## ORTHOGRAPHIC PROJECTIONS OF POINTS, LINES \& PLANES

# To draw projections of any object, one must have following information: 

A) OBJECT
\{With its description, well defined\}
B) OBSERVER
\{Always observing perpendicular to resp. Ref. Plane\}
C) LOCATION OF OBJECT
\{Means its position with reference to H.P. \& V.P.\}

## NOTATIONS

## Following notations should be followed while naming Different views in orthographic projections.

| OBJECT | POINT A | LINE AB |
| :---: | :---: | :---: |
| IT'S TOP VIEW | $a$ | $a b$ |
| IT'S FRONT VIEW | $a^{\prime}$ | $a^{\prime} b^{\prime}$ |
| IT'S SIDE VIEW | $a^{\prime \prime}$ | $a^{\prime \prime} b^{\prime \prime}$ |

Same system of notations should be followed incase numbers, like 1, 2, 3 - are used.

TERMS ‘ABOVE’ \& ‘BELOW’ WITH RESPECT TO H.P. AND TERMS 'INFRONT' \& 'BEHIND' WITH RESPECT TO V.P.


This quadrant pattern, if observed along $x$ - $y$ line (in red arrow direction) will exactly appear as shown on right side and hence it is further used to understand illustration properly.


## PROJECTIONS OF A POINT IN FIRST QUADRANT



## PROJECTIONS OF STRAIGHT LINES

## INFORMATION REGARDING A LINE MEANS: <br> - It's length

- Position of it's ends with HP \& VP
- It's inclinations with HP \& VP will be given.

AIM:- To draw it's projections - means FV \& TV.

## SIMPLE CASES OF THE LINE

1. A vertical line (line perpendicular to HP \& parallel to VP)
2. Line parallel to both HP \& VP.
3. Line inclined to HP \& parallel to VP.
4. Line inclined to VP \& parallel to HP.
5. Line inclined to both HP \& VP.



## EXAMPLE PROBLEMS ON POINTS

## PROBLEM 1:

A point $A$ is $\mathbf{2 0} \mathbf{~ m m}$ above HP and $\mathbf{3 0} \mathbf{~ m m}$ in front of VP. Draw its projections

## Solution steps:

1) Draw reference line $X Y$.
2) Mark a point a' at a distance of 20 mm above XY.
3) Through this point draw a perpendicular line to $X Y$ and mark the top view a at a distance of 30 mm below XY.


Orthographic projection


## PROBLEM 2:

A point D is $\mathbf{2 0} \mathbf{~ m m}$ below HP and $\mathbf{3 0} \mathbf{~ m m}$ in front of VP. Draw its projections.

## Solution steps:

1) Draw reference line $X Y$.
2) Mark a point d' at a distance of 20 mm below XY .
3) Through this point draw a perpendicular line to $X Y$ and mark the top view $d$ at a distance of 30 mm above XY.


Orthographic projection


## PROBLEM 3:

Draw the projections of the following points on the same ground line, keeping the distance between projectors equal to 25 mm .
(i) Point A, 20 mm above HP, 25 mm behind VP;
(ii) Point B, 25 mm below HP, 20 mm behind VP;
(iii) Point C, 20 mm below HP, 30 mm in front of VP;
(iv) Point D, 20 mm above HP, 25 mm in front of VP;
(v) Point E, on HP, 25 mm behind VP;
(vi) Point F, on VP, 30 mm above HP;

Solution:



## Parallel to VP and inclined to HP

## PROBLEM 4:

Draw projections of a 80 mm long line $P Q$. Its end $P$ is 10 mm above HP and 10 mm in front of VP. The line is parallel to VP and inclined to HP at $30^{\circ}$.

## Solution steps:

1) Draw the plan and elevations of the end point $P$.
2) Draw plan PQ of the line at an angle of $30^{\circ}$ to XY .
3) Draw the projector of $Q$.
4) From the elevation of end point $P$ draw a line parallel to $X Y$ meeting projector of $Q$ at $Q^{\prime}$.
5) $P^{\prime} Q^{\prime}$ is the elevation and $P Q$ is the plan of the line.


## Parallel to VP and inclined to HP

## PROBLEM 5:

A straight line AB of 40 mm length has one of its ends A, at 10 mm from the HP and 15 mm from the VP. Draw the projections of the line if it is parallel to the VP and inclined at $30^{\circ}$ to the HP. Assume the line to be located in each of the four quadrants by turns. (EXAMPLE)



## Parallel to HP and inclined to VP

## PROBLEM 6:

A straight line AB of 40 mm length is parallel to the HP and inclined at $30^{\circ}$ to the VP. Its end point $A$ is 10 mm from the HP and 15 mm from the VP. Draw the projections of the line $A B$, assuming it to be located in all the four quadrants by turns.

b


( Quadrant 4)


Orthographic Projections Means FV \& TV of Line AB are shown below, with their apparent inclinations $\alpha \& \beta$


Here TV (ab) is not // to $X Y$ line
Hence it's corresponding FV
$a^{\prime} b^{\prime}$ is not showing True Length \&
True Inclination with HP.

Note the procedure When FV \& TV known, How to find True Length. (Views are rotated to determine True Length \& it's inclinations with HP \& VP).


Note the procedure When True Length is known, How to locate FV \& TV. (Component a- 1 of TL is drawn which is further rotated
 of $T L a b_{1}$ gives length of FV. Hence it is brought upto Locus of a' and further rotated to get point b'. a'b' will be FV. Similarly drawing component of other TL $\left(a^{\prime} b_{1}{ }^{\prime}\right)$ TV can be drawn.

1) True Length (TL) - $a^{\prime} b_{1}{ }^{\prime} \& a b_{1}$

Diagram showing graphical relations among all important parameters of this topic.

2) Angle of TL with HP - $\boldsymbol{\theta}$
3) Angle of TL with VP - $\emptyset$
4) Angle of $F V$ with $X Y-\alpha$
5) Angle of $T V$ with $X Y-\beta$
6) LTV (length of FV) - Component (a-1)
7) LFV (length of TV) - Component (a'-1')
8) Position of A- Distances of a \& a' from XY
9) Position of B- Distances of b \& b' from XY
10) Distance between End Projectors


## INCLINED TO HP \& VP

## PROBLEM 7:

## Line $A B$ is 75 mm long and it is $30^{\circ} \& 40^{\circ}$ inclined to HP \& VP respectively. End $A$ is

 12 mm above HP and 10 mm in front of VP. Draw projections. Line is in $1^{\text {st }}$ quadrant.
## Solution steps:

1) Draw $X Y$ line and one projector.
2) Locate a' 12 mm above $X Y$ line \& a 10 mm below XY line.
3) Take $30^{\circ}$ angle from $a^{\prime} \& 40^{\circ}$ from a and mark TL, i.e., 75 mm on both lines. Name those points $b_{1}{ }^{\prime}$ and $b$ respectively.
4) Draw horizontal component of TL a $b_{1}$ from point $b_{1}$ and name it 1. (the length $a-1$ gives length of FV as we have seen already)
5) Extend it up to locus of a and rotating $a^{\prime}$ as center locate $b^{\prime}$ as shown. Join $a^{\prime} b^{\prime}$ as FV.
6) From b' drop a projector downward \& get point b. Join a \& b, i.e., TV.


## FINDING INCLINATION WITH HP

## PROBLEM 8:

Line AB 75 mm long makes $45^{\circ}$ inclination with VP while it's FV makes $55^{\circ}$. End A is 10 mm above HP and 15 mm in front of VP. If line is in $1^{\text {st }}$ quadrant draw it's projections and find it's inclination with HP.

## Solution Steps:-

1.Draw xy line.
2.Draw one projector for a' \& a
3.Locate $a^{\prime} 10 \mathrm{~mm}$ above XY \& a 15 mm below XY.
4.Draw a line $45^{\circ}$ inclined to $X Y$ from point $a$ and cut TL 75 mm on it and name that point $b_{1}$.
5.Draw locus from point $b_{1}$.
6. Take $55^{\circ}$ angle from $a^{\prime}$ for $F V$ above XY line.
7.Draw a vertical line from $b_{1}$ up to locus of a and name it 1 . It is horizontal component of TL \& is LFV.
8. Continue it to locus of $a^{\prime}$ and rotate upward up to the line of FV and name it $b^{\prime}$. This $a^{\prime} b^{\prime}$ line is FV.
9. Drop a projector from $b^{\prime}$ on locus from point $b_{1}$ and name intersecting point $b$. Line $a b$ is TV of line ab.
10.Draw locus from $b^{\prime}$ and with TL distance cut point $b_{1}{ }^{\prime}$
11.Join $a^{\prime} b_{1}^{\prime}$ as TL and measure it's angle $\alpha$ at $a^{\prime}$. It will be true angle of line with HP.


## FINDING TL AND INCLINATIONS

PROBLEM 9: FV of line $A B$ is $50^{\circ}$ inclined to $X Y$ and measures 55 mm long while it's TV is $60^{\circ}$ inclined to $X Y$ line. If end $A$ is 10 mm above HP and 15 mm in front of VP, draw it's projections, find TL, inclinations of line with HP \& VP.

Solution steps:
1.Draw $X Y$ line and one projector.
2.Locate a' 10 mm above XY and a 15 mm below $X Y$ line. 3.Draw locus from these points.
4.Draw FV $50^{\circ}$ from a' and mark b' cutting 55 mm on it.
5. Similarly draw TV $60^{\circ}$ from a \& drawing projector from b' locate point $b$ and join $a b$. 6. Then rotating views as shown, locate True Lengths $\mathrm{ab}_{1}$ \& $a^{\prime} b_{1}$ ' and their angles with HP and VP.


Line $A B$ is 75 mm long. It's $F V$ and TV measure 50 mm \& 60 mm long respectively. An end is 10 mm above HP and 15 mm in front of VP. Draw projections of line $A B$ if end $B$ is in first quadrant. Find angle with HP and VP.

## SOLUTION STEPS:

1.Draw $X Y$ line and one projector.
2.Locate a' 10 mm above XY and a 15 mm below $X Y$ line.
3.Draw locus from these points.
4.Cut 60 mm distance on locus of a' \& mark 1' on it as it is LTV.
5. Similarly cut 50 mm on locus of a and mark point 1 as it is LFV.
6.From 1' draw a vertical line upward and from a' taking TL ( 75 mm ) in compass, mark b' ${ }_{1}$ point on it. Join a' b' ${ }_{1}$ points.
7. Draw locus from $\mathrm{b}_{1}$
8. With same steps below get $b_{1}$ point and draw also locus from it.
9. Now rotating one of the components i.e., $\mathrm{a}-1$ locate b' and join a' with it to get FV.
10. Locate TV similarly and measure angles $\theta$ and $\Phi$


## FINDING ANGLE WITH HP \& VP

PROBLEM 11:- TV of a $\mathbf{7 5} \mathrm{mm}$ long line CD, measures 50 mm . End C is in HP and 50 mm in front of VP. End D is 15 mm in front of VP and it is above HP. Draw projections of CD and find angles with HP and VP.

## SOLUTION STEPS:

1.Draw XY line and one projector. 2. Locate c' on XY and c 50 mm below XY line.
3.Draw locus from these points.
4. Draw locus of d 15 mm below XY .
5.Cut $50 \mathrm{~mm} \& 75 \mathrm{~mm}$ distances on locus of d from c and mark points $d \& d_{1}$ as these are TV and TL. Join both with c .
6 .From $d_{1}$ draw a vertical line upward up to XY i.e., up to locus of c' and draw an arc as shown.
7 Then draw one projector from d to meet this arc in d' point \& join c' d'
8. Draw locus of d' and cut 75 mm on it from c' as TL
9.Measure angles $\theta$ and $\Phi$


## FINDING TRUE ANGLE

PROBLEM 9:- Two straight lines PQ and QR make an angle of $120^{\circ}$ between them in front and top views. PQ is 60 mm long and is parallel to and 15 mm from both H.P. and V.P. Determine the true angle between PQ and QR, if point $R$ is 50 mm above H.P. (EXAMPLE)

## SOLUTION STEPS:

1. Draw a reference line $x y$. Mark point $p^{\prime}$ at 15 mm above $x y$ and point $p$ at 15 mm below $x y$.
2. Draw 60 mm long lines $p^{\prime} q^{\prime}$ and $p q$, parallel to $x y$.
3. Draw a line from point $\mathrm{q}^{\prime}$, inclined at $120^{\circ}$ to xy such that it meets the horizontal line at 50 mm above xy at point $r^{\prime}$. Join $q^{\prime} r^{\prime}$ and $p^{\prime} r^{\prime}$.
4. Draw a line from point $q$, inclined at $120^{\circ}$ to $x y$ such that it meets the projector from $r^{\prime}$ at a point $r$. Join $q r$ and pr.
5. As lines pq and $p^{\prime} q^{\prime}$ are parallel to $x y$, they represent the true length of side $P Q$. Here $P Q=60 \mathrm{~mm}$.
6. Draw an arc with centre $p$ and radius $p r$ to meet the horizontal line from $p$ at point $r_{1}$. Project point $r_{1}$ to meet horizontal lines from point $r^{\prime}$ at point $r_{1}{ }^{\prime}$. Join $p^{\prime} r_{1}^{\prime}$ to represent the TL of the line PR. Here, $P R=p^{\prime} r_{1}^{\prime}=94$ mm.
7. Draw an arc with centre $q$ and radius qr, to meet the horizontal line at $r_{2}$. Project point $r_{2}$ to meet horizontal lines form point $r^{\prime}$ at point $r^{\prime}{ }_{2}$. Join $q^{\prime} r_{2}{ }^{\prime}$ to represent the $T L$ of line $Q R$. Here, $Q R=q^{\prime} r_{2}^{\prime}=53 \mathrm{~mm}$.
8. Draw actual triangle PQR taking true lengths, i.e., 60 $\mathrm{mm}, 94 \mathrm{~mm}$ and 53 mm . Measure the inclined angle PQR as the actual angle between sides PQ and QR. Here, it is $112^{\circ}$.


## PROBLEMS INVOLVING TRACES OF THE LINE

## TRACES OF THE LINE:-

These are the points of intersections of a line ( or it's extension ) with respect to reference planes.

A line itself or its extension, where ever touches H.P., that point is called TRACE OF THE LINE ON H.P. (It is called H.T.)

Similarly, a line itself or it's extension, where ever touches V.P., that point is called TRACE OF THE LINE ON V.P. (it is called V.T.)
V.T.:- It is a point on VP.

Hence it is called FV of a point in VP.
Hence it's TV comes on XY line.( Here onward denoted as ' V ')
H.T.:- It is a point on HP.

Hence it is called $T V$ of a point in HP.
Hence it's FV comes on XY line.( Here onward denoted as 'h' )

