# EE540 Advance Electromagnetic Theory & Antennas

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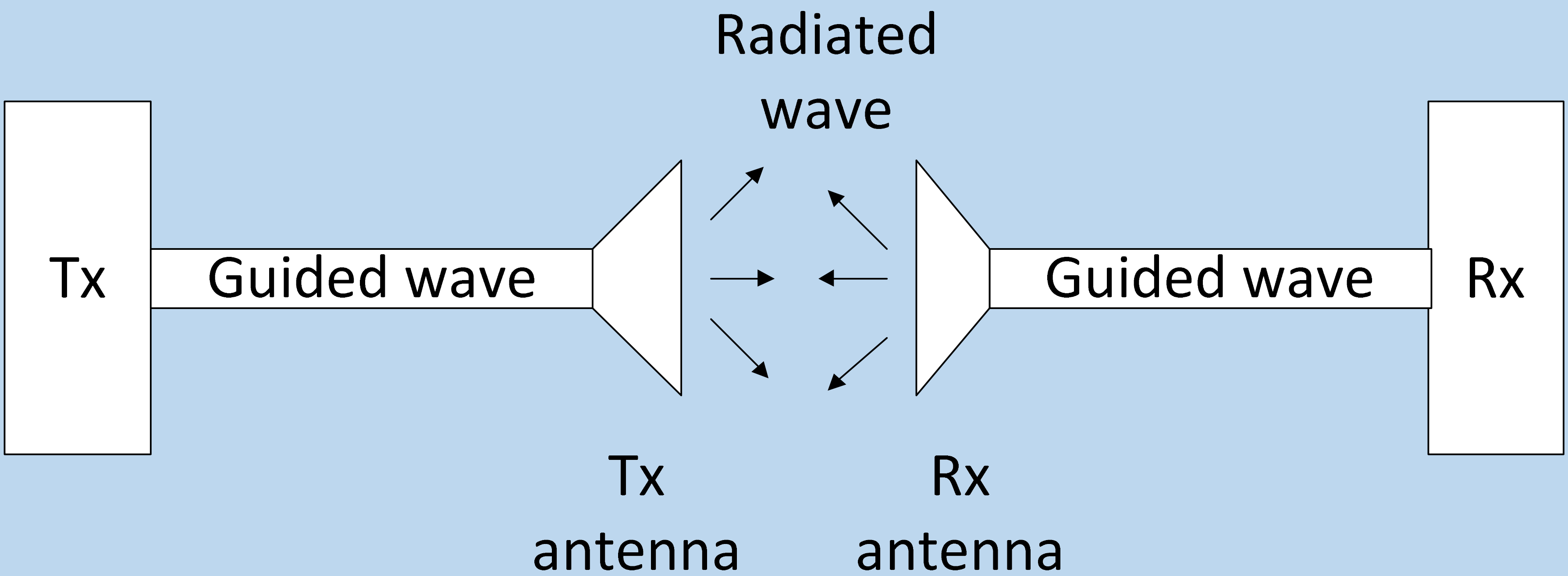
# Antennas

- We use mobile phones everyday
- Mobile phone converts our *voice into electrical signal* using microphone
- This signal is *modulated and radiated* to free space
  - by antennas as EM waves
  - which is picked up by the base station antennas
- We generally use transmission line like tv cables
  - for transferring EM energy from one point to another within a circuit

# Introduction

- This mode of energy transfer is called guided wave propagation
- It basically means that wave inside transmission line like coaxial cable is guided inside it and
  - will not come out from it into free space
- Hence antenna is also called as mode transformer which
- transforms *guided-wave field into a radiated wave field* for transmitting antenna and vice versa

# Introduction

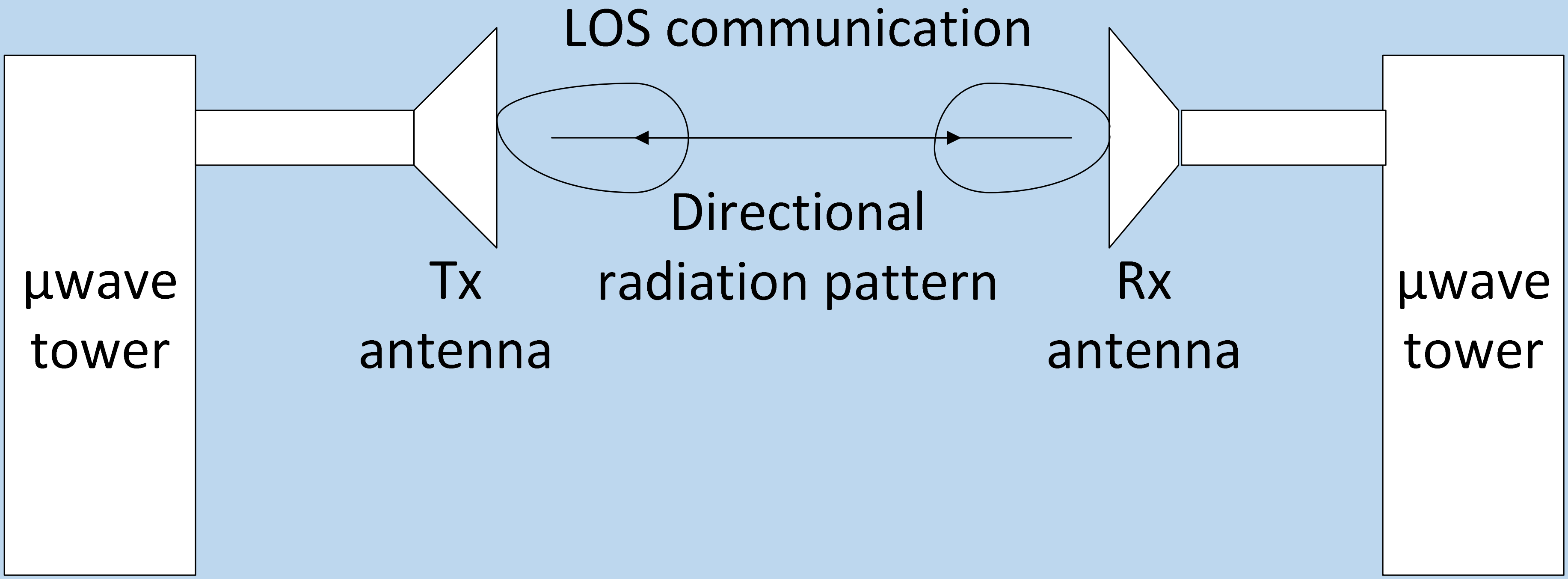



- Fig. Antenna as mode transformer

# Introduction

- An important property of antenna is its ability to transmit power in a preferred direction like in microwave towers
  - where we align the transmitting antenna and receiving antenna
  - for line of sight (LOS) communication

# Introduction

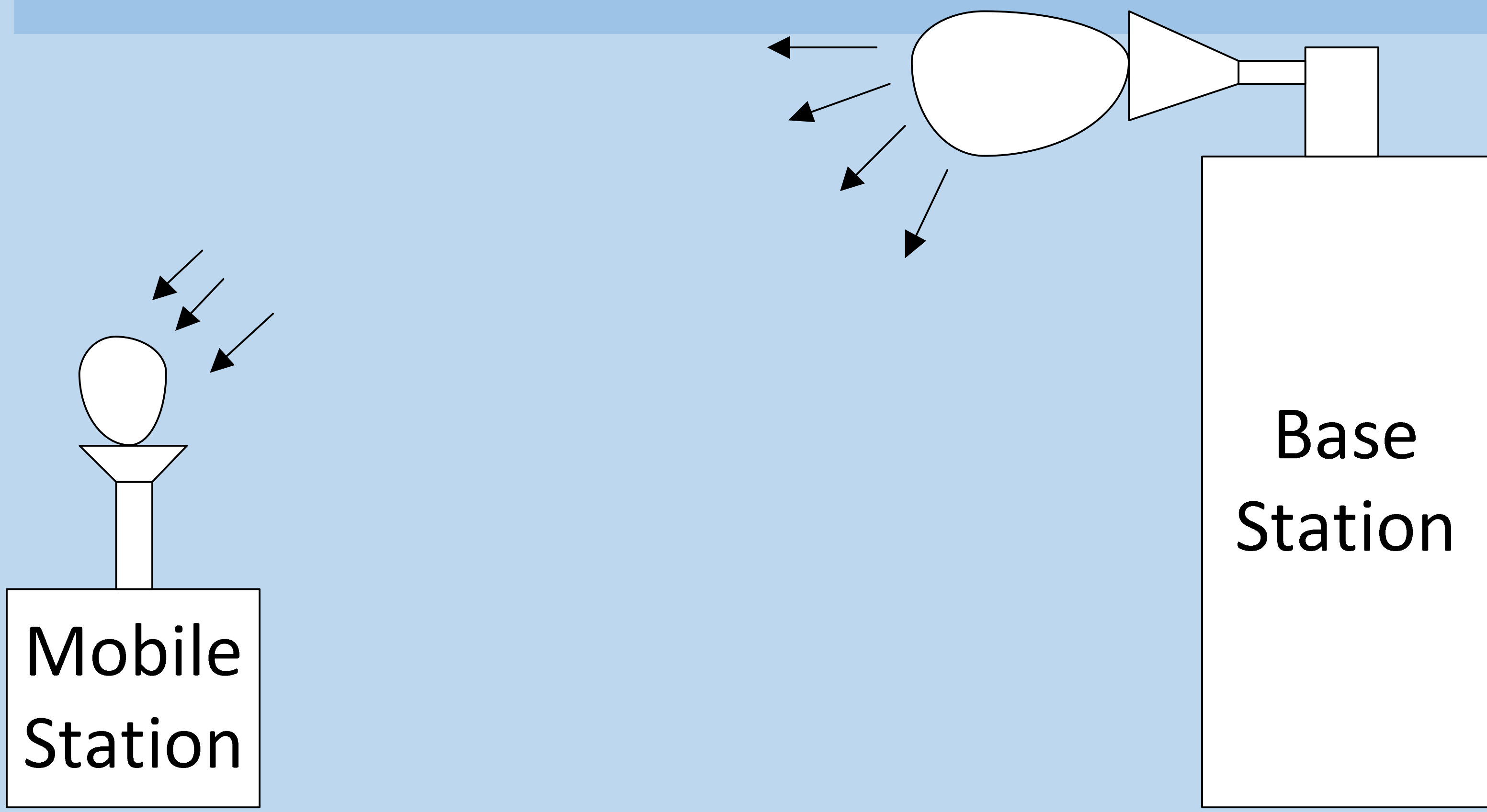



- Fig. Microwave tower: LOS communication

# Introduction

- Radiation pattern shows how power is radiated from the antenna in 3-dimension
- Unlike the previous case, ideally base station (BS) and mobile station (MS) antennas should radiate equally in all directions
  - as well as they can pick up signals from all directions
- Such isotropic antennas do not exist in practice
- Omnidirectional directional antennas are used for such cases

# Introduction




- Fig. Mobile communication

# Radiation fundamentals

**When does a charge radiates?**

- accelerating/decelerating charges
  or
- time-varying currents
  - in a conductor radiate EM waves

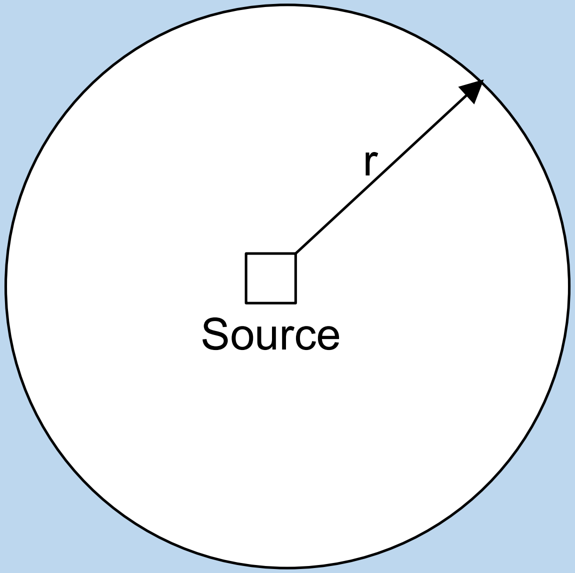



Fig. A giant sphere of radius r with a source of EM wave at its origin

# Radiation fundamentals

- Consider a giant sphere of radius r which encloses the source of EM waves at the origin
- The total power passing out of the spherical surface is given by Poynting theorem,

$$P_{total}(r) = \oint \vec{S}_{avg} \bullet d\vec{s} = \frac{1}{2} \oint \mathrm{Re}\left(\vec{E} \times \vec{H}^*\right) \bullet d\vec{s}$$

$$P_{total} = \lim_{r \to \infty} P(r)$$

# Radiation fundamentals

- This is the energy per unit time that is radiated into infinity and
  - it never comes back to the source
- The signature of radiation is irreversible flow of energy away from the source
- Let us analyze the following three cases:

CASE 1: *A stationary charge will not radiate*

  - no flow of charge =>no current=>no magnetic field=>no radiation  (for EM waves we need both E and H)

# Radiation fundamentals

CASE 2: *A charge moving with constant velocity will not radiate*

- The area of the giant sphere is $4\pi r^2$
- So for the radiation to occur Poynting vector must decrease no faster than $1/r^2$
- power remains constant in that case
  - irrespective of the distance from the source

# Radiation fundamentals

- From Coloumb's law, electrostatic fields decrease as $1/r^2$,
- whereas Biot Savart's law also states that magnetic fields decrease as $1/r^2$

- So the total decrease in the Poynting vector is proportional to $1/r^4$
- Hence power decreases as $1/r^2$
  - It dies out after some distance from the source
  - implies no radiation

# Radiation fundamentals

CASE 3: *A time varying current or acceleration (or deceleration) of charge will radiate*

- To create radiation
  - there must be a time varying current or
  - acceleration (or deceleration) of charge

# Radiation fundamentals

- Basic radiation equation:

$$L\frac{di}{dt} = Q\frac{dv}{dt}$$

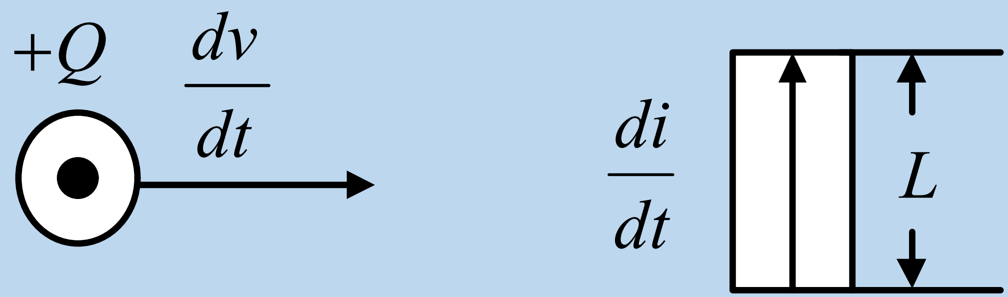



Fig. Fundamental law of radiation

- where
- L=length of current carrying element, m
- $\frac{di}{dt}$ =time changing current, $As^{-1}$(units)
- Q=charge, C
- $\frac{dv}{dt}$ =acceleration of charge, $ms^{-2}$

# Radiation fundamentals

- For this case (we will show this later),
  - a time varying field (both E and H) is produced
    - which varies as 1/r
  - whose field direction is along $\hat{\theta}$ and $\hat{\phi}$
- Hence the direction of Poynting vector is radially outward
- Since Poynting vector varies as $1/r^2$, total power is always constant
  - It can go to infinite distance

# Radiation fundamentals

- Two conditions for EM waves:
  - 1) *Fields produced by the EM source should have components varying as 1/r*
  - 2) *Field direction should not be radial but transversal so that the power flow or Poynting vector should be radial*
- It can be shown that for an infinitesimally small current carrying element (Hertz dipole)
  - which is the building block for antenna,
  - it indeed produces such fields
    - when supplied with time varying currents

# Radiation fundamentals

- But it is a difficult process to find such fields directly from current density and
  - calculations are highly complex
- A major simplification is possible when
  - we find the magnetic vector potential first and
    - find the fields from it
- It is similar to
  - find electric field from electric potential than
  - directly finding electric field
- This way it is easier

# Radiation fundamentals

**Wave equation for potential functions**

- One of the Maxwell's divergence equation

$$\nabla \bullet \vec{B} = 0$$

- Hence, we can write

$$\vec{B} = \nabla \times \vec{A}$$

- It means that we can find magnetic flux density
  - from the curl of magnetic vector potential

# Radiation fundamentals

**Wave equation for potential functions**

- Putting this in the following Maxwell's curl equation

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad \Rightarrow \nabla \times \vec{E} = -\frac{\partial\left(\nabla \times \vec{A}\right)}{\partial t}$$

  - which can be rewritten as

$$\Rightarrow \nabla \times \left( \vec{E} + \frac{\partial \vec{A}}{\partial t} \right) = 0$$

  - For time varying fields,

$$\vec{E} = -\nabla V - \frac{\partial \vec{A}}{\partial t}$$

# Radiation fundamentals

- Putting this in the following Maxwell's divergence equation

$$\nabla \bullet \vec{E} = \frac{\rho}{\varepsilon}$$

$$\nabla \bullet \left( \frac{\partial \vec{A}}{\partial t} + \nabla V \right) = -\frac{\rho}{\varepsilon}$$

$$\Rightarrow \nabla \bullet \frac{\partial \vec{A}}{\partial t} + \nabla^2 V = -\frac{\rho}{\varepsilon}$$

# Radiation fundamentals

- Applying Lorentz Gauge condition

$$\nabla \bullet \vec{A} + \mu\varepsilon \frac{\partial V}{\partial t} = 0$$

- Applying above condition

$$\Rightarrow \nabla\left(-\mu\varepsilon \frac{\partial V}{\partial t}\right) - \nabla^2 \vec{A} = \mu\vec{J} - \mu\varepsilon\left(\frac{\partial^2 \vec{A}}{\partial t^2} + \frac{\partial}{\partial t}\nabla V\right)$$

$$\Rightarrow \nabla^2 \vec{A} - \mu\varepsilon \frac{\partial^2 \vec{A}}{\partial t^2} = -\mu\vec{J}$$

# Radiation fundamentals

- Another Maxwell's curl equation

$$\nabla \times \vec{B} = \mu \vec{J} + \mu\varepsilon \frac{\partial \vec{E}}{\partial t}$$

- Simplifies to

$$\nabla \times \nabla \times \vec{A} = \nabla\left(\nabla \bullet \vec{A}\right) - \nabla^2 \vec{A} = \mu \vec{J} - \mu\varepsilon \frac{\partial\left(\frac{\partial \vec{A}}{\partial t} + \nabla V\right)}{\partial t}$$

# Radiation fundamentals

- Applying Lorentz Gauge condition

$$\nabla \bullet \vec{A} + \mu\varepsilon \frac{\partial V}{\partial t} = 0$$

- Applying above condition

$$\frac{\partial\left(\nabla \bullet \vec{A}\right)}{\partial t} + \nabla^2 V = -\frac{\rho}{\varepsilon}$$

$$\Rightarrow \boxed{\nabla^2 V - \mu\varepsilon \frac{\partial^2 V}{\partial t^2} = -\frac{\rho}{\varepsilon}}$$

# Radiation fundamentals

**Wave equation for potential functions**

- From Maxwell's equations for time varying fields,
- we have derived the two wave equations for potential functions
  - magnetic vector and
  - electric potentials

$$\nabla^2 \vec{A} - \mu\varepsilon \frac{\partial^2 \vec{A}}{\partial t^2} = -\mu \vec{J} \quad ; \nabla^2 V - \mu\varepsilon \frac{\partial^2 V}{\partial t^2} = -\frac{\rho}{\varepsilon}$$

# Radiation fundamentals

**Solution of wave equation for potential functions**

- For time harmonic functions of potentials,

$$\nabla^2 \vec{A} + \beta^2 \vec{A} = -\mu \vec{J}$$

  - where $\beta = \omega\sqrt{\mu\varepsilon}$
  - To solve the above equation, we can apply Green's function technique

# Radiation fundamentals

**Solution of wave equation for potential functions**

- Green's function G is the solution of the above equation with the R.H.S equal to a delta function

$$\nabla^2 G + \beta^2 G = \delta(space)$$

$$-\mu\vec{J}(\vec{r}) \rightarrow \vec{A}(\vec{r}); \delta(\vec{r}) \rightarrow G(\vec{r})$$

- Once we obtain the Green's function,
  - we can obtain the solution for any arbitrary current source by applying the convolution theorem

# Radiation fundamentals

- Since the medium surrounding the source is linear,
  - we can obtain the potential for any arbitrary current input
    - by the convolution of the impulse function (Green's function) with the input current

$$\vec{A}(\vec{r}) = G(\vec{r}) * \left(-\mu \vec{J}(\vec{r}\,')\right) = \frac{\mu}{4\pi}\int_V J(\vec{r}\,') \frac{e^{-j\beta|\vec{r}-\vec{r}\,'|}}{|\vec{r}-\vec{r}\,'|} dv'$$

# Radiation fundamentals

- Notation:
  - The prime coordinates denote the source variables
  - unprimed coordinates denote the observation points
- The modulus sign in  is to make sure that

  $\left|\vec{r}-\vec{r}'\right|$ is positive

  - since the distance in spherical coordinates is always positive

# Radiation fundamentals

Digression:

- LTI system
- For such system with an impulse response h(t) and input signal x(t),
  - the output signal is given by y(t)=h(t)*x(t)
- Note that is LTI system,
  - we consider x(t), h(t) and y(t) are functions of time
- In magnetic vector calculation, $-\mu\vec{J}(\vec{r}), G(\vec{r}), \vec{A}(\vec{r})$
  - are functions of space

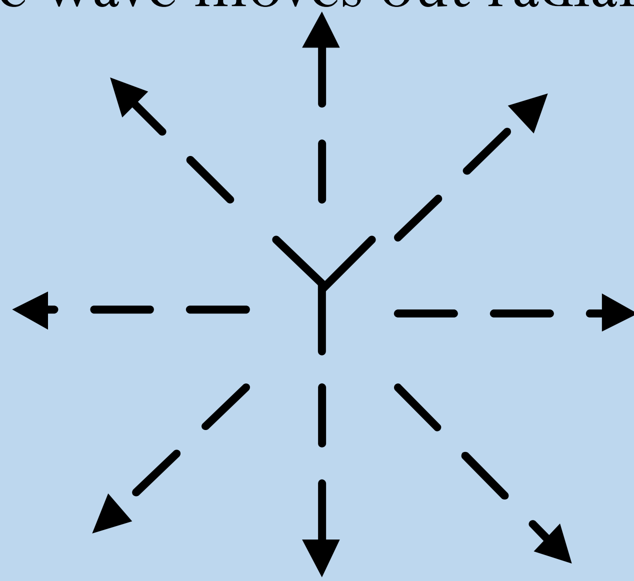
# Radiation fundamentals

Table: Analogy of LTI and Magnetic vector potential calculation

| Sl. No. System | LTI | Magnetic vector potential calculation |
|---|---|---|
| 1 | x(t) | $-\mu\vec{J}(\vec{r})$ |
| 2 | h(t) | $G(\vec{r})$ |
| 3 | y(t) | $\vec{A}(\vec{r})$ |

# Radiation fundamentals

- For radiation problems,
  - the most appropriate coordinate system is spherical
    - since the wave moves out radially in all directions



- Fig. An antenna radiating equally in all directions

# Radiation fundamentals

- It has also symmetry along θ and φ directions

$$\frac{\partial G}{\partial \theta} = \frac{\partial G}{\partial \phi} = 0$$

- Hence, the above equation reduces to

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial G}{\partial r}\right) + \beta^2 G = \delta(r)$$

# Radiation fundamentals

- Putting $\Psi$= G r,

$$\frac{\partial^2}{\partial r^2}\Psi + \beta^2\Psi = r\delta(r)$$

- For r not equal to 0 (field should not be obtained at the source itself),

$$\frac{\partial^2}{\partial r^2}\Psi + \beta^2\Psi = 0$$

- Therefore,

$$\Psi = Ae^{-j\beta r} + Be^{+j\beta r}$$

# Radiation fundamentals

- Since the radiation moves radially in positive r direction
- negative r direction is not physically feasible for a source of a field, we get,

$$\Psi = Ae^{-j\beta r} \qquad G = \frac{Ae^{-j\beta r}}{r}$$

- we can find the constant A and hence

$$A = -\frac{1}{4\pi}; G = -\frac{e^{-j\beta r}}{4\pi r}$$

  - which is magnetic vector potential produced by a delta source