EE540 Advance Electromagnetic Theory & Antennas

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Antenna pattern and parameters

Polarization

- Let us consider antenna is placed at the origin of a spherical coordinate system
  - and *wave is propagating radially outward in all directions*
- In the far field region of an antenna, the electric field of the antenna is given by

$$\vec{E}(\theta,\phi) = E_\theta(\theta,\phi)\hat{\theta} + E_\phi(\theta,\phi)\hat{\phi}$$

Antenna pattern and parameters

- Putting the time dependence, we have,

$$\vec{E}(\theta,\phi,t)=E_\theta(\theta,\phi)\cos(\omega t)\hat{\theta}+E_\phi(\theta,\phi)\cos(\omega t+\delta)\hat{\phi}=E_\theta(\theta,\phi,t)\hat{\theta}+E_\phi(\theta,\phi,t)\hat{\phi}$$

  - where δ is the phase difference between the elevation and azimuthal components of the electric field
- The figure traced out by the tip of the radiated electric field vector
  - as a function of time for a fixed position of space can be defined as antenna polarization

Antenna pattern and parameters

a) LP

- When δ=0, the two transversal electric field components are in time phase
- The total electric field vector
- $\vec{E}(\theta,\phi,t) = E_\theta(\theta,\phi)cos\omega t\hat{\theta} + E_\phi(\theta,\phi)cos\omega t\hat{\phi}$
- makes an angle θ_{LP} with the θ-axis

$$\theta_{LP} = tan^{-1}\left[\frac{E_\theta(\theta,\phi,t)}{E_\phi(\theta,\phi,t)}\right] = tan^{-1}\left[\frac{E_\theta(\theta,\phi)}{E_\phi(\theta,\phi)}\right]$$

Antenna pattern and parameters

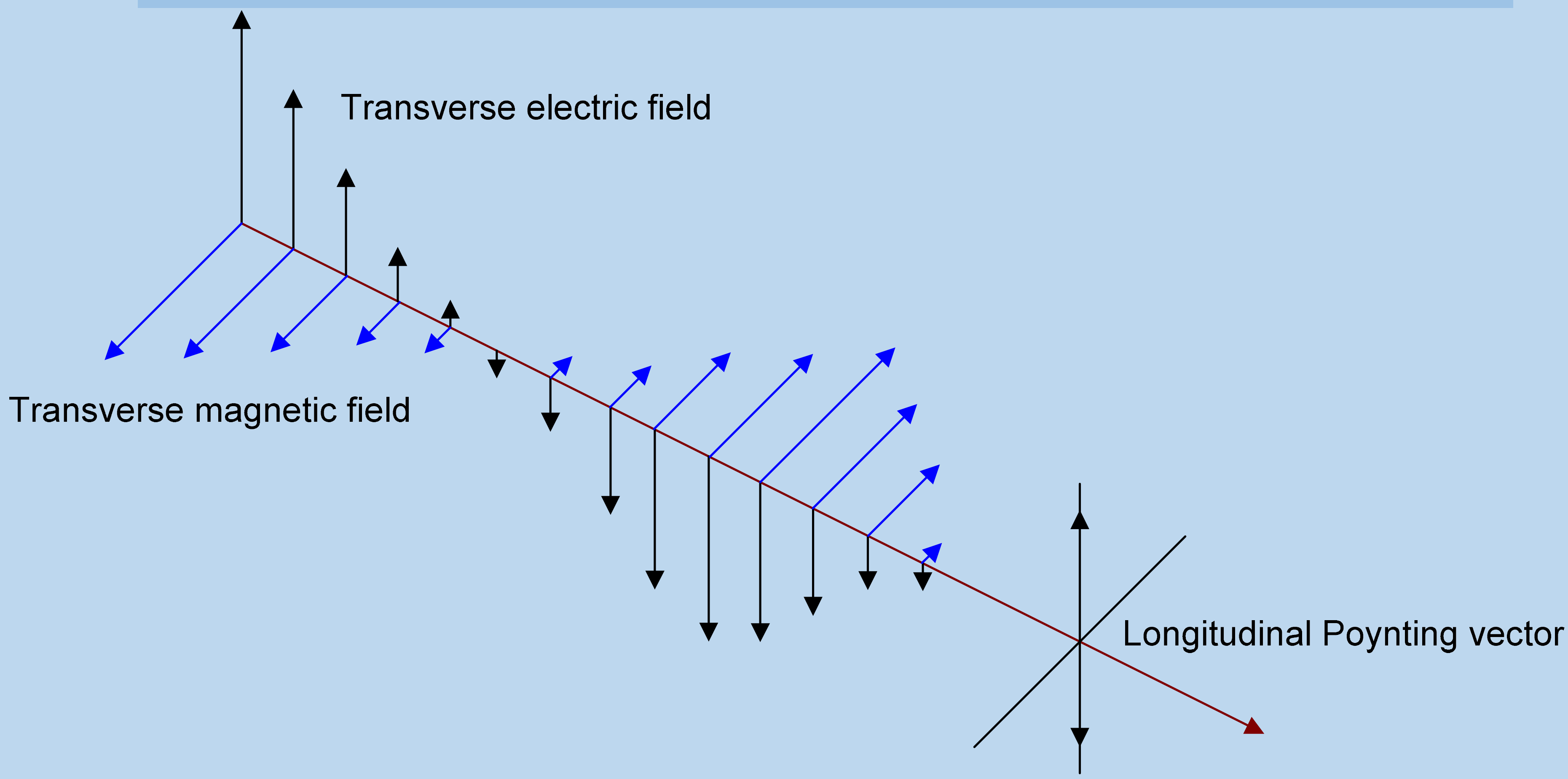


Fig. LP wave

Antenna pattern and parameters

- The tip of the total radiated electric field vector traces out a line
- Therefore, the antenna's polarization is LP

b) CP

- When $\delta = \pm \frac{\pi}{2}$,
  - the two transversal electric field components are out of phase in time
- and if the two transversal electric field components are of equal amplitude $E_\theta(\theta,\phi) = E_\phi(\theta,\phi) = E_0(\theta,\phi)$

Antenna pattern and parameters

- The total electric field vector
- $\vec{E}(\theta, \phi, t) = E_\theta(\theta, \phi) cos\omega t\hat{\theta} + E_\phi(\theta, \phi) cos\left(\omega t \pm \frac{\pi}{2}\right)\hat{\phi}$
- $\vec{E}(\theta, \phi, t) = E_\theta(\theta, \phi) cos\omega t\hat{\theta} \mp E_\phi(\theta, \phi) sin(\omega t)\hat{\phi}$
- makes an angle θ_{CP} with the θ-axis
- $\theta_{CP} = tan^{-1}\left[\mp \frac{sin\omega t}{cos\omega t}\right] = tan^{-1}[\mp tan\omega t] = \mp \omega t$
- This implies that the total radiated electric field vector of the antenna traces out a circle as time progresses from 0 to $2\frac{\pi}{\omega}$ and so on
- If the vector rotates counterclockwise for which θ_{CP} is positive (clockwise for which θ_{CP} is negative), then the antenna polarization is RHCP (LHCP)

Antenna pattern and parameters

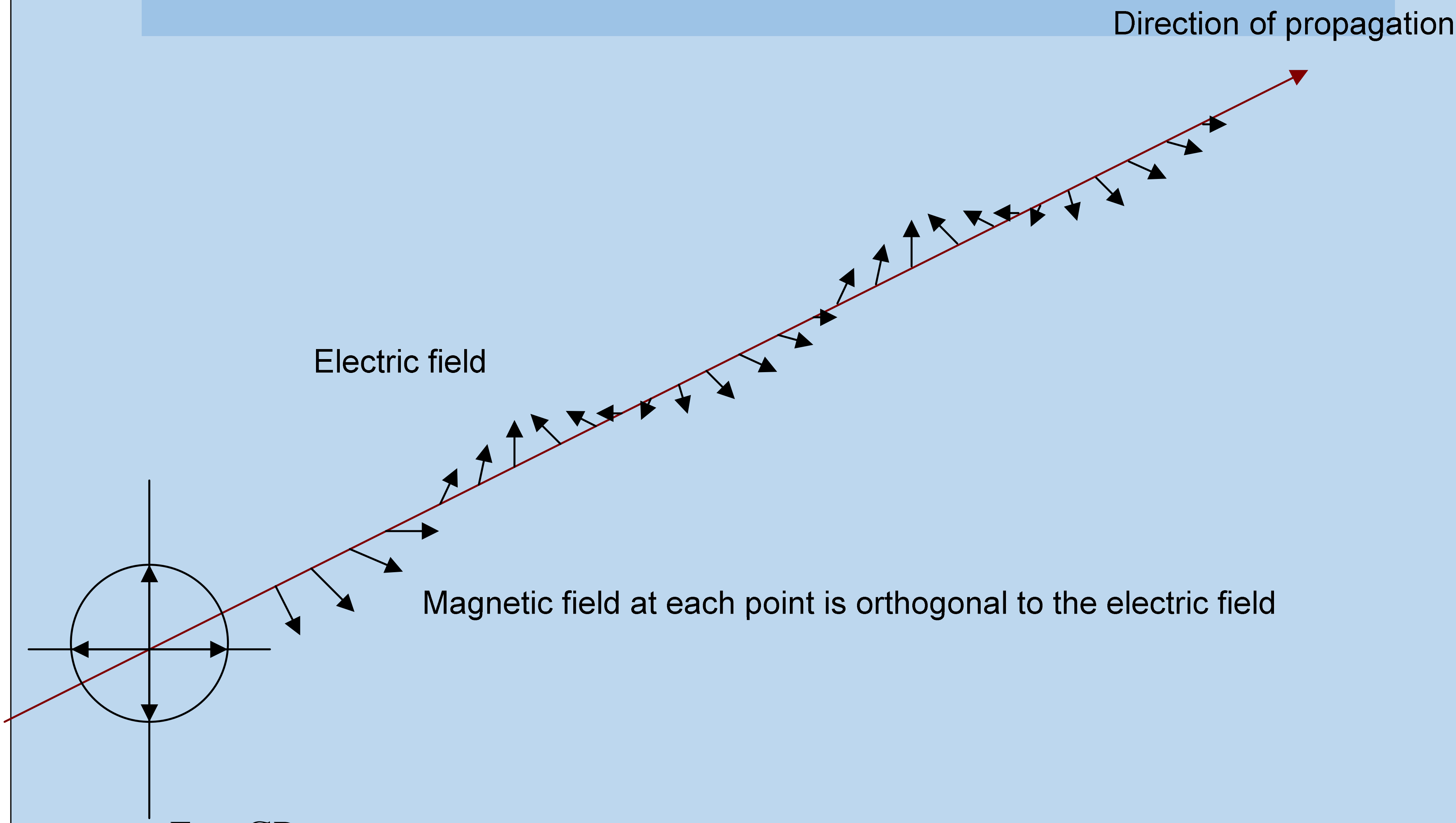


Fig. CP wave

Antenna pattern and parameters

- The total electric field vector for CP can be also expressed as
- $\vec{E}(\theta,\phi,t) = Re\left\{E_0(\theta,\phi)e^{j\omega t}\hat{\theta} + E_0(\theta,\phi)e^{j\left(\omega t \pm \frac{\pi}{2}\right)}\hat{\phi}\right\}$
- For RHCP,
- $\vec{E}(\theta,\phi,t) = Re\left\{E_0(\theta,\phi)e^{j\omega t}\hat{\theta} + E_0(\theta,\phi)e^{j\left(\omega t - \frac{\pi}{2}\right)}\hat{\phi}\right\}$
- $\vec{E}(\theta,\phi,t) = Re\left[E_0(\theta,\phi)e^{j\omega t}\left\{\hat{\theta} + e^{j\left(-\frac{\pi}{2}\right)}\hat{\phi}\right\}\right]$
- $\vec{E}(\theta,\phi,t) = Re\left[E_0(\theta,\phi)e^{j\omega t}\{\hat{\theta} - j\hat{\phi}\}\right]$
- For LHCP,
- $\vec{E}(\theta,\phi,t) = Re\left\{E_0(\theta,\phi)e^{j\omega t}\hat{\theta} + E_0(\theta,\phi)e^{j\left(\omega t + \frac{\pi}{2}\right)}\hat{\phi}\right\}$
- $\vec{E}(\theta,\phi,t) = Re\left[E_0(\theta,\phi)e^{j\omega t}\{\hat{\theta} + j\hat{\phi}\}\right]$

Antenna pattern and parameters

- The total electric field vector for CP in Cartesian coordinates assuming wave propagation along z-axis can be also expressed as
- $\vec{E} = Re\left\{E_0 e^{j\omega t}\hat{x} + E_0 e^{j\left(\omega t \pm \frac{\pi}{2}\right)}\hat{y}\right\}$
- For RHCP,
- $\vec{E} = Re\left\{E_0 e^{j\omega t}\hat{x} + E_0 e^{j\left(\omega t - \frac{\pi}{2}\right)}\hat{y}\right\}$
- $\vec{E} = Re\left[E_0 e^{j\omega t}\left\{\hat{x} + e^{j\left(-\frac{\pi}{2}\right)}y\right\}\right]$
- $\vec{E} = Re\left[E_0 e^{j\omega t}\{\hat{x} - j\hat{y}\}\right]$
- For unit vectors for RHCP can be expressed as $\hat{e} = \frac{1}{\sqrt{2}}\{\hat{x} - j\hat{y}\}$
- For unit vectors for LHCP can be expressed as $\hat{e} = \frac{1}{\sqrt{2}}\{\hat{x} + j\hat{y}\}$

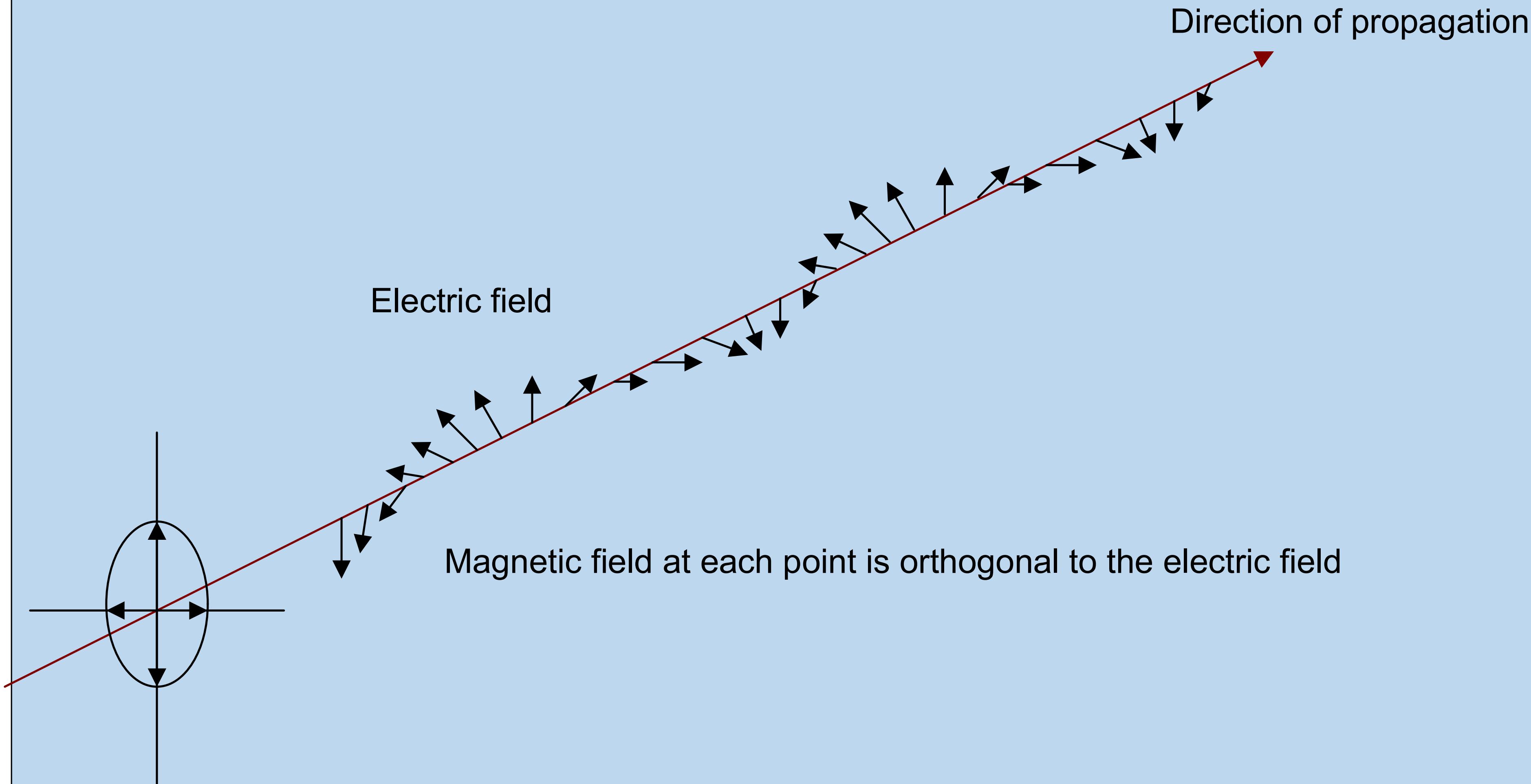
Antenna pattern and parameters

- For $E_\theta(\theta,\phi) \neq E_\phi(\theta,\phi) \quad and \quad \delta \neq 0, \pi$
- the total electric field vector traces out an ellipse and
  - hence it is elliptically polarized (EP)

$$\vec{E}_{EP} = \left(\hat{\theta} + Ae^{j\delta}\hat{\phi}\right)e^{-j\beta r}e^{j\omega t}$$

- Unit vector for EP, $\vec{e} = \frac{\hat{\theta}+Ae^{j\delta}\hat{\phi}}{\sqrt{1+A^2}}$
- If δ is in the upper half of the complex plane, LHEP and vice versa LP $(\delta = 0, \pi)$, RHCP $\left(A = 1, \delta = -\frac{\pi}{2}\right)$& LHCP $\left(A = 1, \delta = \frac{\pi}{2}\right)$
- The ratio of the major and minor axes of the ellipse is called axial ratio (AR)
- For instance, AR=0 dB for CP and AR= ∞ dB for LP

Antenna pattern and parameters



- Fig. EP wave

Antenna pattern and parameters

- Antenna polarization:
- To sum up,
- The polarization of a radiated wave in the far field region of an antenna defines antenna polarization
- Assume a transmitting antenna transmitting electric field of maximum magnitude E_m in the far field region of an antenna

$$\vec{E}^t = E_m \hat{e}_t$$

- where $\hat{e}_t$ is the unit vector which gives polarization for the transmitting antenna

Antenna pattern and parameters

- It is the complex vector representation of field normalized to unity
- Assume that there is a receiving antenna in the far field region of this antenna
- The field of some magnitude E^m that would give maximum received power at the received antenna terminal is

$$\vec{E}^r = E^m \hat{e}_r$$

- where $\hat{e}_r$ is the unit vector which gives polarization for the receiving antenna

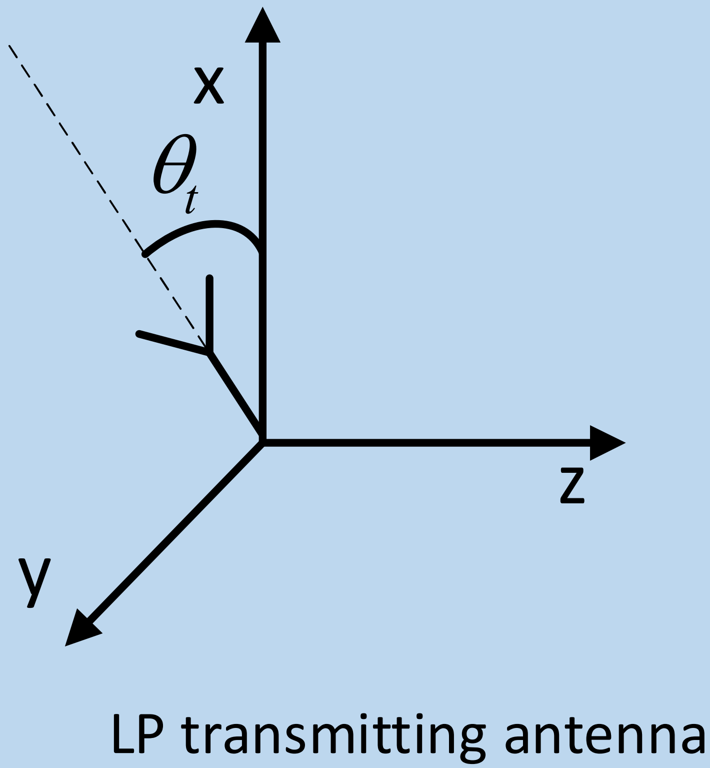
Antenna pattern and parameters

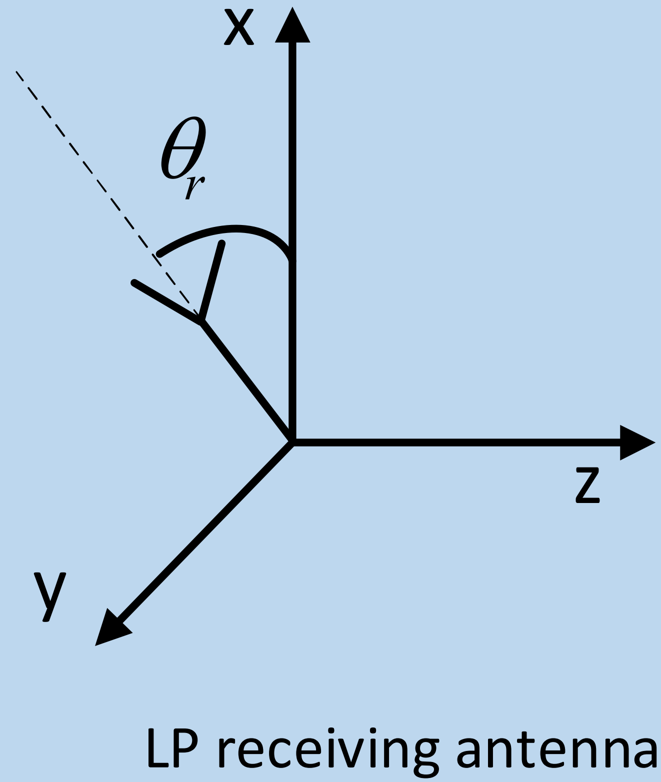
- Polarization efficiency is defined as

$$P.E. = \left| \hat{e}_t \bullet \hat{e}_r^* \right|^2$$

- Receiving LP wave with an LP receiving antenna:
- Let us consider a case where the transmitting antenna is placed at an angle θ_t w.r.t x-axis
- and the receiving antenna is placed at an angle θ_r w.r.t x-axis
- Both transmitting and receiving antennas are LP antennas (assume wave propagation along z-axis)

Antenna pattern and parameters





- Fig. LP transmitting and receiving antenna

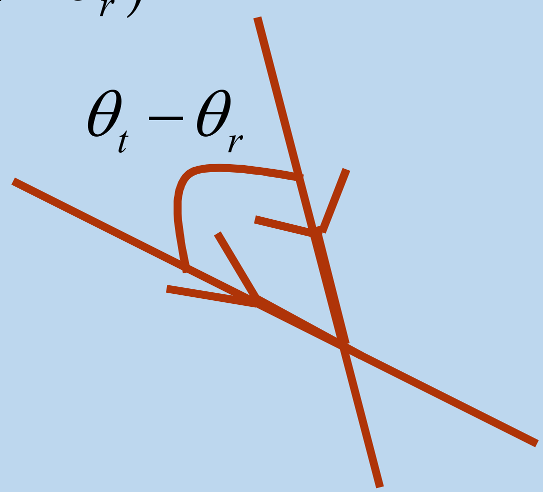
Antenna pattern and parameters

- Therefore, polarization unit vectors for the transmitting and receiving antennas are

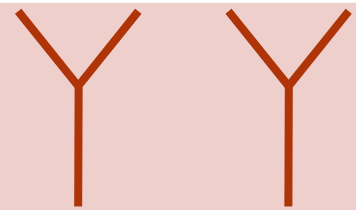
$$\hat{e}_t = \cos\theta_t \hat{x} + \sin\theta_t \hat{y}; \hat{e}_r = \cos\theta_r \hat{x} + \sin\theta_r \hat{y}$$

- Polarization efficiency can be calculated as

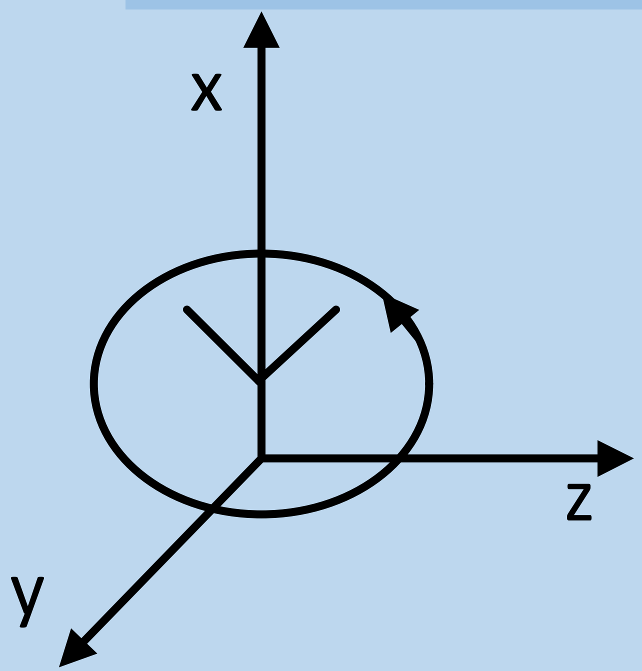
$$P.E. = \left| \hat{e}_t \bullet \hat{e}_r^* \right|^2 = \cos^2\left(\theta_t - \theta_r\right)$$



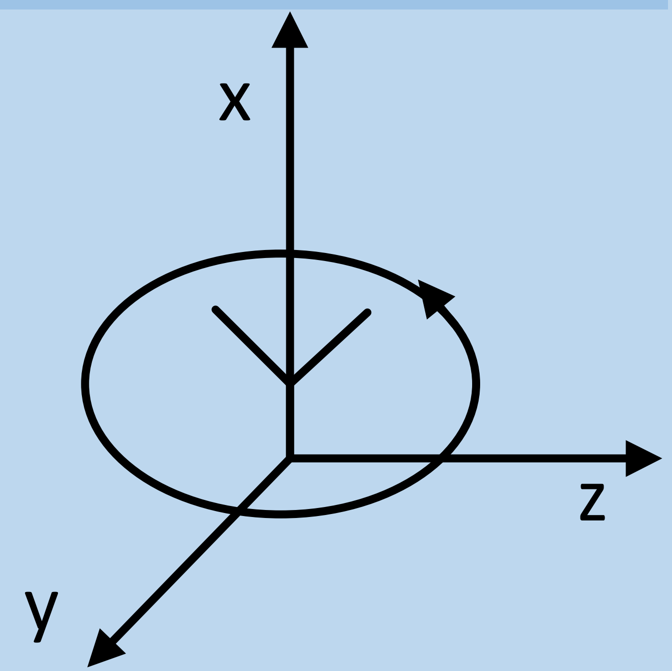
Antenna pattern and parameters

θ_t-θ_r	P.E.	Illustration
0	1	
$\pi/2$	0	

Antenna pattern and parameters



RHCP transmitting antenna



RHCP receiving antenna

- Fig. Receiving a CP wave with a CP receiving antenna

Antenna pattern and parameters

- Receiving a CP wave with a CP receiving antenna
- Consider a RHCP receiving antenna is kept in the far field region of RHCP transmitting antenna
- Assume wave propagation along positive z-axis
- Hence the polarization unit vectors for transmitting and receiving antennas are

$$\hat{e}_t = \frac{1}{\sqrt{2}}\left(\hat{x} - j\hat{y}\right); \hat{e}_r = \frac{1}{\sqrt{2}}\left(\hat{x} - j\hat{y}\right)$$

Antenna pattern and parameters

- Polarization efficiency

$$P.E. = \left| \hat{e}_t \bullet \hat{e}_r^* \right|^2 = 1$$

- For instance, the receiving antenna is LHCP

$$\hat{e}_t = \frac{1}{\sqrt{2}}\left(\hat{x} - j\hat{y}\right); \hat{e}_r = \frac{1}{\sqrt{2}}\left(\hat{x} + j\hat{y}\right)$$

- Polarization efficiency

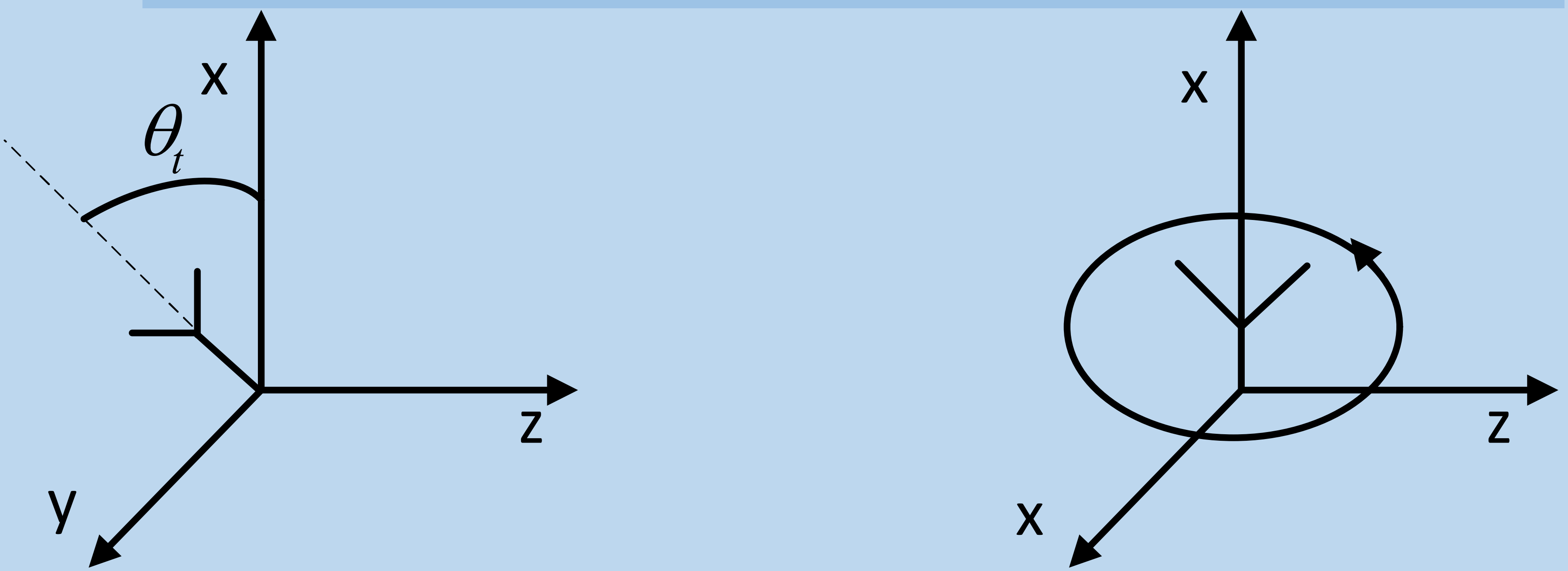
$$P.E. = \left| \hat{e}_t \bullet \hat{e}_r^* \right|^2 = 0$$

Antenna pattern and parameters

- It is like two persons shaking hands
- Right to right hand shake is matched but right to left hand shake is not matched
- Receiving an LP wave by a CP receiving antenna:
- Consider an LP wave transmitting antenna which is oriented at an angle θ_t with the x-axis
- Receiving antenna is a RHCP antenna
- Hence

$$\hat{e}_t = \cos\theta_t \hat{x} + \sin\theta_t \hat{y}; \hat{e}_r = \frac{1}{\sqrt{2}}\left(\hat{x} - j\hat{y}\right)$$

Antenna pattern and parameters



LP transmitting antenna

RHCP receiving antenna

Fig. Receiving an LP wave by a CP receiving antenna

Antenna pattern and parameters

- Therefore
- Polarization efficiency is

$$P.E. = \left|\hat{e}_t \bullet \hat{e}_r^*\right|^2 = \frac{1}{2}\left|\cos\theta_t + j\sin\theta_t\right|^2 = \frac{1}{2}$$

- Hence there is a 3-dB signal loss
- The same case when one use a RHCP/LHCP transmitting antenna and a LP receiving antenna

Antenna pattern and parameters

- Let us try to understand two terms (co- and cross-polarization) which are important for the antenna radiation pattern
- Co-polarization means you measure the antenna with another antenna oriented
  - in the same polarization with the antenna under test (AUT)
- Cross-polarization means that you measure the antenna with antenna oriented
  - perpendicular w.r.t. the main polarization

Antenna pattern and parameters

- Cross-polarization is the polarization orthogonal
  - to the polarization under consideration
- For example,
  - if the field of an antenna is horizontally polarized,
  - the cross-polarization for this case is vertical polarization
- If the polarization is RHCP,
  - the cross-polarization is LHCP
- Let us put this into mathematical expressions:
- We may write the total electric field propagating along z-axis as

$$\vec{E} = \left(E_{co}\hat{u}_{co} + E_{cr}\hat{u}_{cr}\right)e^{-j\beta z}$$

Antenna pattern and parameters

- where the co- and cross-polarization unit vectors satisfy the orthonormality condition

$$\hat{u}_{co} \bullet \hat{u}_{co}^{*} = 1, \hat{u}_{cr} \bullet \hat{u}_{cr}^{*} = 1, \hat{u}_{co} \bullet \hat{u}_{cr}^{*} = 0, \hat{u}_{cr} \bullet \hat{u}_{co}^{*} = 0$$

- Therefore,
- the co- and cross-polarization components of the electric fields can be obtained as

$$E_{co} = \vec{E} \bullet \hat{u}_{co}^{*}, E_{cr} = \vec{E} \bullet \hat{u}_{cr}^{*}$$

Antenna pattern and parameters

a) LP

- For a general LP wave along ϕ_{LP}, we can write,

$$\hat{u}_{co} = \cos\phi_{LP}\hat{x} + \sin\phi_{LP}\hat{y}, \hat{u}_{cr} = \sin\phi_{LP}\hat{x} - \cos\phi_{LP}\hat{y}$$

- For a x-directed LP wave, ϕ_{LP} =0, hence,

$$\hat{u}_{co} = \hat{x}, \hat{u}_{cr} = -\hat{y}; E_{co} = \vec{E} \bullet \hat{u}_{co}^{*} = E_x, E_{cr} = \vec{E} \bullet \hat{u}_{cr}^{*} = -E_y$$

- For a y-directed LP wave, $\phi_{LP}=90^0$, hence,

$$\hat{u}_{co} = \hat{y}, \hat{u}_{cr} = \hat{x}; E_{co} = \vec{E} \bullet \hat{u}_{co}^{*} = E_y, E_{cr} = \vec{E} \bullet \hat{u}_{cr}^{*} = E_x$$

Antenna pattern and parameters

b) CP

- For a RHCP wave propagating along z-axis, we can write,

$$\hat{u}_{co} = \frac{\hat{x} - j\hat{y}}{\sqrt{2}}, \hat{u}_{cr} = \frac{\hat{x} + j\hat{y}}{\sqrt{2}}$$

$$E_{co} = \vec{E} \bullet \hat{u}_{co}^* = \frac{E_x + jE_y}{\sqrt{2}}, E_{cr} = \vec{E} \bullet \hat{u}_{cr}^* = \frac{E_x - jE_y}{\sqrt{2}}$$

- For a LHCP wave, co- and cross-polarization unit vectors and components of the electric field will interchange

Antenna pattern and parameters

c) EP

- For a EP wave, we can write,

$$\hat{u}_{co} = \frac{\hat{x} + Ae^{j\phi_{EP}}\,\hat{y}}{\sqrt{1+A^2}}, \hat{u}_{cr} = \frac{-Ae^{-j\phi_{EP}}\,\hat{x} + \hat{y}}{\sqrt{1+A^2}}$$

- In order to determine the far-field radiation pattern of an AUT, two antennas are required
- The one being tested (AUT) is normally free to rotate and it is connected in receiving mode

Antenna pattern and parameters

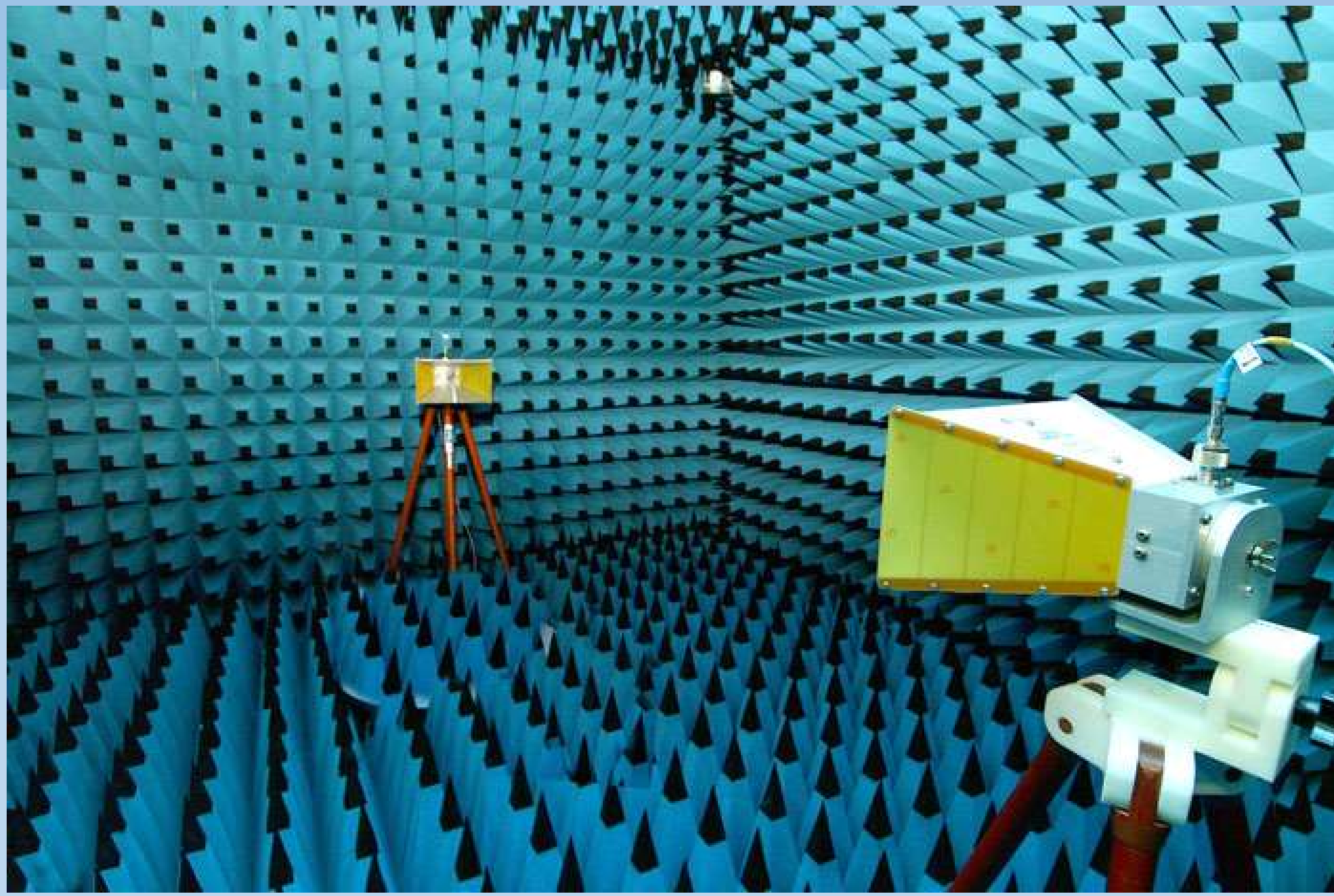


Fig. Antenna measurement set up

Antenna pattern and parameters

- Note that AUT as a receiving antenna measurement will generate the same radiation pattern
  - to that of AUT used as a transmitting antenna
- Another antenna is usually fixed and it is connected in transmitting mode
- The AUT is rotated by a positioner and
  - it can rotate 1-, 2- and 3-degrees of freedom of rotation

Antenna pattern and parameters

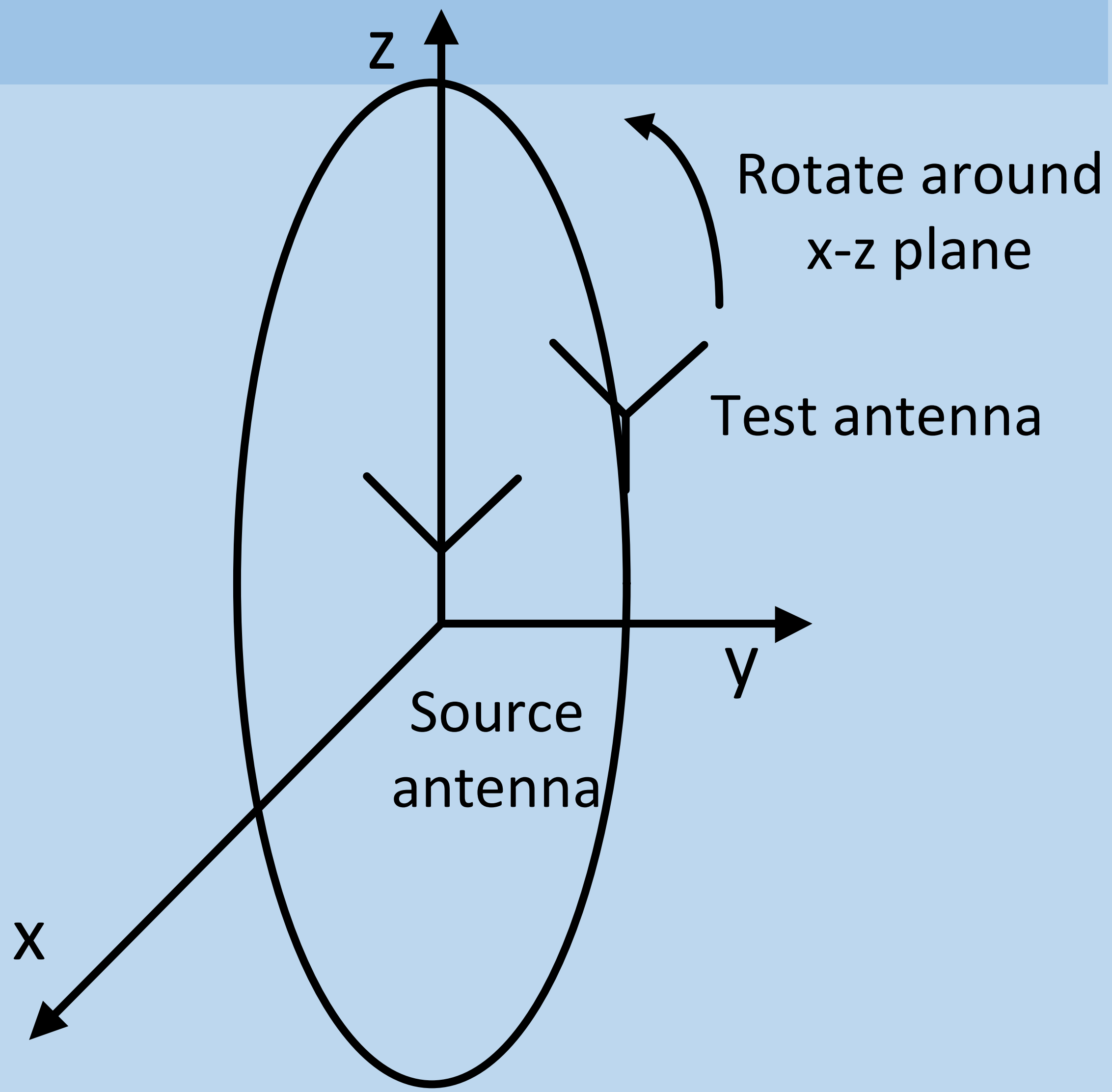
- The AUT is rotated in usually two principal planes (elevation and azimuthal)
- The received field strength is measured by a spectrum analyzer or power meter
  - which will be used to generate the antenna radiation pattern in two principal planes also known as E- and H- planes
- The antenna radiation patterns in these two principal planes can be used to generate the 3-D radiation pattern of an antenna

Antenna pattern and parameters

- Let us consider Hertz dipole which is a vertically polarized antenna
- For E-plane **co-polar measurements**,
  - source and test antenna are oriented vertically
  - so that the main beam of the test antenna (receiving antenna) is directed towards y-axis
- Source antenna is kept stationary and
  - we rotate the test antenna in the x-z plane
  - and record the electric fields variation over θ

Antenna pattern and parameters

- Fig. E-plane co-polar measurements

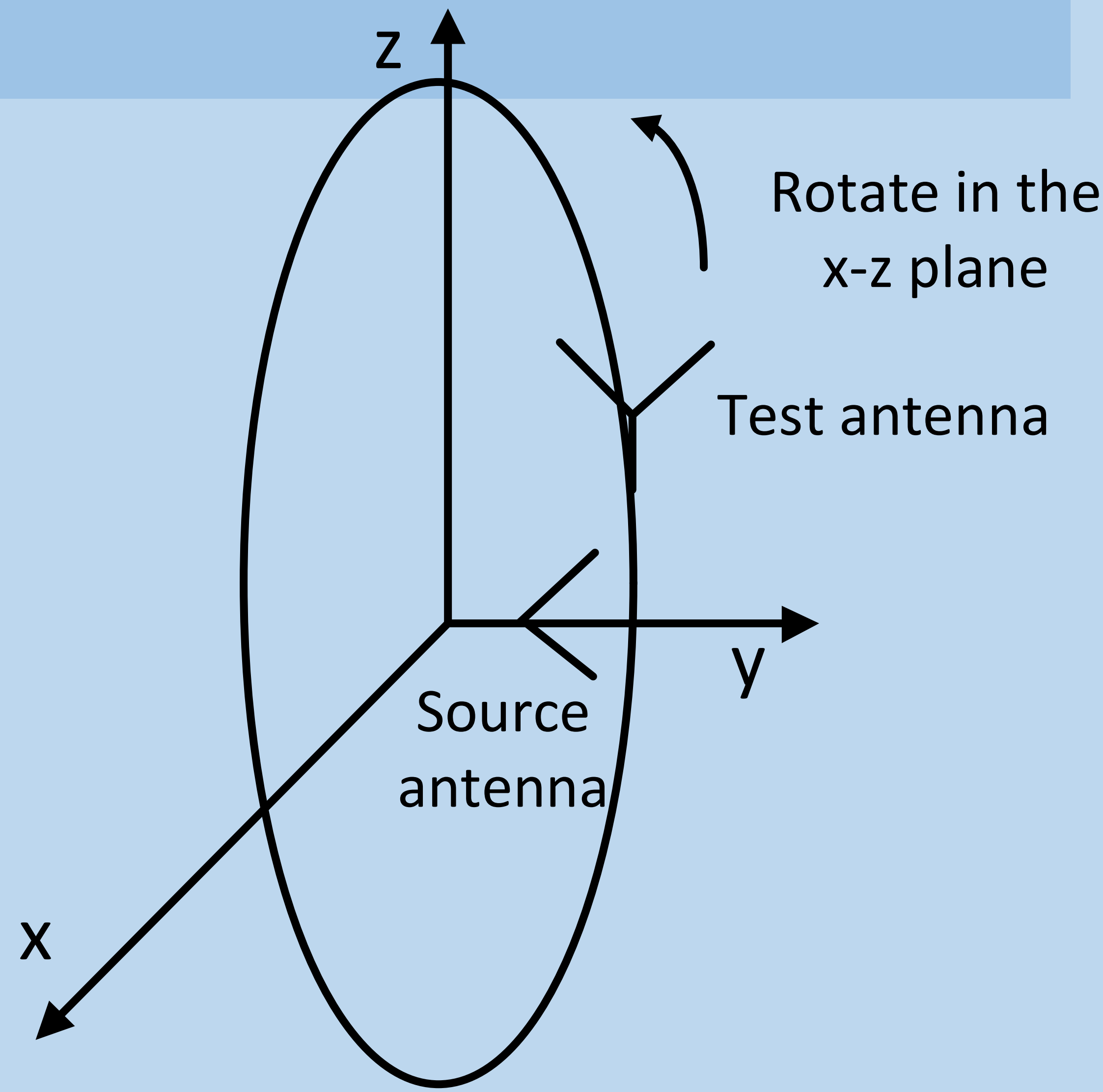


Antenna pattern and parameters

- For E-plane **cross-polar measurements**,
  - source is oriented horizontally and test antenna is oriented vertically
- Source antenna is kept stationary and
  - we rotate the test antenna in the x-z plane
  - and record the electric fields variation over θ

Antenna pattern and parameters

- Fig. E-plane cross-polar measurements

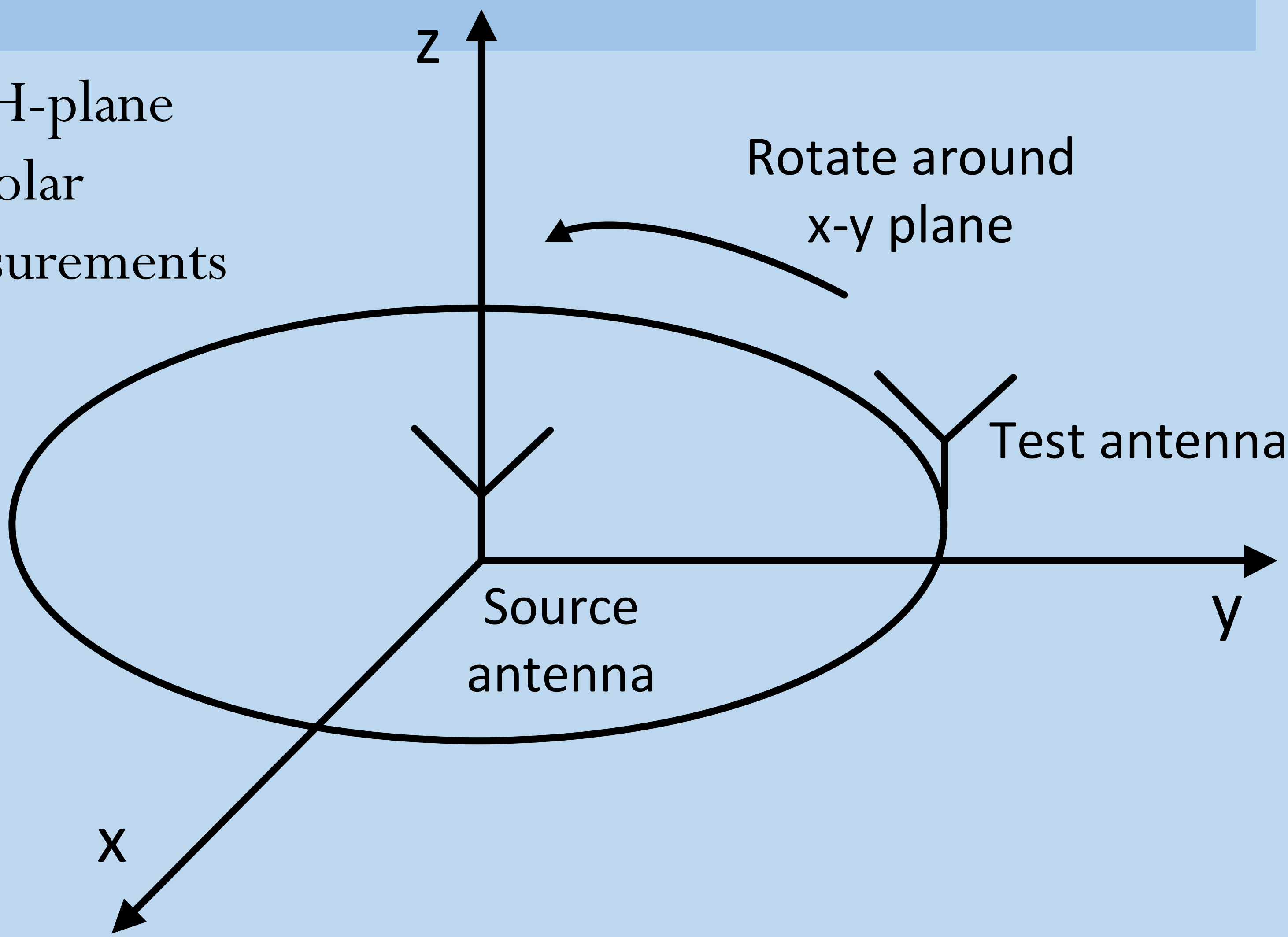


Antenna pattern and parameters

- For H-plane **co-polar measurements,**
  - source and test antenna are oriented vertically
- Source antenna is kept stationary and
  - we rotate the test antenna in the x-y plane
  - and record the electric fields variation over φ

Antenna pattern and parameters

- Fig. H-plane co-polar measurements

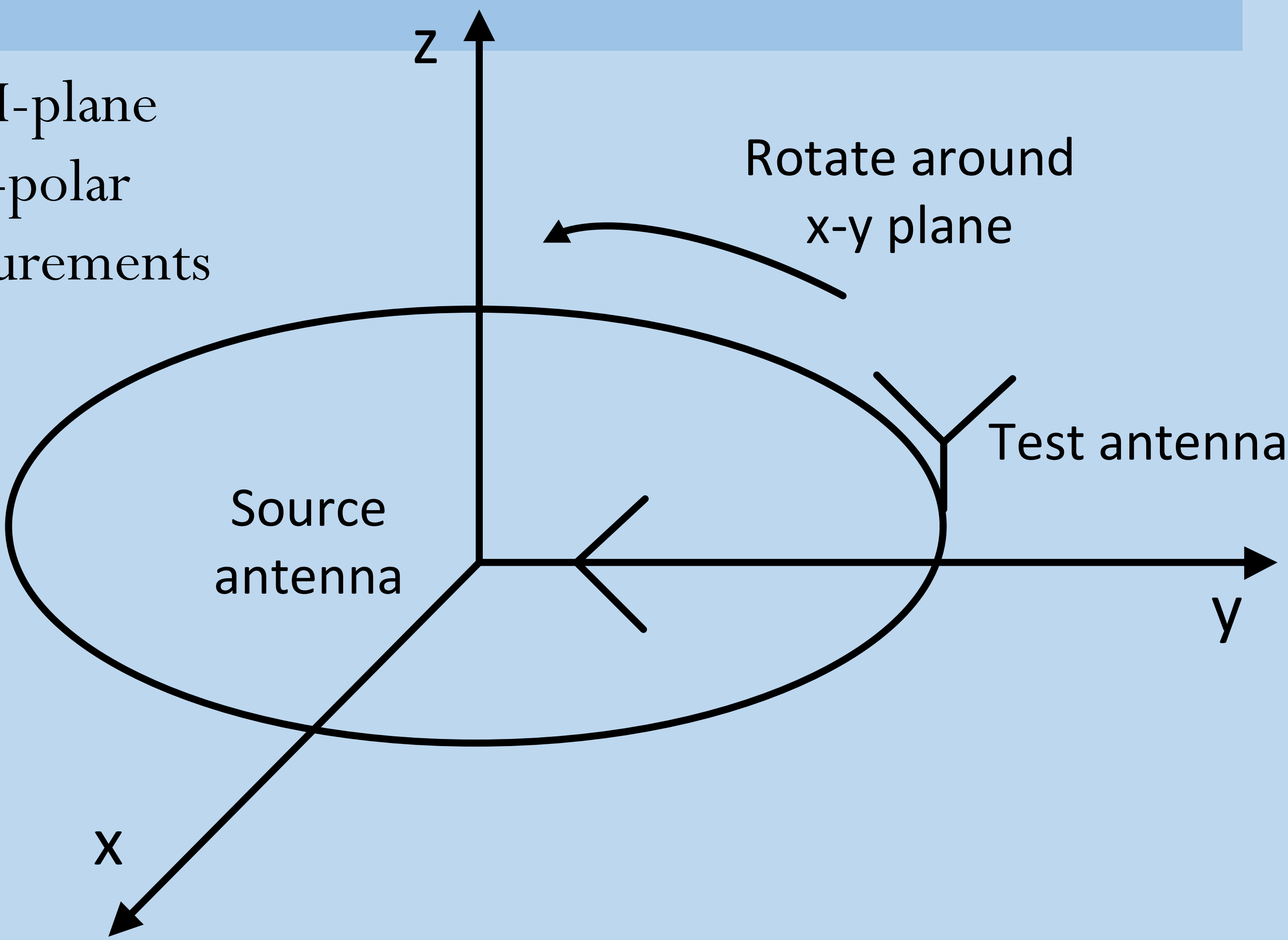


Antenna pattern and parameters

- For H-plane **cross-polar measurements**,
  - source is oriented horizontally and test antenna is oriented vertically
- Source antenna is kept stationary and
  - we rotate the test antenna in the x-y plane
  - and record the electric fields variation over φ

8.3 Antenna pattern and parameters

- Fig. H-plane cross-polar measurements



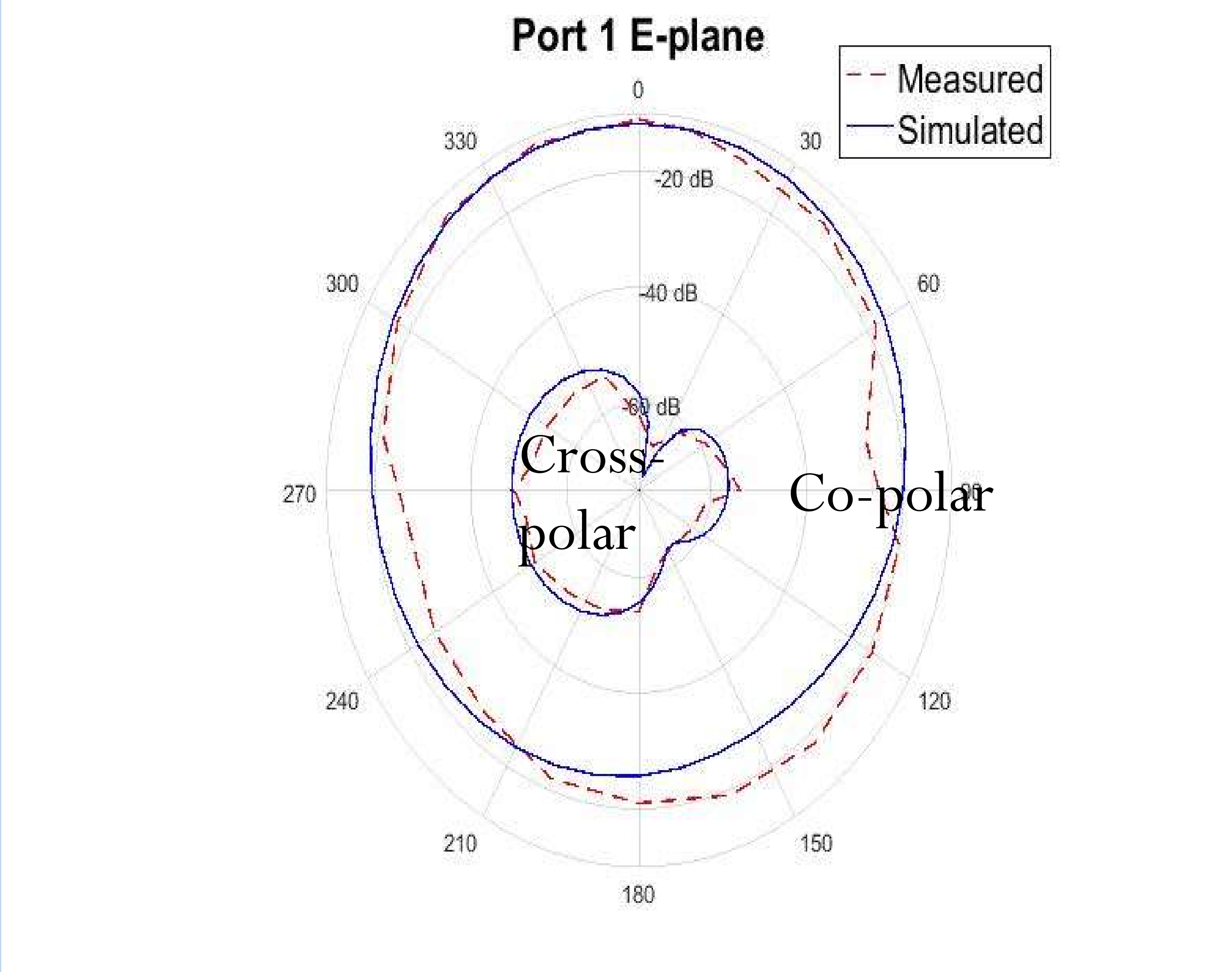


Fig. Typical E-plane pattern

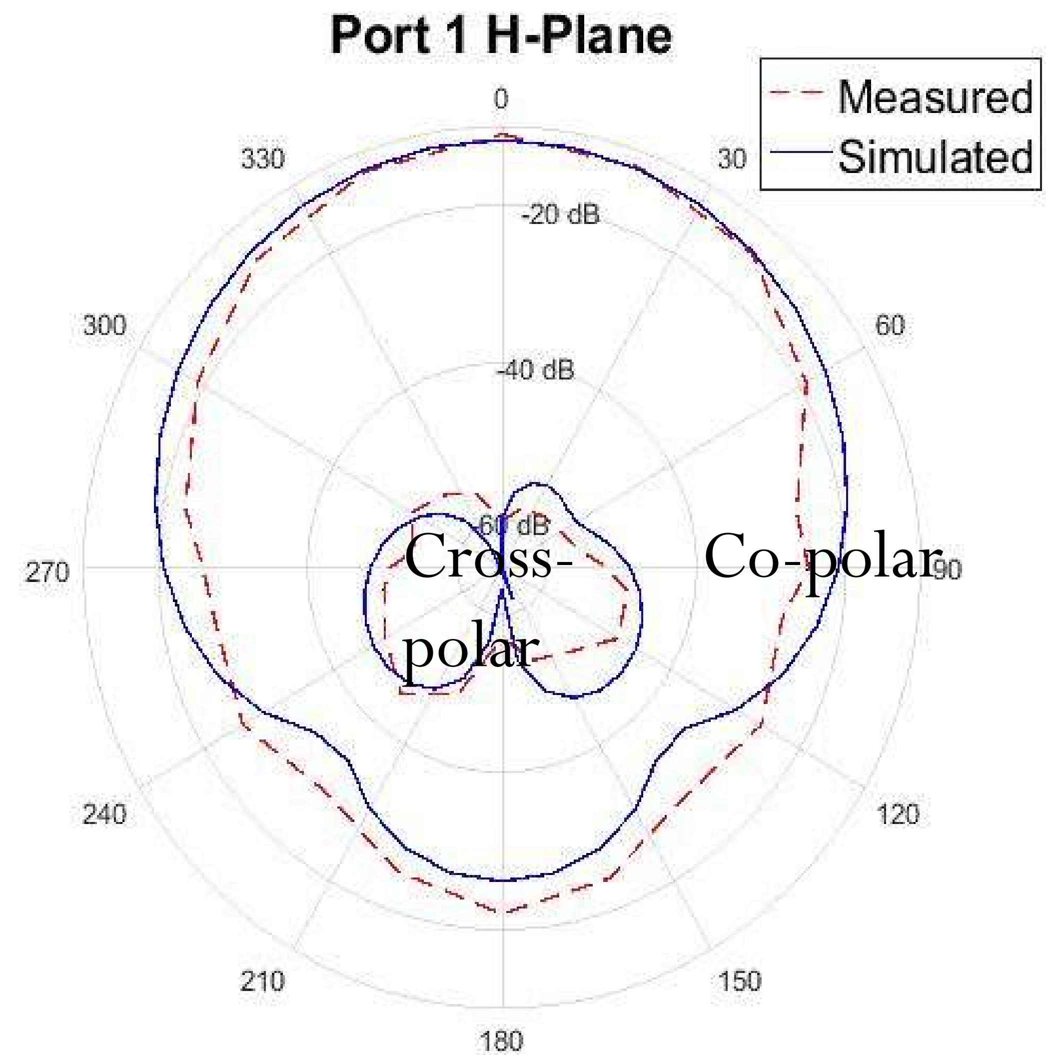


Fig. Typical H-plane pattern

Antenna pattern and parameters

- Let us look at the antenna input characteristics
- Antenna is considered as a load to the transmission line
- Hence one can find out the equivalent circuit of an antenna
- Antenna is a resonator
- It can be equivalently modelled as a series RLC circuit (simplest model)

Antenna pattern and parameters

Quality factor and bandwidth

- The equivalent circuit of a resonant antenna can be
  - approximated by a series RLC resonant circuit
  - where $R=R_r+R_L$ are the radiation and loss resistances,
  - L is the inductance and
  - C is the capacitance of the antenna

Antenna pattern and parameters

- For a resonant antenna like dipoles,
  - the FBW is related to the radiation efficiency and quality factor Q (FBW=1/Q)
- The quality factor of an antenna is defined as $2\pi f_0$ (f_0 is the resonant frequency) times the energy stored over the power radiated and Ohmic losses

$$Q = 2\pi f_0 \frac{\frac{1}{4}|I|^2 L + \frac{1}{4}|I|^2 \frac{1}{(2\pi f_0)^2 L}}{\frac{1}{2}|I|^2 (R_r + R_L)} = \frac{2\pi f_0 L}{R_r + R_L} = \frac{1}{2\pi f_0 (R_r + R_L) C}$$

$$= \frac{1}{2\pi f_0 R_r C} \frac{R_r}{(R_r + R_L)} = Q_{lossless} \bullet e_{rad}$$

Antenna pattern and parameters

- where $Q_{lossless}$ is the quality factor when the antenna is lossless (R_L=0) and
- e_{rad} is the antenna radiation efficiency.
- Note that the radiation efficiency of an antenna is defined
  - as the ratio of the power delivered
  - to the radiation resistance R_r to the power delivered to R_r and R_L

$$e_{rad} = \frac{\frac{1}{2}|I|^2 R_r}{\frac{1}{2}|I|^2 (R_r + R_L)} = \frac{R_r}{(R_r + R_L)}$$

Antenna pattern and parameters

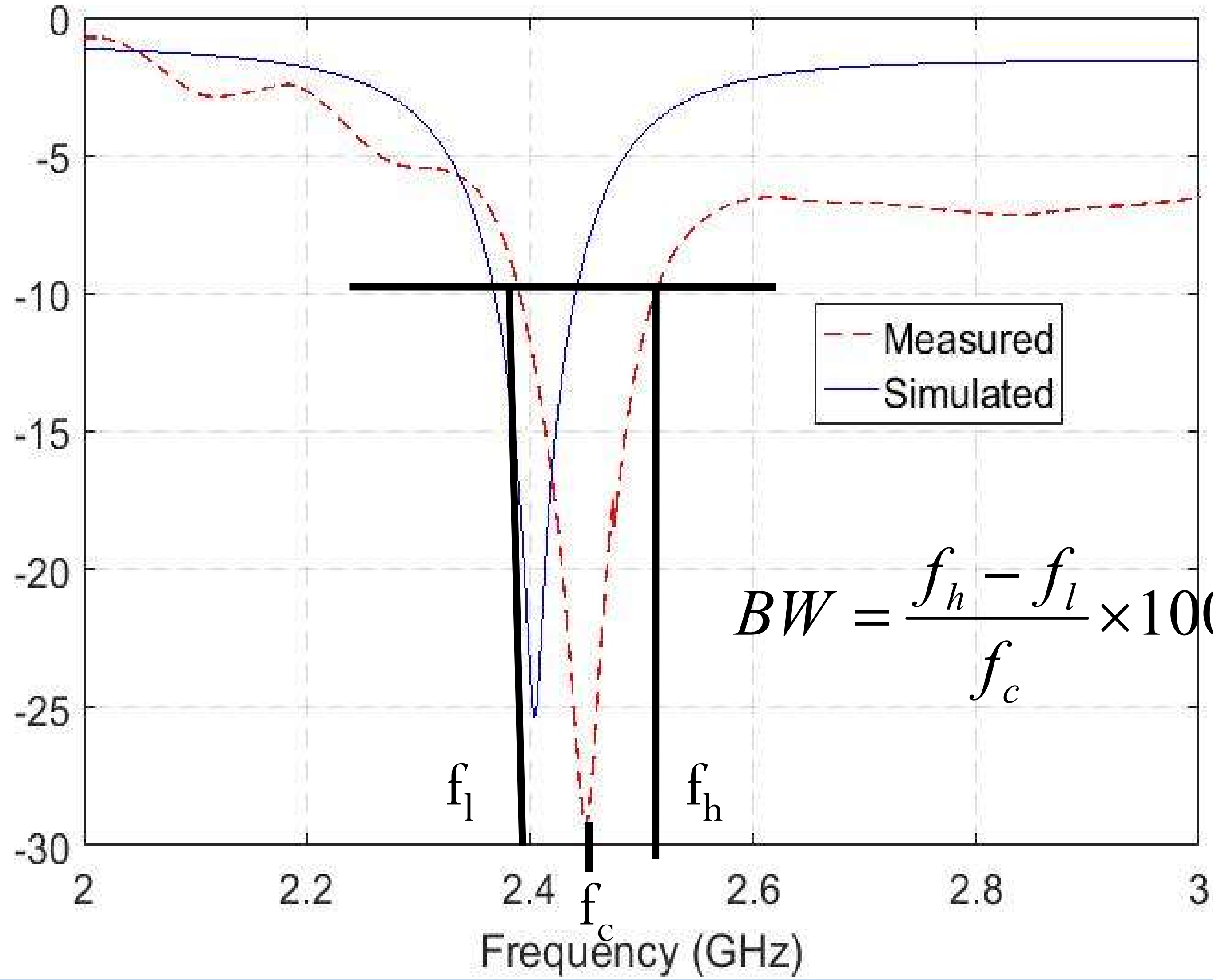
- The feed line also has a characteristic impedance Z_0 usually of 50 Ohm
- At the input of the antenna, the impedance seen by the feed line can be assumed as Z_L
- Then the reflection coefficient and VSWR may be calculated as

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}; VSWR = \frac{1+|\Gamma|}{1-|\Gamma|}$$

$$0 \le |\Gamma| \le 1, 1 \le VSWR \le \infty$$

$$BW : VSWR \le 2$$

$$|\Gamma|(dB)$$



$$BW = \frac{f_h - f_l}{f_c} \times 100\%$$