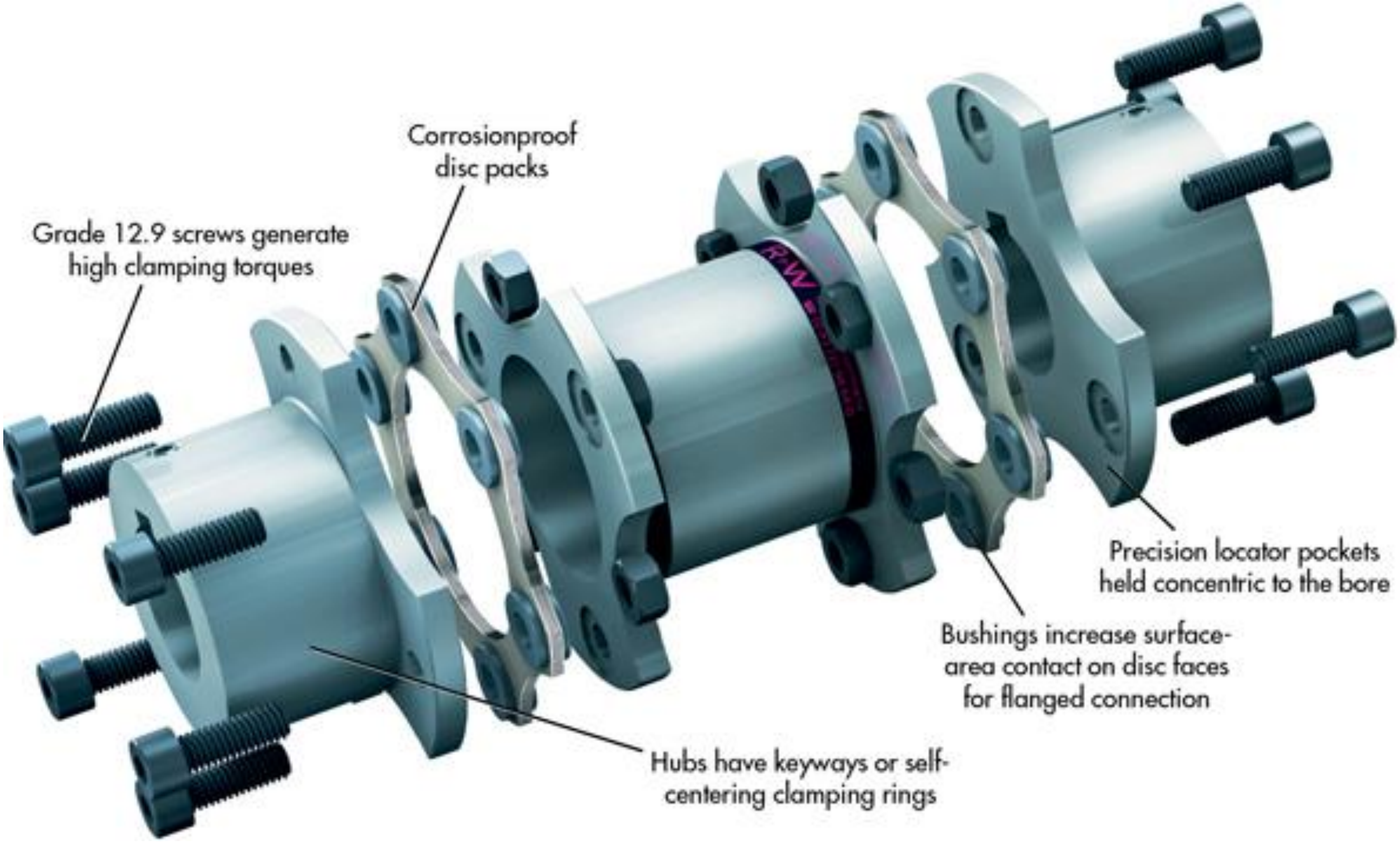
Friction in Machines :: Disc Friction

- Thrust Bearings

Sanding Machine

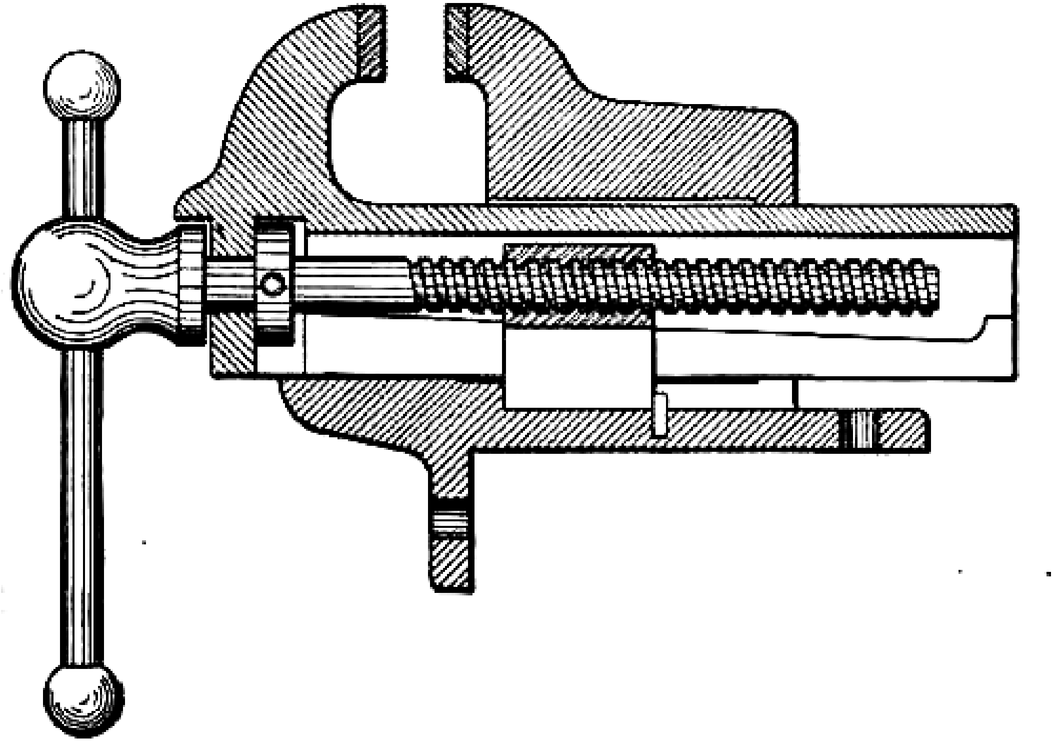
Friction in Machines :: Disc Friction

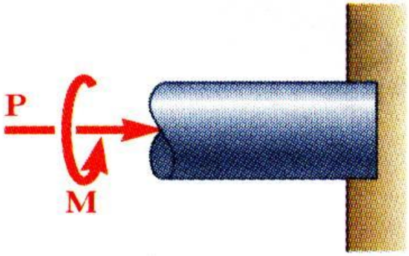
- Disc-Pack Coupling



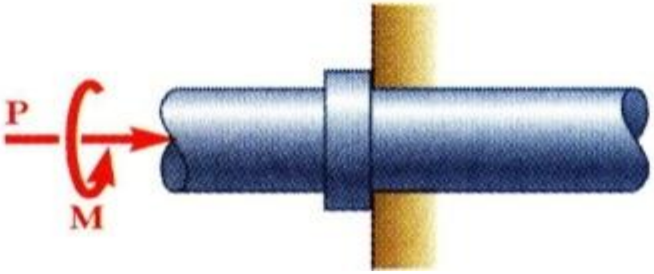
Friction in Machines :: Disc Friction

- Disc friction during screw motion???





End/Pivot Bearing



Collar Bearing

Friction in Machines :: Disc Friction

Thrust Bearings (Disk Friction)

- Thrust bearings provide axial support to rotating shafts.
- Axial load acting on the shaft is *P*.
- Friction between circular surfaces under distributed normal pressure (Ex: clutch plates, disc brakes)

Consider two flat circular discs whose shafts are mounted in bearings: they can be brought under contact under *P*

Max torque that the clutch can transmit =

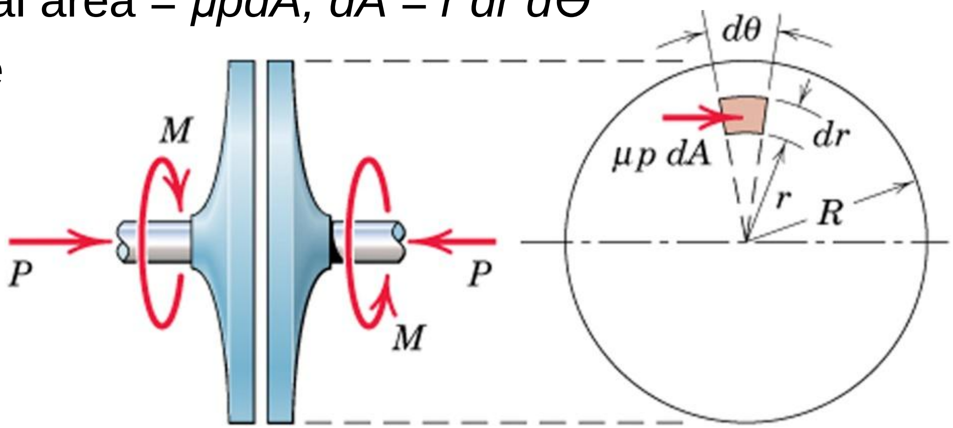
M required to slip one disc against the other

p is the normal pressure at any location between the plates

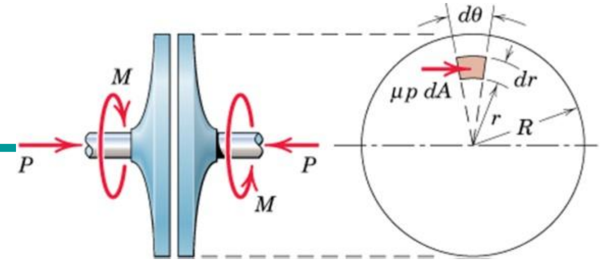
→ Frictional force acting on an elemental area = $\mu p dA$; $dA = r\, dr\, d\Theta$

Moment of this elemental frictional force about the shaft axis = $\mu p r dA$

Total $M = \int \mu p r dA$ over the area of disc



Friction in Machines :: Disc Friction



Thrust Bearings (Disk Friction)

Assuming that *μ* and *p* are uniform over the entire surface → $P = \pi R^2 p$

→ Substituting the constant *p* in $M = \int \mu p r dA$ → $M = \frac{\mu P}{\pi R^2} \int_0^{2\pi} \int_0^{R} r^2 dr\, d\theta$

→ $M = \frac{2}{3} \mu P R$ ➔ Magnitude of moment reqd for impending rotation of shaft

≈ moment due to frictional force ***μp*** acting a distance **⅔ *R*** from shaft center

Frictional moment for worn-in plates is only about ¾ of that for the new surfaces

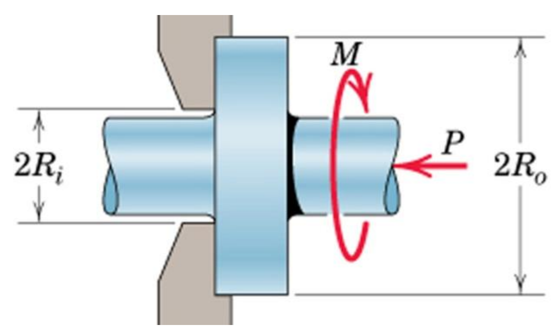
→ *M* for worn-in plates = $\frac{1}{2}(\mu P R)$

If the friction discs are rings (Ex: Collar bearings) with outside and inside radius as R_o and R_i, respectively (limits of integration R_o and R_i) → $P = \pi(R_o^2 - R_i^2)p$

→ The frictional torque: $M = \frac{\mu P}{\pi\left(R_o^2 - R_i^2\right)} \int_0^{2\pi} \int_{R_i}^{R_o} r^2 dr\, d\theta$

→ $M = \frac{2}{3} \mu P \frac{R_o^3 - R_i^3}{R_o^2 - R_i^2}$

Frictional moment for worn-in plates → $M = \frac{1}{2} \mu P(R_o + R_i)$

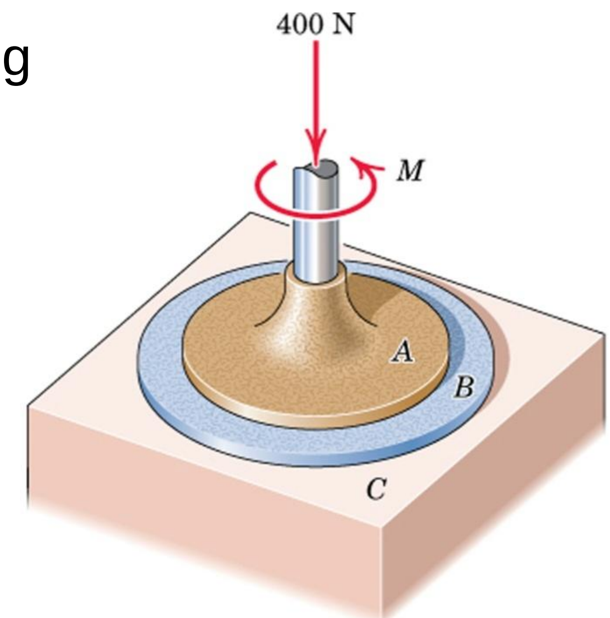


Example (1) on Disc Friction

Circular disk *A* (225 mm dia) is placed on top of disk *B* (300 mm dia) and is subjected to a compressive force of 400 N. Pressure under each disk is constant over its surface. Coeff of friction betn *A* and *B* = 0.4. Determine:

(a) the couple *M* which will cause *A* to slip on *B*.

(b) min coeff of friction *μ* between *B* and supporting surface *C* which will prevent *B* from rotating.



Solution: $M = \frac{2}{3}\mu P R$

(a) Impending slip between *A* and *B*:

μ=0.4, *P*=400 N, R=225/2 mm

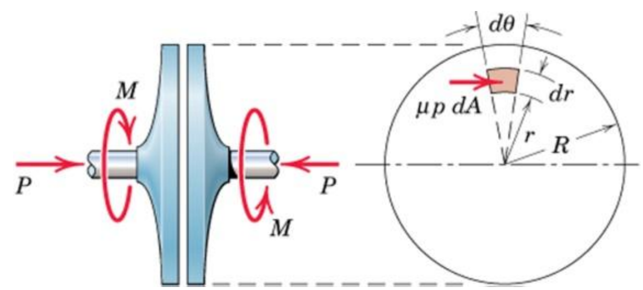
M = 2/3 x 0.4 x 400 x 0.225/2 → *M* = 12 Nm

(b) Impending slip between *B* and *C* :

Slip between *A* and *B* → *M* = 12 Nm

μ=? *P*=400 N, R=300/2 mm

12 = 2/3 x *μ* x 400 x 0.300/2 → *μ* = 0.3



Applications of Friction in Machines

Belt Friction

Impending slippage of flexible cables, belts, ropes over sheaves, wheels, drums

→ It is necessary to estimate the frictional forces developed between the belt and its contacting surface.

Consider a drum subjected to two belt tensions (T_1 and T_2)

M is the torque necessary to prevent rotation of the drum

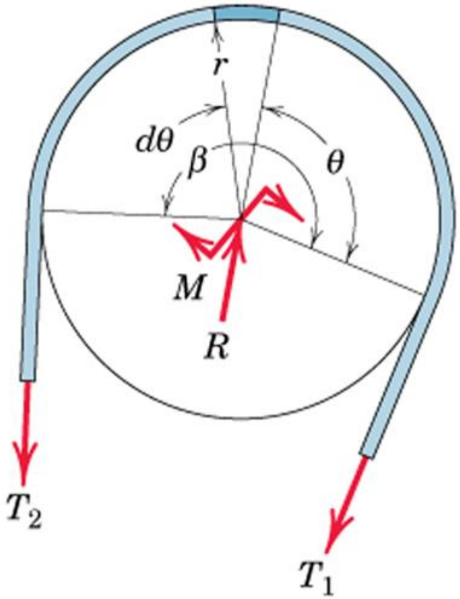
R is the bearing reaction

r is the radius of the drum

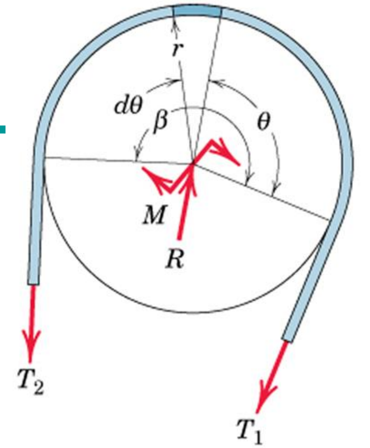
β is the total contact angle between belt and surface

(β in radians)

$T_2 > T_1$ since M is clockwise



Applications of Friction in Machines



Belt Friction: Relate T_1 and T_2 when belt is about to slide to left

Draw FBD of an element of the belt of length *r dθ*
Frictional force for impending motion = *μ dN*

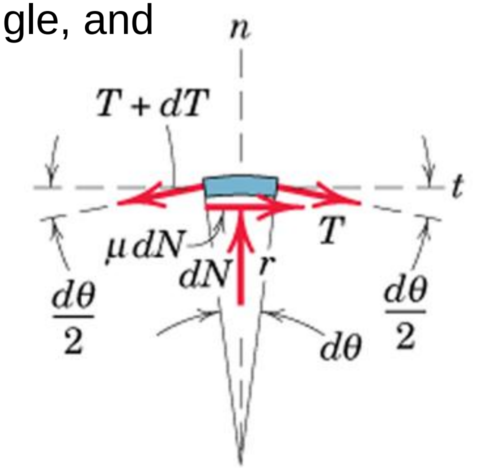
Equilibrium in the *t*-direction: $T \cos \frac{d\theta}{2} + \mu\, dN = (T + dT) \cos \frac{d\theta}{2}$

→ $\mu dN = dT$ (cosine of a differential quantity is unity in the limit)

(For the fig., $T_1 > T_2$)

Equilibrium in the *n*-direction: $dN = (T + dT) \sin \frac{d\theta}{2} + T \sin \frac{d\theta}{2}$

→ $dN = 2Td\theta/2 = Td\theta$ (sine of a differential in the limit equals the angle, and product of two differentials can be neglected)



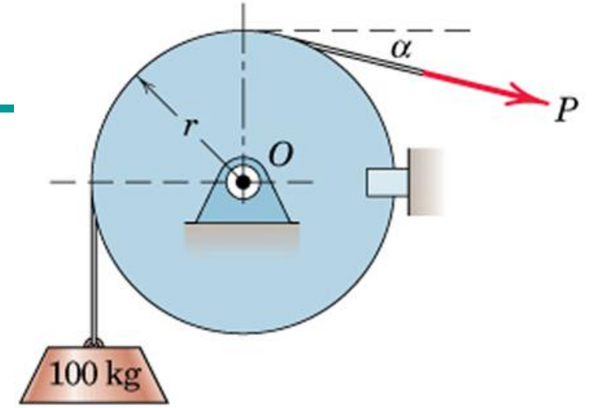
Combining two equations: $\frac{dT}{T} = \mu\, d\theta$

Integrating between corresponding limits: $\int_{T_1}^{T_2} \frac{dT}{T} = \int_0^{\beta} \mu\, d\theta$

→ $\ln \frac{T_2}{T_1} = \mu\beta$ → $T_2 = T_1 e^{\mu\beta}$ ($T_2 > T_1$; e = 2.718…; β in radians)

- Rope wrapped around a drum *n* times → $\beta = 2\pi n$ radians
- *r* not present in the above eqn → eqn valid for non-circular sections as well
- In belt drives, belt and pulley rotate at constant speed → the eqn describes condition of impending slippage.

Example on Belt Friction



A force P is reqd to be applied on a flexible cable that supports 100 kg load using a **fixed** circular drum.

μ between cable and drum = 0.3

(a)For $\alpha = 0$, determine the max and min P in order not to raise or lower the load

(b) For $P = 500$ N, find the min α before the load begins to slip

Solution: Impending slippage of the cable over the fixed drum is given by: $T_2 = T_1 e^{\mu\beta}$

Draw the FBD for each case

(a) $\mu = 0.3$, $\alpha = 0$, $\beta = \pi/2$ rad

For impending upward motion of the load: $T_2 = P_{max}$; $T_1 = 981$ N

$P_{max}/981 = e^{0.3(\pi/2)} \rightarrow P_{max} = 1572$ N

For impending downward motion: $T_2 = 981$ N; $T_1 = P_{min}$

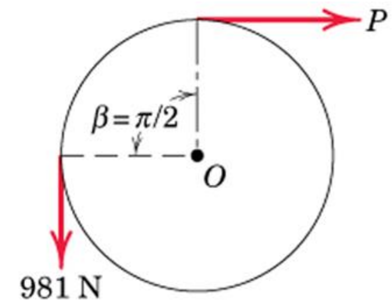
$981/P_{min} = e^{0.3(\pi/2)} \rightarrow P_{min} = 612$ N

(b) $\mu = 0.3$, $\alpha = ?$, $\beta = \pi/2+\alpha$ rad, $T_2 = 981$ N; $T_1 = 500$ N

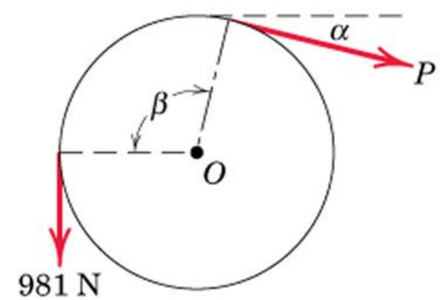
$981/500 = e^{0.3\beta} \rightarrow 0.3\beta = \ln(981/500) \rightarrow \beta = 2.25$ rad

$\rightarrow \beta = 2.25\text{x}(360/2\pi) = 128.7^o$

$\rightarrow \alpha = 128.7 - 90 = 38.7^o$



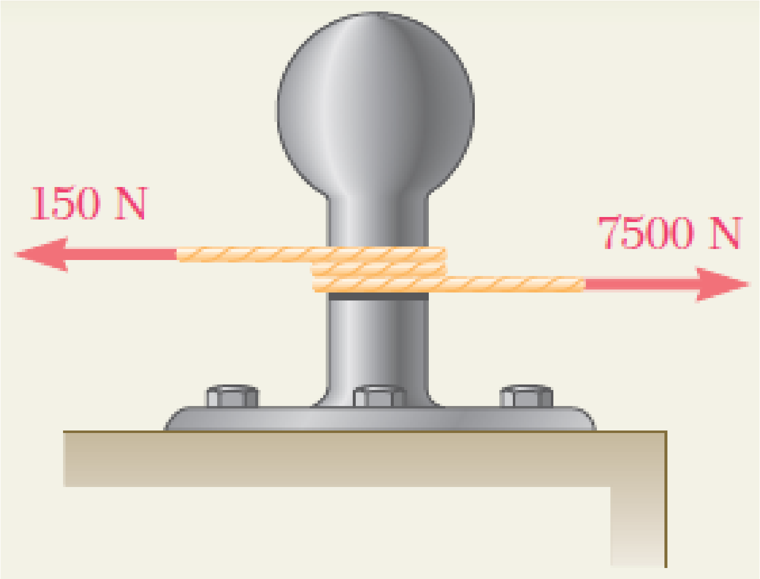
(a) α = 0



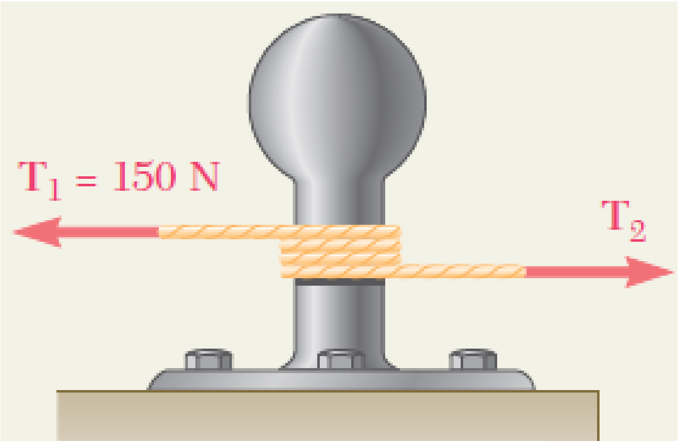
(b) P = 500 N

Example (2) on Belt Friction

A hawser thrown from a ship to a pier is wrapped two full turns around a bollard. The tension in the hawser is 7500 N; by exerting a force of 150 N on its free end, a dockworker can just keep the hawser from slipping. (*a*) Determine the coefficient of friction between the hawser and the bollard. (*b*) Determine the tension in the hawser that could be resisted by the 150-N force if the hawser were wrapped three full turns around the bollard.



Example (2) on Belt Friction



(a) Impending slippage of the hawser gives the application of the equation
$\mathbf{T_2} = \mathbf{T_1}\, e^{\mu\beta}$

$\mathbf{T_1}$ = 150 N, $\mathbf{T_2}$ **=** 7,500 N**,**
$\beta = 2\times 2\pi$ rad = 12.57 rad

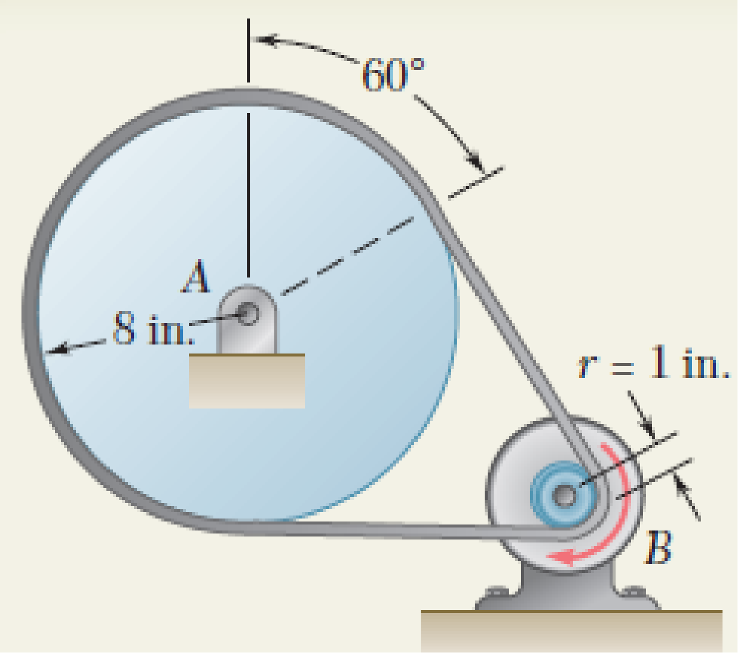
$\therefore\ \mu = 0.311$

(b) For 3 turns of the hawser, $\beta = 3\times 2\pi$ rad = 18.85 rad

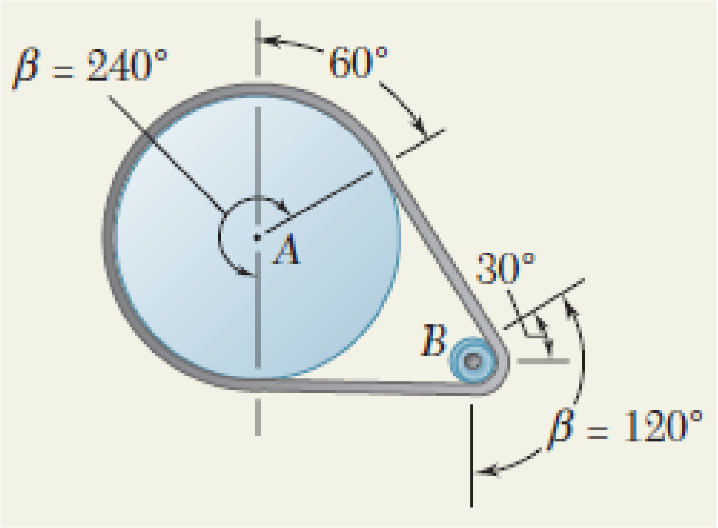
$\mathbf{T_1}$ = 150 N, $\mu = 0.311$
Using $\mathbf{T_2} = \mathbf{T_1}\, e^{\mu\beta}$,
$\mathbf{T_2}$ = 52.73 kN

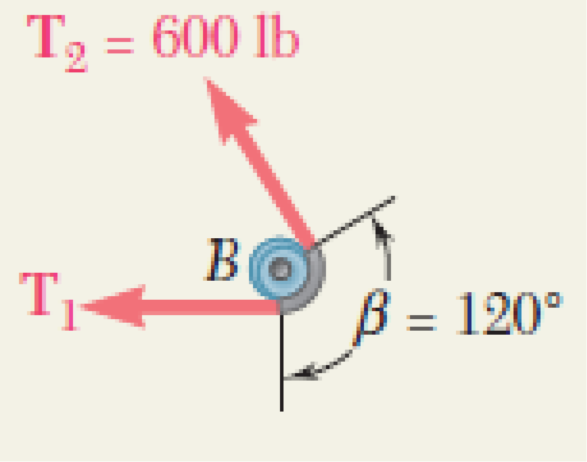
Example (3) on Belt Friction

A flat belt connects pulley *A*, which drives a machine tool, to pulley *B*, which is attached to the shaft of an electric motor. The coefficients of friction are $\mu_s = 0.25$ and $\mu_k = 0.20$ between both pulleys and the belt. Knowing that the maximum allowable tension in the belt is 600 lb, determine the largest torque which can be exerted by the belt on pulley *A*.



Example (3) on Belt Friction





Slippage will first occur for pulley B since the angle β is smaller as compared to pulley A

(for the same μ)

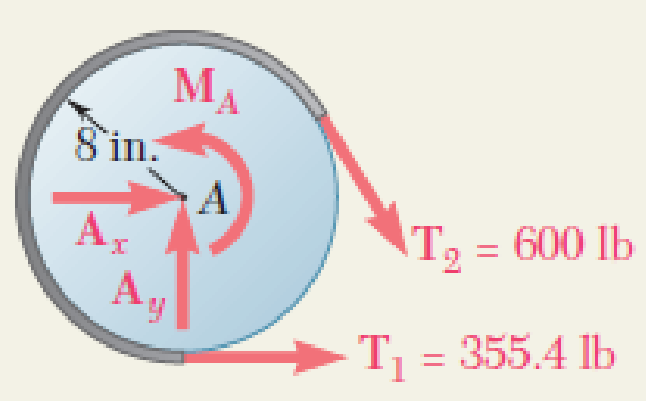
For pulley B,

$\mathbf{T_2}$ **=** 600 lb**,** β = 120° = $2\pi/3$ rad

μ = 0.25

$\therefore$ $\mathbf{T_1}$ = 355.4 lb

Example (3) on Belt Friction



Free body diagram of pulley A

$$+\circlearrowleft \Sigma M_A = 0: \qquad M_A - (600 \text{ lb})(8 \text{ in.}) + (355.4 \text{ lb})(8 \text{ in.}) = 0$$

$$M_A = 1957 \text{ lb} \cdot \text{in.} \qquad M_A = 163.1 \text{ lb} \cdot \text{ft}$$

Note. We may check that the belt does not slip on pulley A by computing the value of μ_s required to prevent slipping at A and verifying that it is smaller than the actual value of μ_s. From Eq. (8.13) we have

$$\mu_s \beta = \ln \frac{T_2}{T_1} = \ln \frac{600 \text{ lb}}{355.4 \text{ lb}} = 0.524$$

and, since $\beta = 240° = 4\pi/3$ rad,

$$\frac{4\pi}{3}\mu_s = 0.524 \qquad \mu_s = 0.125 < 0.25$$