EE540 Advance Electromagnetic Theory & Antennas

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Hertz dipole

- Let us find the fields of a small current carrying element of length dl
- The procedure involves
  - Determining the current on the antenna
  - Then compute

$$\vec{J} \to \vec{A} \to \vec{H} \to \vec{E}$$

Hertz dipole

- The infinitesimal time-varying current in the Hertz dipole is assumed as

$$I(t) = I_0 e^{j\omega t} \hat{z}$$

  - where ω is the angular frequency of the current
- Since the current is assumed along the z-direction,
  - the magnetic vector potential at the observation point P is along z-direction

$$\vec{A} = A_z \hat{z} = \frac{\mu_0 I_0 dl e^{-j\beta r} e^{j\omega t}}{4\pi r} \hat{z}$$

Hertz dipole

- Note that for this case

$$\vec{J}\left(\vec{r}^{'}\right)dv^{'} = I_0 dl e^{j\omega t}$$

- For infinitesimally small current element at the origin

$$\vec{r} - \vec{r}^{'} = \vec{r}$$

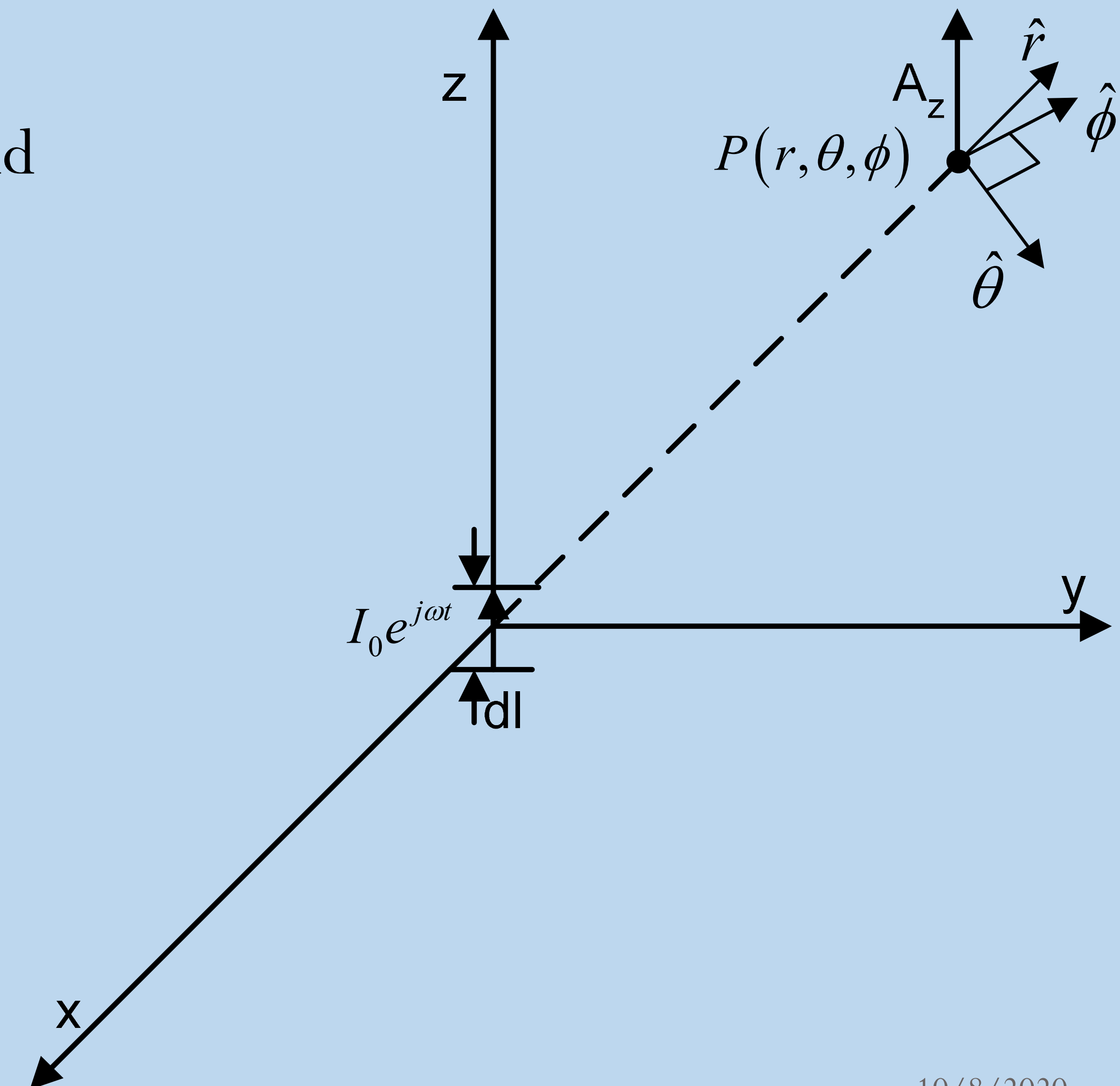
$$\vec{A}(\vec{r}) = G(\vec{r}) * \left(-\mu\vec{J}\left(\vec{r}^{'}\right)\right) = \frac{\mu}{4\pi}\int_V J\left(\vec{r}^{'}\right)\frac{e^{-j\beta\left|\vec{r}-\vec{r}^{'}\right|}}{\left|\vec{r}-\vec{r}^{'}\right|}dv^{'}$$

$$\Rightarrow \vec{A} = A_z\hat{z} = \frac{\mu_0 I_0 dl e^{-j\beta r}e^{j\omega t}}{4\pi r}\hat{z}$$

Hertz dipole

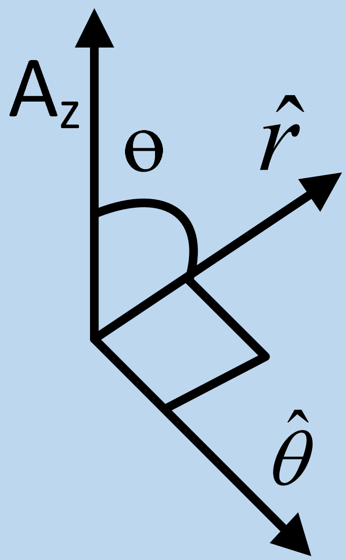
- Fig. Hertz dipole

located at the origin and oriented along z-axis



Hertz dipole

$$\begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\phi & \cos\phi & 0 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\phi & \cos\phi & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ A_z \end{bmatrix}$$



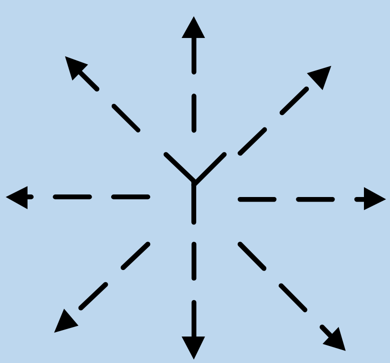
- Coordinate transformation

$$\Rightarrow A_r = A_z \cos\theta; A_\theta = -A_z \sin\theta; A_\phi = 0$$

Hertz dipole

$$\because \vec{H} = \frac{\nabla \times \vec{A}}{\mu_0} = \frac{1}{\mu_0 r^2 \sin\theta} \begin{vmatrix} \hat{r} & r\hat{\theta} & r\sin\theta\hat{\phi} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ A_r & rA_\theta & r\sin\theta A_\phi \end{vmatrix}$$

- Using the symmetry of the problem (no variation in ϕ), we have,



$$\vec{H} = \frac{1}{\mu_0 r^2 \sin\theta} \begin{vmatrix} \hat{r} & r\hat{\theta} & r\sin\theta\hat{\phi} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \equiv 0 \\ A_z\cos\theta & -rA_z\sin\theta & 0 \end{vmatrix}$$

Hertz dipole

$$\Rightarrow H_r = 0; H_\theta = 0; H_\phi = \frac{r\sin\theta}{\mu_0 r^2 \sin\theta}\left\{\frac{\partial}{\partial r}\left(-rA_z \sin\theta\right) - \frac{\partial}{\partial\theta}\left(A_z \cos\theta\right)\right\}$$

$$\Rightarrow H_\phi = \frac{dle^{+j\omega t}}{4\pi r}\left\{\frac{\partial}{\partial r}\left(-e^{-j\beta r}\sin\theta\right) - \frac{\partial}{\partial\theta}\left(\frac{e^{-j\beta r}\cos\theta}{r}\right)\right\} = \frac{I_0 dle^{+j\omega t}}{4\pi r}\left\{j\beta e^{-j\beta r}\sin\theta + \frac{e^{-j\beta r}\sin\theta}{r}\right\}$$

$$= \frac{I_0 dle^{+j\omega t}e^{-j\beta r}\sin\theta}{4\pi}\left\{\left(\frac{j\beta}{r} + \frac{1}{r^2}\right)\right\}$$

- The Hertz dipole has only ϕ component of the magnetic field,
- i.e., the magnetic field circulates the dipole

Hertz dipole

- The electric field for ($\vec{J}=0$ in free space, we don't have any conduction current flowing) can be obtained as

$$\vec{E}=\frac{\nabla\times\vec{H}}{j\omega\varepsilon}$$

- Using the symmetry of the problem (no variation in ϕ) like before, we have,

$$\vec{E}=\frac{1}{j\omega\varepsilon r^{2}\sin\theta}\begin{vmatrix}\hat{r} & r\hat{\theta} & r\sin\theta\hat{\phi}\\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\theta} & \frac{\partial}{\partial\phi}\equiv 0\\ 0 & 0 & r\sin\theta H_{\phi}\end{vmatrix}=\frac{1}{j\omega\varepsilon r^{2}\sin\theta}\left\{\frac{\partial}{\partial\theta}\left(r\sin\theta H_{\phi}\right)\hat{r}-\frac{\partial}{\partial r}\left(r\sin\theta H_{\phi}\right)r\hat{\theta}\right\}$$

Hertz dipole

$$E_r = \frac{I_0 dl e^{j\omega t} e^{-j\beta r}}{j\omega\varepsilon r^2 \sin\theta 4\pi}\left(\frac{j\beta}{r}+\frac{1}{r^2}\right)\left\{\frac{\partial}{\partial\theta}\left(r\sin^2\theta\right)\right\} = \frac{I_0 dl e^{j\omega t} e^{-j\beta r}}{j\omega\varepsilon r^2 \sin\theta 4\pi}\left(\frac{j\beta}{r}+\frac{1}{r^2}\right) r 2\sin\theta\cos\theta$$

- Note that E_r has only $1/r^2$ and $1/r^3$ variation with r

$$E_\theta = -\frac{r}{j\omega\varepsilon r^2 \sin\theta}\frac{\partial}{\partial r}\left(r\sin\theta H_\phi\right) = -\frac{I_0 dl e^{j\omega t}\sin\theta}{j\omega\varepsilon r 4\pi}\frac{\partial}{\partial r}\left(\left\{j\beta+\frac{1}{r}\right\}e^{-j\beta r}\right)$$

$$= \frac{I_0 dl e^{j\omega t}\sin\theta}{4\pi\varepsilon} e^{-j\beta r}\left\{\frac{j\beta^2}{\omega r}+\frac{\beta}{\omega r^2}-\frac{j}{\omega r^3}\right\}$$

- We see that electric field is in the (r, θ) plane
  - whereas the magnetic field has ϕ component only

Hertz dipole

- Therefore, the electric field and magnetic field are
  - perpendicular to each other
- *Points to be noted:*
  - Fields can be classified into three types
    - Radiation fields (spatial variation $1/r$)
    - Induction fields (spatial variation $1/r^2$) and
    - Electrostatic fields (spatial variation $1/r^3$)

Hertz dipole

1) Field variation with frequency since $\beta = \omega\sqrt{\mu\varepsilon}$

- Electrostatic fields ($1/r^3$) are also inversely proportional to the frequency ($\frac{1}{\omega}$)
- Induction field ($1/r^2$) is independent of frequency ($\frac{\beta}{\omega}$)
- Radiation field ($1/r$) is proportional to frequency ($\frac{\beta^2}{\omega}$)

Hertz dipole

2) Field variation with r

- For small values of r,
  - electrostatic field is the dominant term and
- Induction field is the
  - transition from electrostatic field to radiation fields
- For large values of r,
  - radiation field is the dominant term

Hertz dipole

- Near fields:
- We can show that Hertz dipole has reactive near field

$$H_{\phi_{nf}} = \frac{I_0 dl e^{j\omega t} e^{-j\beta r} \sin\theta}{4\pi r^2}$$

$$E_{\theta_{nf}} = \frac{I_0 dl e^{j\omega t} e^{-j\beta r} \sin\theta}{4\pi\varepsilon\omega} \left\{ \frac{\beta}{r^2} - \frac{j}{r^3} \right\}$$

$$E_{r_{nf}} = \frac{2 I_0 dl e^{j\omega t} e^{-j\beta r} \cos\theta}{j 4\pi\varepsilon\omega} \left\{ \frac{j\beta}{r^2} + \frac{1}{r^3} \right\}$$

Hertz dipole

- It looks like the static magnetic field produced by a current carrying element using Biot Savart's law

$$H_{\phi_{nf}} = \frac{I_0 dl e^{j\omega t} e^{-j\beta r} \sin\theta}{4\pi r^2}$$

- It resembles the electric field produced by an electric dipole

$$E_{r_{nf}} = \frac{2 I_0 dl e^{j\omega t} e^{-j\beta r} \cos\theta}{j4\pi\varepsilon\omega}\left\{\frac{1}{r^3}\right\} \quad E_{\theta_{nf}} = -\frac{I_0 dl e^{j\omega t} e^{-j\beta r} \sin\theta}{4\pi\varepsilon\omega}\left\{\frac{j}{r^3}\right\}$$

Hertz dipole

- Note that $1/r^5$ term in Poynting vector is purely reactive
- Note that the $1/r^4$ term in the Poynting vector is
  - due to induction fields
  - which will die out after some distance from the source
- Far fields:

$$H_{\phi_{ff}} = \frac{j\beta I_0 dl e^{j\omega t} e^{-j\beta r} \sin\theta}{4\pi r}$$

$$E_{\theta_{ff}} = \frac{j\beta^2 I_0 dl e^{j\omega t} e^{-j\beta r} \sin\theta}{4\pi\varepsilon\omega r}$$

Hertz dipole

- Note that the $1/r^2$ term in the Poynting vector
  - gives total power over a giant sphere always constant
- Hence such waves from the radiation fields
  - will go to infinite distance in free space
- They also satisfy two conditions for EM waves:
  - Transversal fields (E_θ and H_φ gives Poynting vector along radial direction)
  - Fields vary as $1/r$ (Poynting vector varies as $1/r^2$)

Hertz dipole

- We can also observe that the three types of fields are equal in magnitude when

  $\beta^2/r = \beta/r^2 = 1/r^3$

  $\Rightarrow r = 1/\beta = \lambda/2\pi$

$$E_r = \frac{I_0 dl e^{j\omega t} e^{-j\beta r} \sin\theta}{4\pi\varepsilon\omega}\left\{\frac{j\beta^2}{r} + \frac{\beta}{r^2} - \frac{j}{r^3}\right\}$$

- For $r < \lambda/2\pi$,
  - $1/r^3$ term dominates
- For $r >> \lambda/2\pi$,
  - the $1/r$ term dominates

Hertz dipole

- *Near field region:*
- For r<< $\lambda/2\pi$
  - in fact the near field region distance r= $\lambda/2\pi$ is for D<<λ
  - for an ideal infinitesimally small Hertz dipole
- electrostatic fields dominate

Hertz dipole

- as r<< λ/2π

$$\because e^{-j\beta r} \to 1$$

$$E_r \approx -2j\frac{I_0 dl\cos\theta e^{j\omega t}}{4\pi\varepsilon\omega r^3}; \quad E_\theta \approx -j\frac{I_0 dl\sin\theta e^{j\omega t}}{4\pi\varepsilon\omega r^3}$$

- The magnitude of the near field is

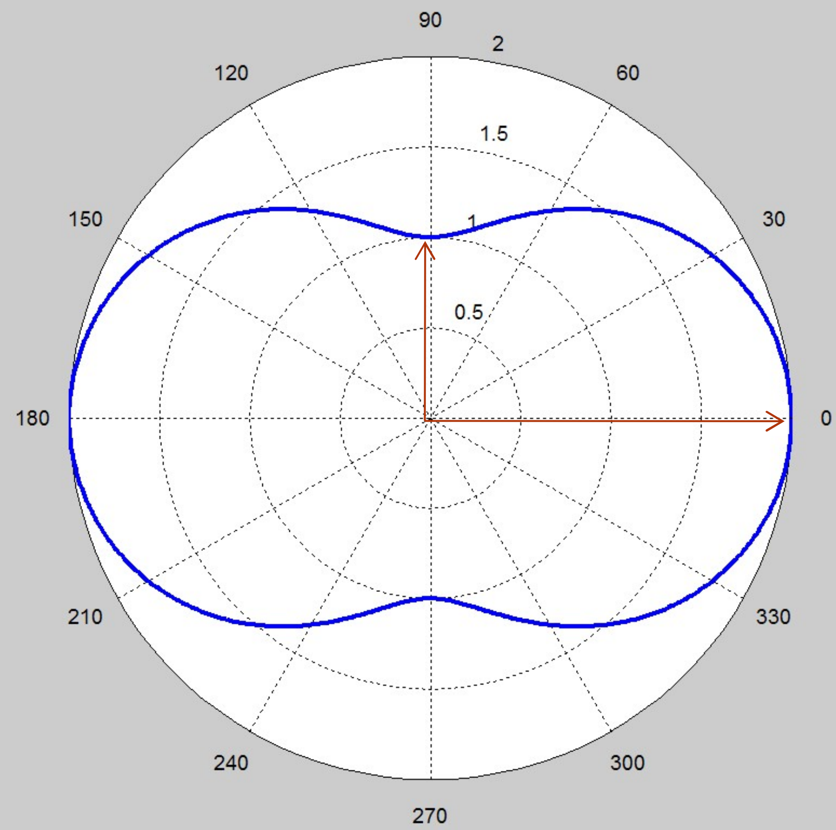
$$|E| = \sqrt{|E_r|^2 + |E_\theta{}^2|} = \frac{I_0 dl}{4\pi\varepsilon\omega r^3}\sqrt{4\cos^2\theta + \sin^2\theta}$$

Hertz dipole

- A polar plot of the near field can be generated by writing a MATLAB program for plotting

$$F(\theta)=\sqrt{4\cos^2\theta+\sin^2\theta}$$

- Maximum field is along
  - $\theta=0^0$, $\theta=180^0$ and
- minimum is along
  - $\theta=90^0$, $\theta=270^0$ (see Fig. (a))



- Fig. (a) Near field pattern plot of a Hertz dipole located at the origin and oriented along z-axis (maximum radiation along z-axis)

Hertz dipole

- *Far field region:*
- For r>> λ/2π (in between reactive near field and Fraunhofer far field region, there exists the Fresnel near field region that's why we have chosen an r>> λ/2π), radiation field is the dominant term
- In other words kr>>1, we have,

$$E_\theta = j\frac{I_0 dl \sin\theta e^{j\omega t} e^{-j\beta r}\beta^2}{4\pi\varepsilon\omega r}; H_\phi = j\frac{I_0 dl \sin\theta e^{j\omega t} e^{-j\beta r}\beta}{4\pi r}$$

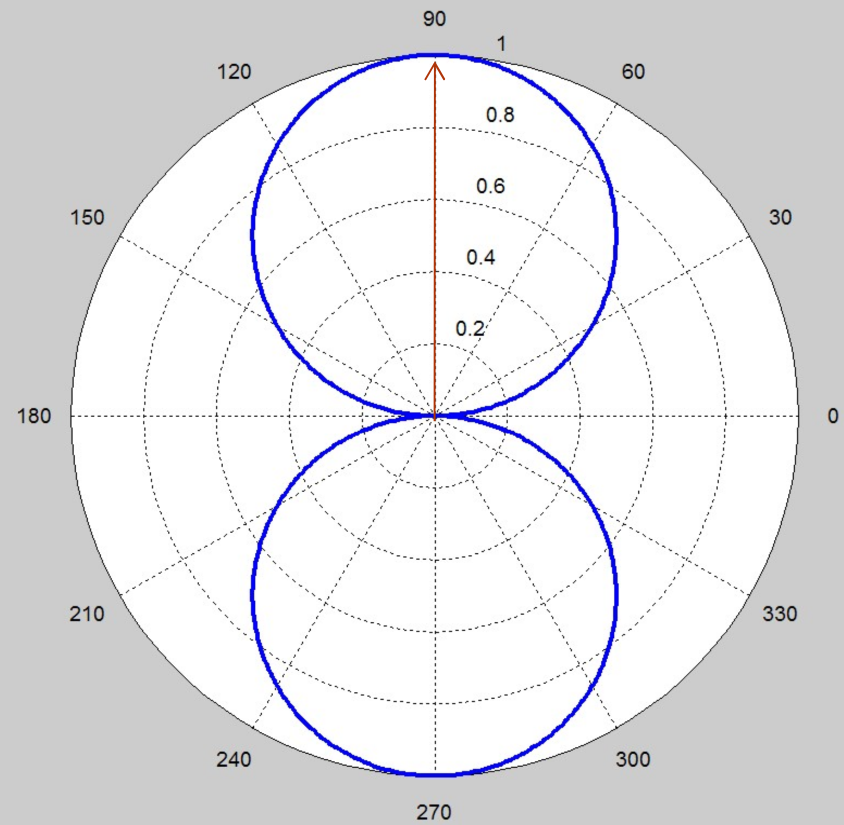
Hertz dipole

- The electric fields and magnetic fields are in phase with each other
- They are 90° out of phase with the current
  - due to the (j) term in the expressions of E_θ and H_φ
- It is interesting to note that the ratio of electric field and magnetic field is constant

$$\frac{E_\theta}{H_\phi} = \frac{\beta}{\omega\varepsilon} = \frac{\omega\sqrt{\mu\varepsilon}}{\omega\varepsilon} = \sqrt{\frac{\mu}{\varepsilon}} = \eta$$

Hertz dipole

- Hence, the fields have sinusoidal variations with θ
  - They are zero along $\theta=0$
- No radiation along z-axis unlike near field case
  - They are maximum along $\theta=\pi/2$



- Fig. (b) E-plane radiation pattern of a Hertz dipole in far field (H-plane radiation will look like a circle)

Hertz dipole

- *Power flow:*

$$S_{avg} = \frac{1}{2}\mathrm{Re}\left\{\vec{E} \times \vec{H}^*\right\} = \frac{1}{2}\mathrm{Re}\left\{\left(E_r\hat{r} + E_\theta\hat{\theta}\right) \times H_\varphi{}^*\hat{\phi}\right\}$$

$$S_{avg} = \frac{1}{2}\mathrm{Re}\left\{E_\theta H_\phi{}^*\right\}\hat{r} = \frac{1}{2}\left(\frac{I_0 dl \sin\theta}{4\pi r}\right)^2 \frac{\beta^3}{\omega\varepsilon}\hat{r}$$

- Antenna power flows radially outward
- Power density is not same in all directions
- The net real power is only due to
  - the radiations fields (i.e. $j\beta^2/r$ and $j\beta/r$) of electric and magnetic fields

Hertz dipole

- *Total radiated power:*
- The total radiated power from a Hertz dipole

$$W = \iint S_{avg} r^2 \sin\theta d\theta d\phi$$

$$\therefore W = 40\pi^2 I_0{}^2 \left(\frac{dl}{\lambda} \right)^2$$

Hertz dipole

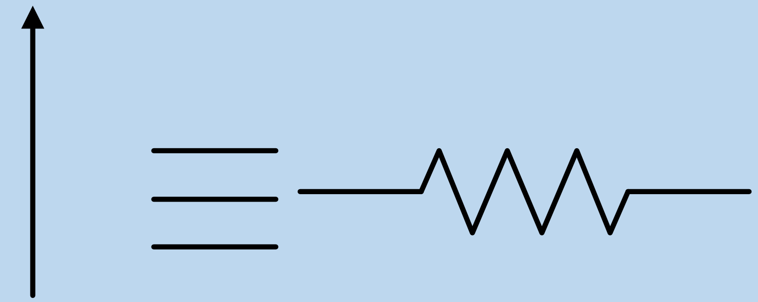
- Power radiated by the Hertz dipole is proportional to
  - the square of the dipole length and
  - inversely proportional to the dipole wavelength
- It implies more and more power is radiated as
  - the frequency and
  - the length
- of the Hertz dipole increases

Hertz dipole

- *Radiation resistance of a Hertz Dipole:*
- Hertz dipole can be equivalently modeled as a radiation resistance

Since $W = 1/2 \, I_0^2 \, R_{rad}$

$$\therefore W = 40\pi^2 I_0^2 \left(\frac{dl}{\lambda}\right)^2$$

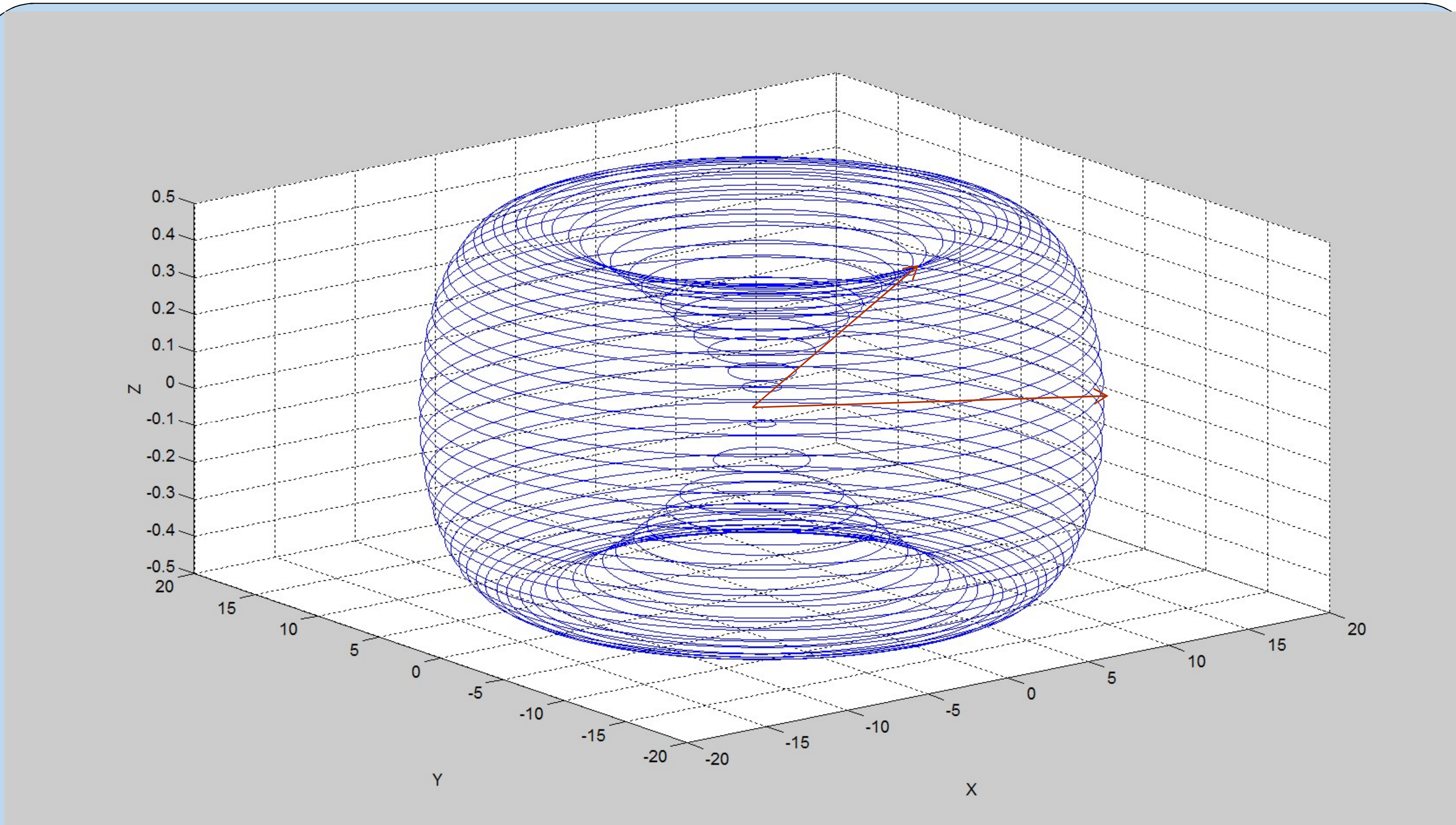


- implies that $R_{rad} = 80\pi^2 \left(\frac{dl}{\lambda}\right)^2$

Fig. Equivalence of Hertz dipole and radiation resistance

Hertz dipole

- *Radiation pattern of a Hertz Dipole:*
- $F(\theta)=\sin\theta$ for a Hertz dipole
- The 3D plot of $\sin\theta$ looks like an apple (see Figure (c))

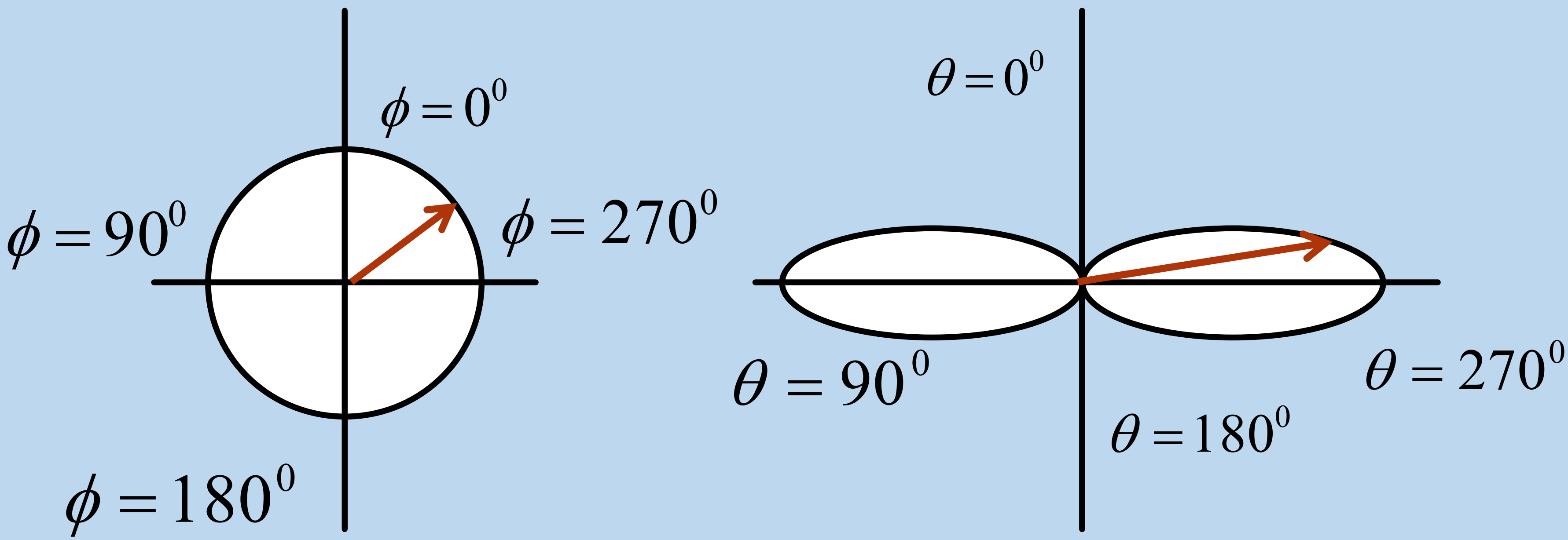


- Fig. (c) A typical 3-D radiation pattern of a Hertz dipole in the far field

Hertz dipole

- Two principal planes radiation patterns (2-D) are normally plotted
  - E-plane (vertical cut)
  - H-plane (horizontal cut) radiations patterns
- are sufficient to describe the radiation pattern of a Hertz dipole
- H-plane (xy-plane) radiation pattern is in the form of circle of radius 1 since $F(\theta, \phi)$ is independent of ϕ
- E-plane (xz-lane) radiation pattern looks like 8 shape

Hertz dipole



- Fig. H-plane and E-plane radiation patterns of Hertz dipole

Hertz dipole

- To get the 3-D plot from the 2-D plot
  - you need to rotate the E-plane pattern along the H-plane pattern
- For this case it will give the shape of an apple
- Note that θ is also known as elevation angle and ϕ as azimuth angle
- E-plane pattern for a dipole is also known as elevation pattern
- H-plane pattern as azimuthal pattern