EE540 Advance Electromagnetic Theory & Antennas

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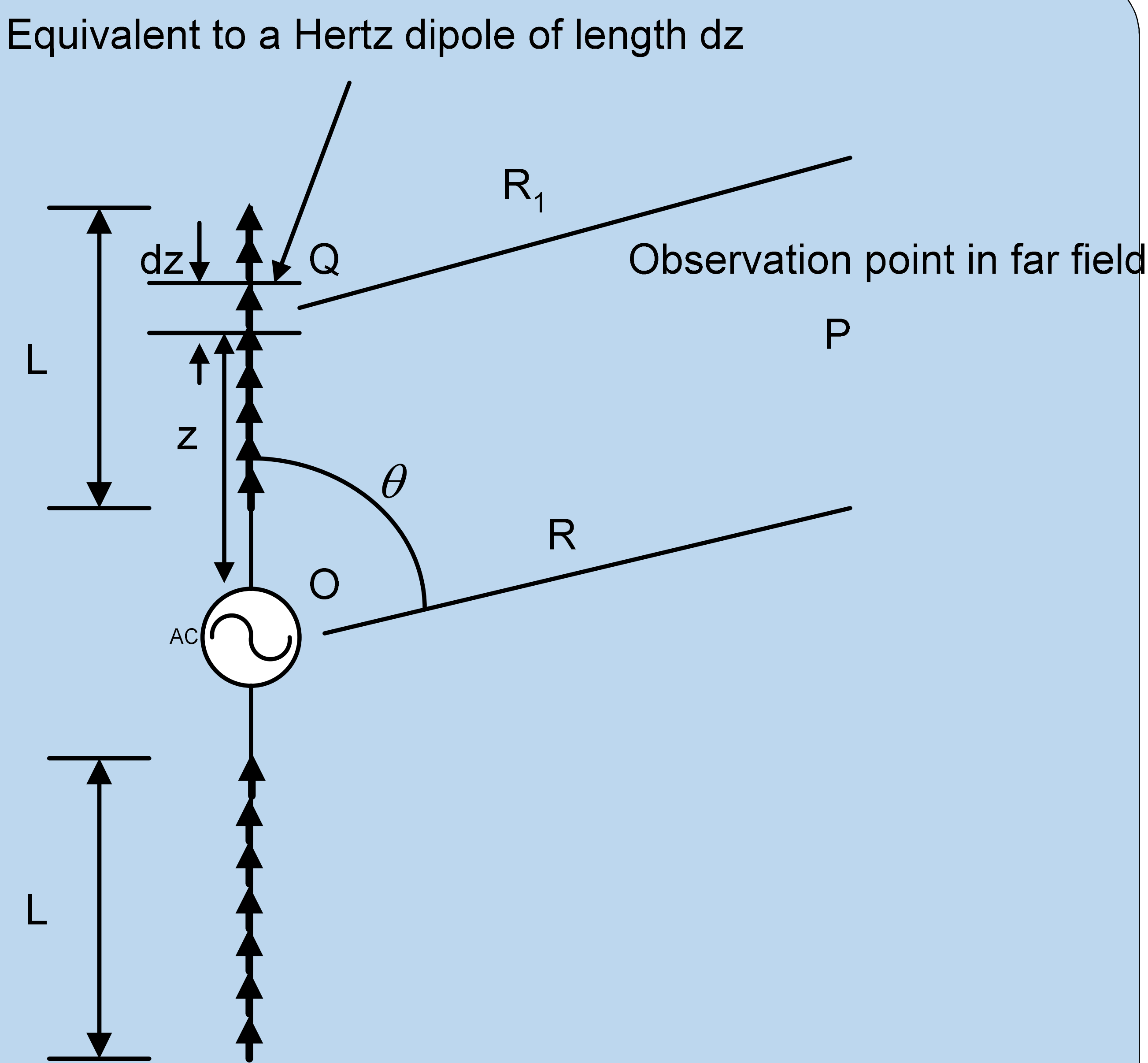
Kinds of antennas

- **Dipole antenna**
- The next extension of a Hertz dipole is
  - a linear antenna or a dipole of finite length as depicted in Fig.
- It consists of a conductor of length 2L
  - fed by a voltage or current source at its center
- We will first find the current distribution on such antennas

- Fig. (a) Dipole of length 2L (Dipole can be assumed to composed of many Hertz dipoles)

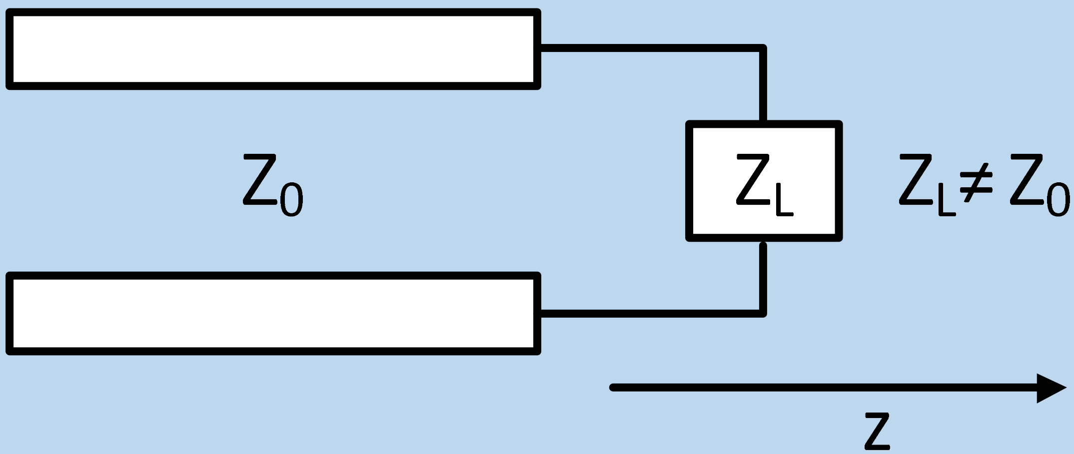


Kinds of antennas

- Let us assume a transmission line loaded with a load Z_L
- Since the line impedance and load impedance may be different
- Any wave incident from the line to the load will be reflected
- Hence at any location along the line,
  - there will be both reflected and incident wave which may be expressed as

$$V(z) = V_0^+ e^{-j\beta z} + V_0^- e^{+j\beta z}$$

Kinds of antennas



- Fig. A transmission line loaded with an impedance of Z_L

Kinds of antennas

- Corresponding electric current wave along the line may be expressed as

$$I(z)=\frac{V_0^+}{Z_0}e^{-j\beta z}-\frac{V_0^-}{Z_0}e^{+j\beta z}=\frac{V_0^+}{Z_0}\left(e^{-j\beta z}-\Gamma e^{+j\beta z}\right);\Gamma=\frac{V_0^-}{V_0^+}=\frac{Z_L-Z_0}{Z_L+Z_0}$$

- The characteristic impedance of the line is defined as

$$Z_0=\frac{V_0^+}{I_0^+}=-\frac{V_0^-}{I_0^-}$$

- For o.c. transmission line

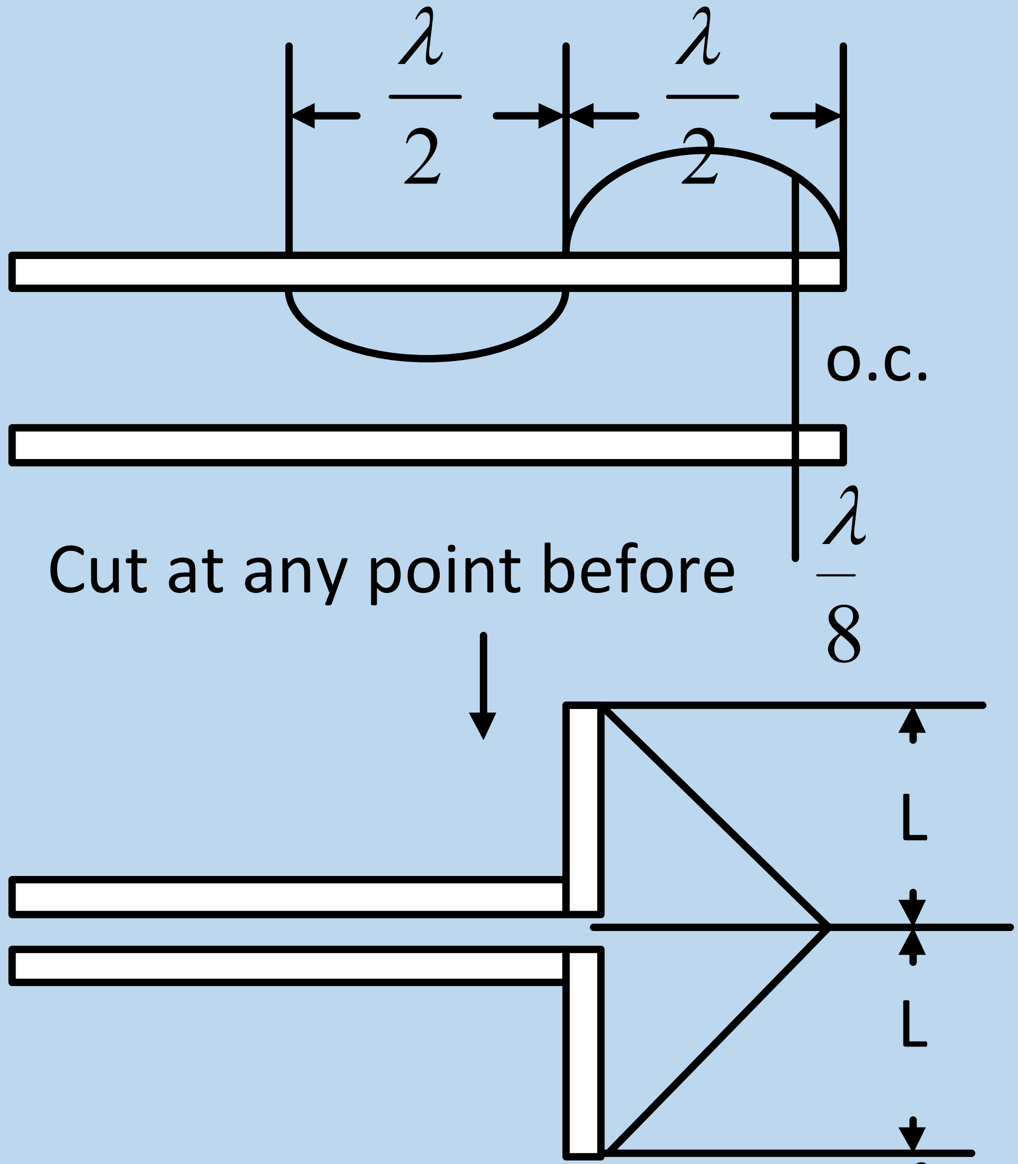
$$I(z)=\frac{V_0^+}{Z_0}\left(e^{-j\beta z}-e^{+j\beta z}\right)=-2j\frac{V_0^+}{Z_0}\sin\beta z$$

Kinds of antennas

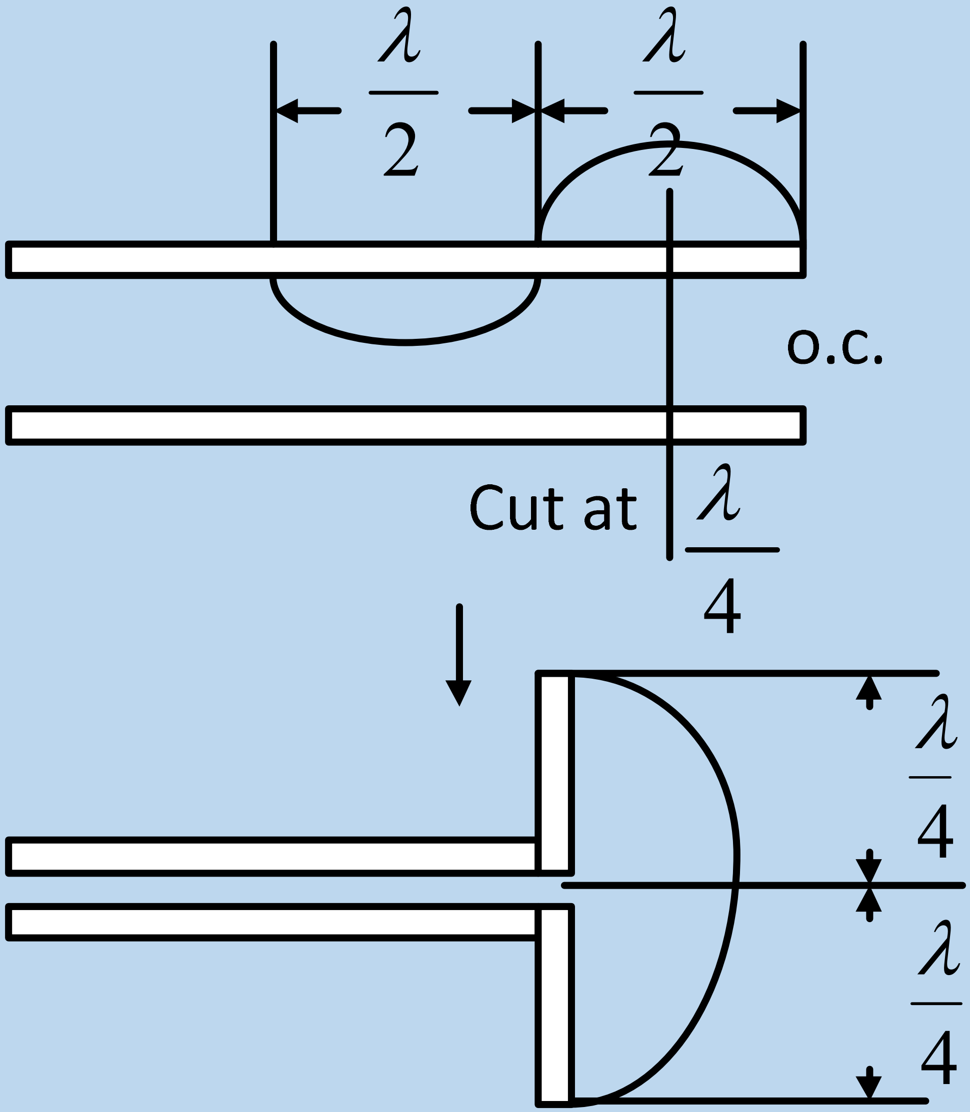
- The current is zero at z= L
- since at the ends, there is no path for the current to flow
  - so, we can write,

$$I(z) = -2j\frac{V_0^+}{Z_0}\sin(\beta(L-|z|)) = I_0 \sin\left[\beta\left(L-|z|\right)\right]; I_0 = -2j\frac{V_0^+}{Z_0}$$

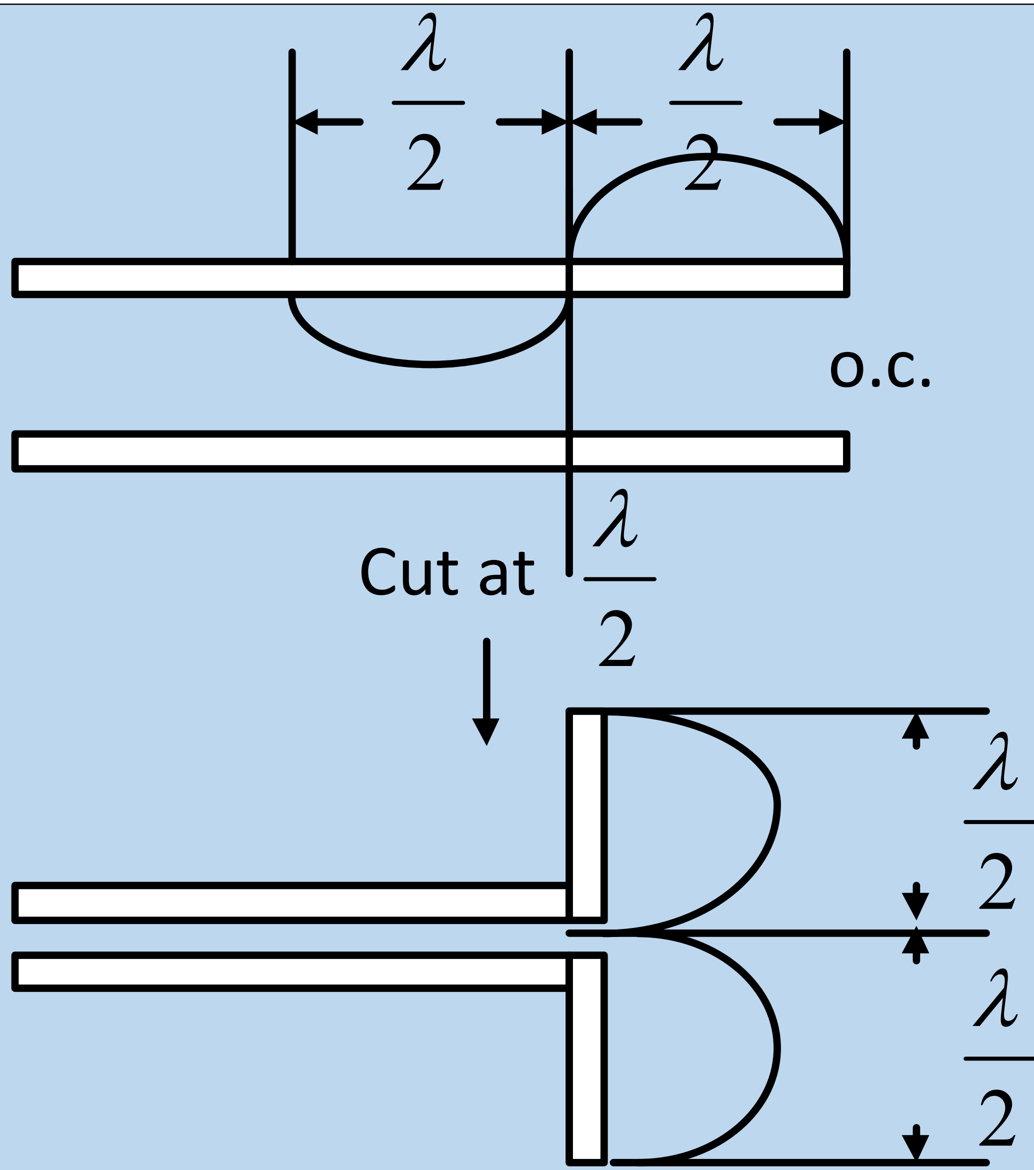
- How to plot the current distribution of dipoles of various length?



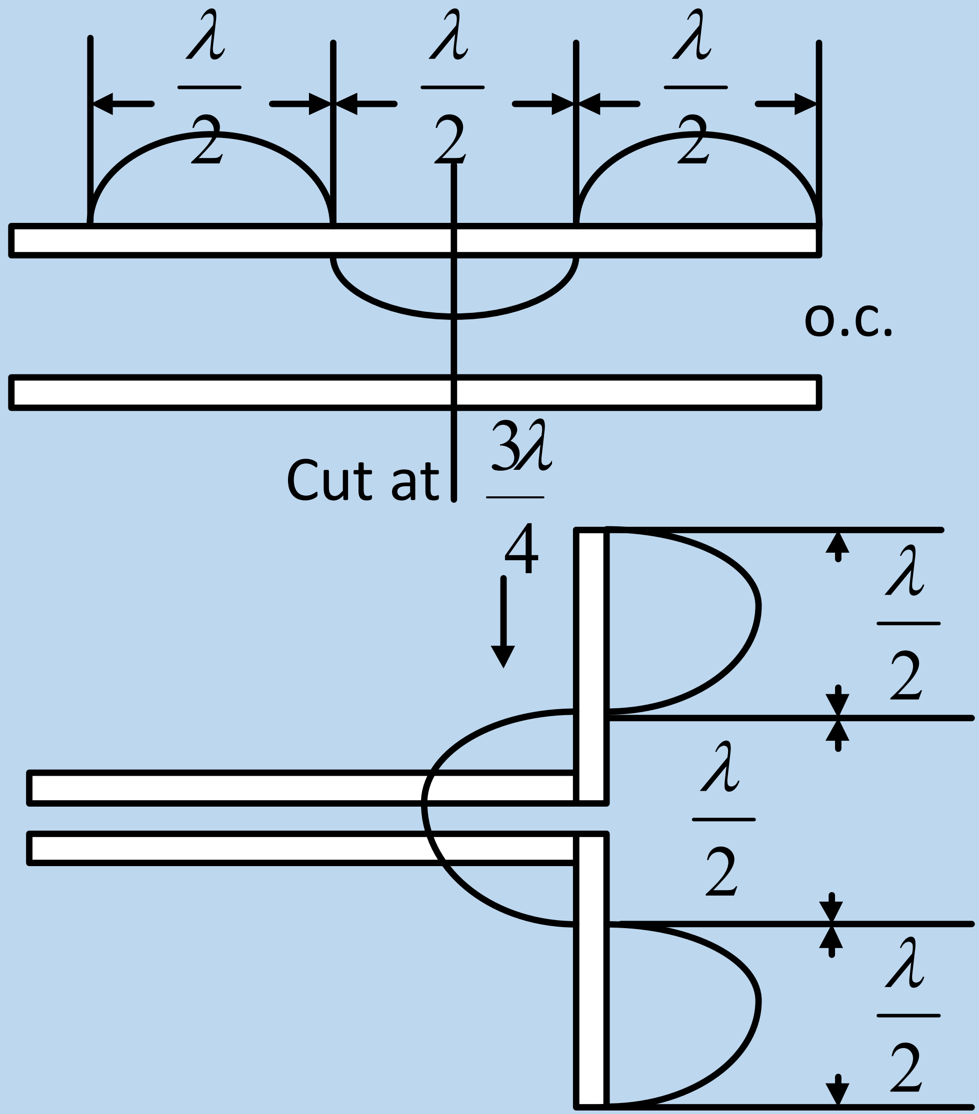
- Fig. Current distribution of a $2L<<\lambda$ dipole & $L<\lambda/8$



- Fig. Current distribution of a λ/2 dipole



- Fig. Current distribution of a λ dipole



- Fig. Current distribution of a $3\lambda/2$ dipole

Kinds of antennas

How to find the electric field?

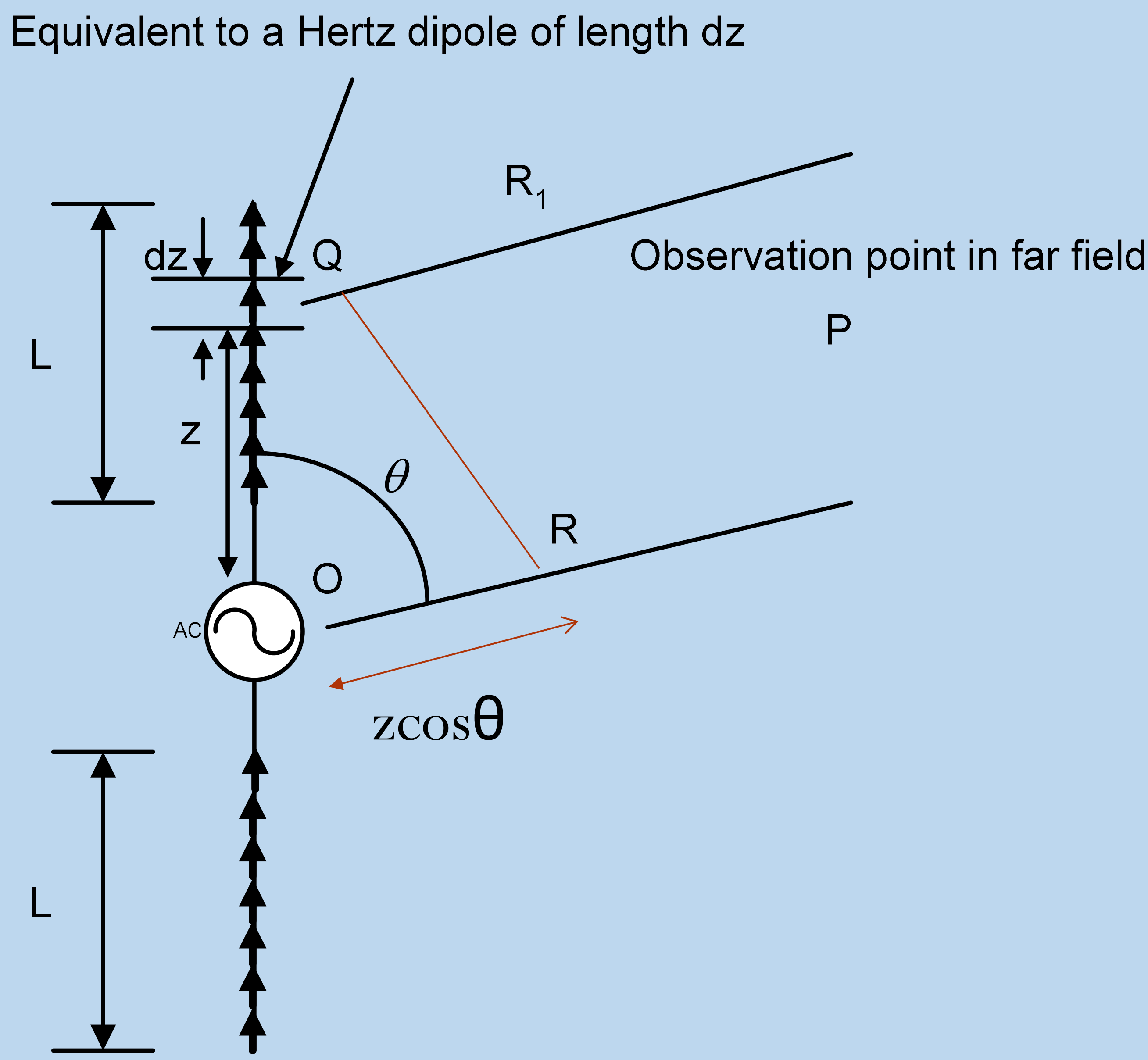
- The electric field due to the current element dz
  - *it has the same expression of the Hertz dipole of the previous section*
    - *except that now we have a length of dz and current of I(z)*
- at far away observation point or in the ***far field*** can be written as

$$dE_\theta = \frac{j\beta^2 \sin\theta I(z) dz e^{-j\beta R_1}}{4\pi\varepsilon\omega R_1}; dH_\phi = \frac{dE_\theta}{\eta_0}$$

- Fig. (a) Dipole of length 2L

(Dipole can be assumed

to composed of many Hertz dipoles)



Kinds of antennas

- Since the observation point P is at a very far distance, the lines OP and QP are parallel and therefore $\therefore R_1 \cong R - z\cos\theta$
- Note that for the amplitude, we can approximate $\dfrac{1}{R_1} \cong \dfrac{1}{R}$
  - since the dipole size is quite small in comparison to the distance of the observation point P from the origin
- Hence

$$dE_\theta \cong \frac{j\beta^2 \sin\theta I(z) dz e^{-j\beta R} e^{j\beta Z\cos\theta}}{4\pi\varepsilon\omega R}$$

Kinds of antennas

- Since we have assumed that the dipole of length 2L is composed of many Hertz dipoles as depicted in Fig. (a)
  - (this is one of the reasons why we say that Hertz dipole or infinitesimal dipole is the building block for many antennas),
- we can write the total radiated electric field as

$$E_\theta = \int_{z=-L}^{L} dE_\theta = \int_{z=-L}^{L} \frac{j\beta^2 \sin\theta I(z) e^{-j\beta R} e^{j\beta Z\cos\theta}}{4\pi\varepsilon\omega R} dz = \int_{z=-L}^{L} \frac{j\beta^2 \sin\theta I_0 \sin(\beta(L-|z|)) e^{-j\beta R} e^{j\beta Z\cos\theta}}{4\pi\varepsilon\omega R} dz$$

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- It can be shown that (see textbook for derivations)

$$\Rightarrow E_\theta \cong j60I_0 \frac{e^{-j\beta R}}{R}\left[\frac{\cos(\beta L\cos\theta)-\cos\beta L}{\sin\theta}\right] = j60I_0 \frac{e^{-j\beta R}}{R} F(\theta)$$

- In the previous equation,
  - the term under bracket is F(θ)
  - and it is the variation of electric field as a function of θ and
- it is the E-plane radiation pattern

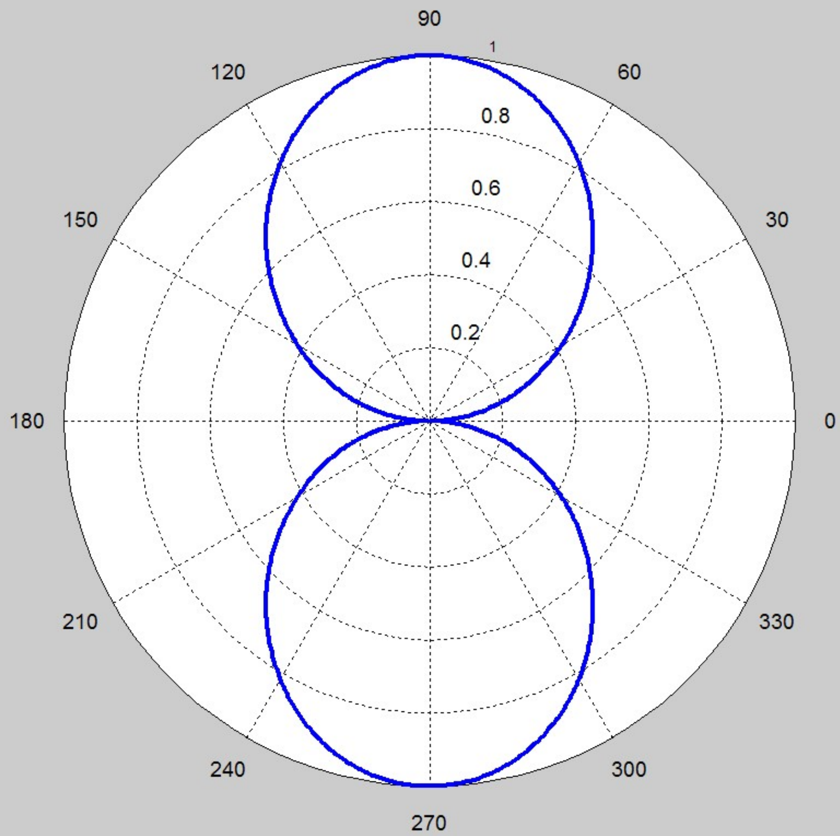
Kinds of antennas

- In the H-plane like that of Hertz dipole,
  - E_θ is a constant and it is not a function of ϕ
- hence it is a circle
- The E–plane radiation pattern of the dipole
  - varies with the length of the dipole as depicted in Fig.

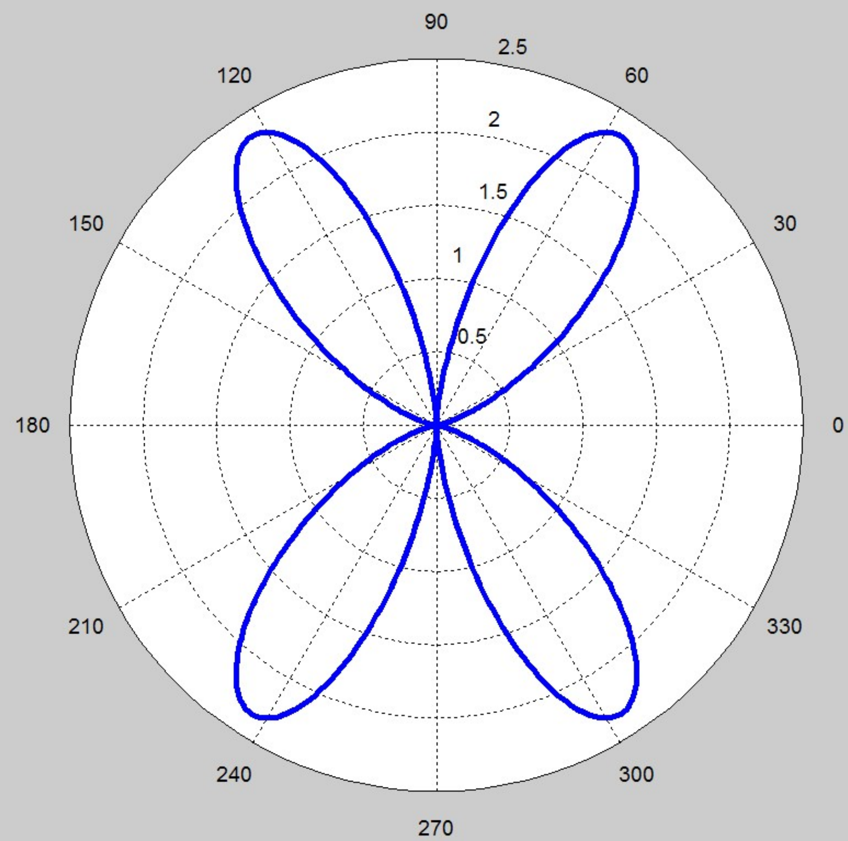
Kinds of antennas

- Note that
  - we have considered the total dipole length is 2L
- Fig. E-plane radiation pattern for dipole of length
  - (a) 2L=2×λ/4= λ/2
  - (b) 2L=2×λ=2λ
  - (c) 2L=2×2λ=4λ

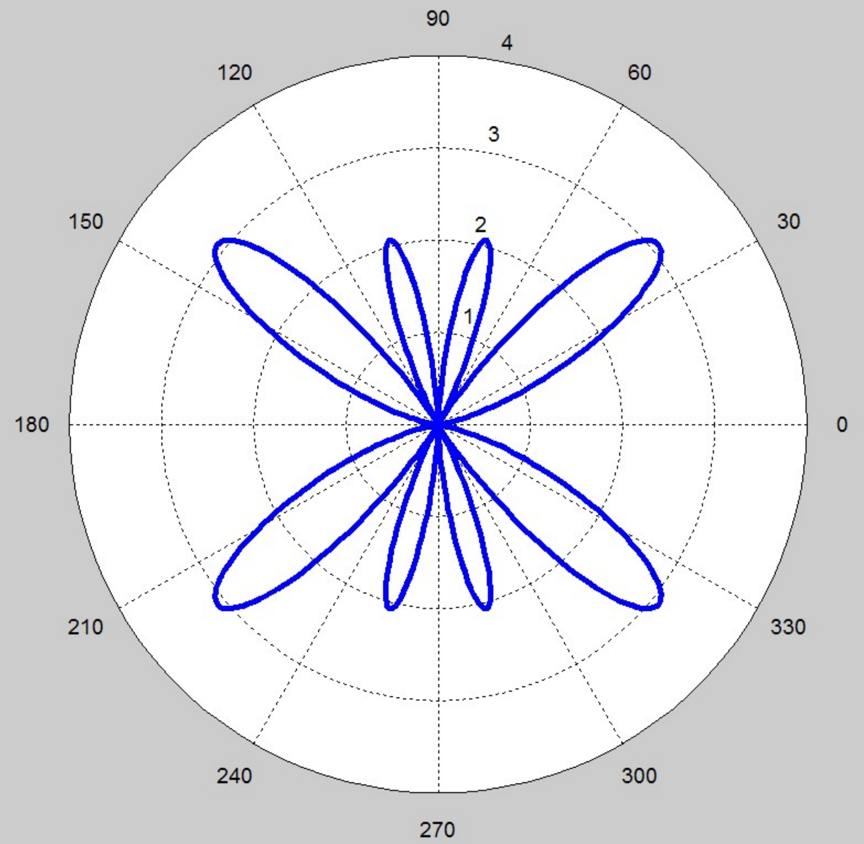
$$\Rightarrow E_\theta \cong j60I_0 \frac{e^{-j\beta R}}{R}\left[\frac{\cos(\beta L\cos\theta)-\cos\beta L}{\sin\theta}\right] = j60I_0 \frac{e^{-j\beta R}}{R} F(\theta)$$



- (a) $2L = 2 \times \lambda/4 = \lambda/2$



- (b) $2L = 2\times\lambda = 2\lambda$



- (c) $2L=2\times2\lambda=4\lambda$

Kinds of antennas

Points to be noted:

1. Input impedance of the dipole (z=0, center of dipole)

$$Z_{in} = \frac{V_{in}}{I_{in}} = \frac{V_{in}}{I_0 \sin \beta L}$$

- For dipole of length 2L, where L =odd multiples of $\frac{\lambda}{4}, \beta L = \frac{m\pi}{2}, \sin \beta L = 1, \quad Z_{in} = \frac{V_{in}}{I_0}$

- For dipole of length 2L, where L =even multiples of . $\frac{\lambda}{4}, \beta L = m\pi, \sin \beta L = 0 \Rightarrow Z_{in} = \infty$

Kinds of antennas

- That's why
  - it is preferable to have dipoles of length odd multiples of $\lambda/2$,
- otherwise
  - it is difficult to have a source with infinite impedance

2. Since increasing the dipole length more and more current is available for radiation,
   - the total power radiated increases monotonically

Kinds of antennas

3. The electric field has only $\hat{\theta}$ component and hence it is linearly polarized
4. The radiation pattern have nulls and it can be calculated by equating $F(\theta)=0$

Kinds of antennas

$$\frac{\cos(\beta L\cos\theta_{null})-\cos\beta L}{\sin\theta_{null}}=0$$

$$\Rightarrow \cos\theta_{null} = \pm 1 \pm \frac{m\lambda}{\pi}$$

- For m=0, $\cos\theta_{null} = \pm 1, \theta_{null} = 0, \pi$
- But, in denominator $\sin\theta_{null}$ is also zero
- So let us take the limit of F(θ) as θ→0, and see

Kinds of antennas

$$\underset{\theta\to 0,\pi}{Lim}\left\{\frac{\cos(\beta L\cos\theta)-\cos(\beta L)}{\sin\theta}\right\}\cong\underset{\theta\to 0,\pi}{Lim}\frac{1}{\sin\theta}\left[\left\{1-\frac{(\beta L\cos\theta)^2}{2!}+\frac{(\beta L\cos\theta)^4}{4!}\right\}-\left\{1-\frac{(\beta L)^2}{2!}+\frac{(\beta L)^4}{4!}\right\}\right]$$

$$=\underset{\theta\to 0,\pi}{Lim}\left\{\frac{(\beta L)^2\sin^2\theta}{2!}-\frac{(\beta L)^4\left(1-\cos^4\theta\right)}{4!}\right\}\times\frac{1}{\sin\theta}=\underset{\theta\to 0,\pi}{Lim}\left\{\frac{(\beta L)^2\sin\theta}{2!}-\frac{(\beta L)^4\sin\theta\left(1+\cos^2\theta\right)}{4!}\right\}=0$$

5. To find θ for maximum radiation, we have to find the solution of

-
- $$\frac{dF(\theta)}{d\theta}=0$$

- We can also take the mean of the first two nulls to approximate $\theta_{\max}$

Kinds of antennas

- What is the radiation resistance of a dipole antenna?

$$R_r = \frac{2P_{rad}}{I_0^2}$$

- Radiated power can be obtained from the radiation intensity as

$$P_{rad} = \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} U(\theta,\phi) \sin\theta \, d\theta \, d\phi$$

Kinds of antennas

- Radiation intensity can be obtained from the Poynting vector as $$U(\theta,\phi)=r^2 S(\theta,\phi)=\frac{30}{\pi}I_0^2\left|F(\theta)\right|^2$$
- For λ/2 dipole antenna

$$\because \beta L=\frac{2\pi}{\lambda}\times\frac{\lambda}{4}=\frac{\pi}{2},\therefore F(\theta)=\frac{\cos(\beta L\cos\theta)-\cos(\beta L)}{\sin\theta}=\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}$$

- Therefore,

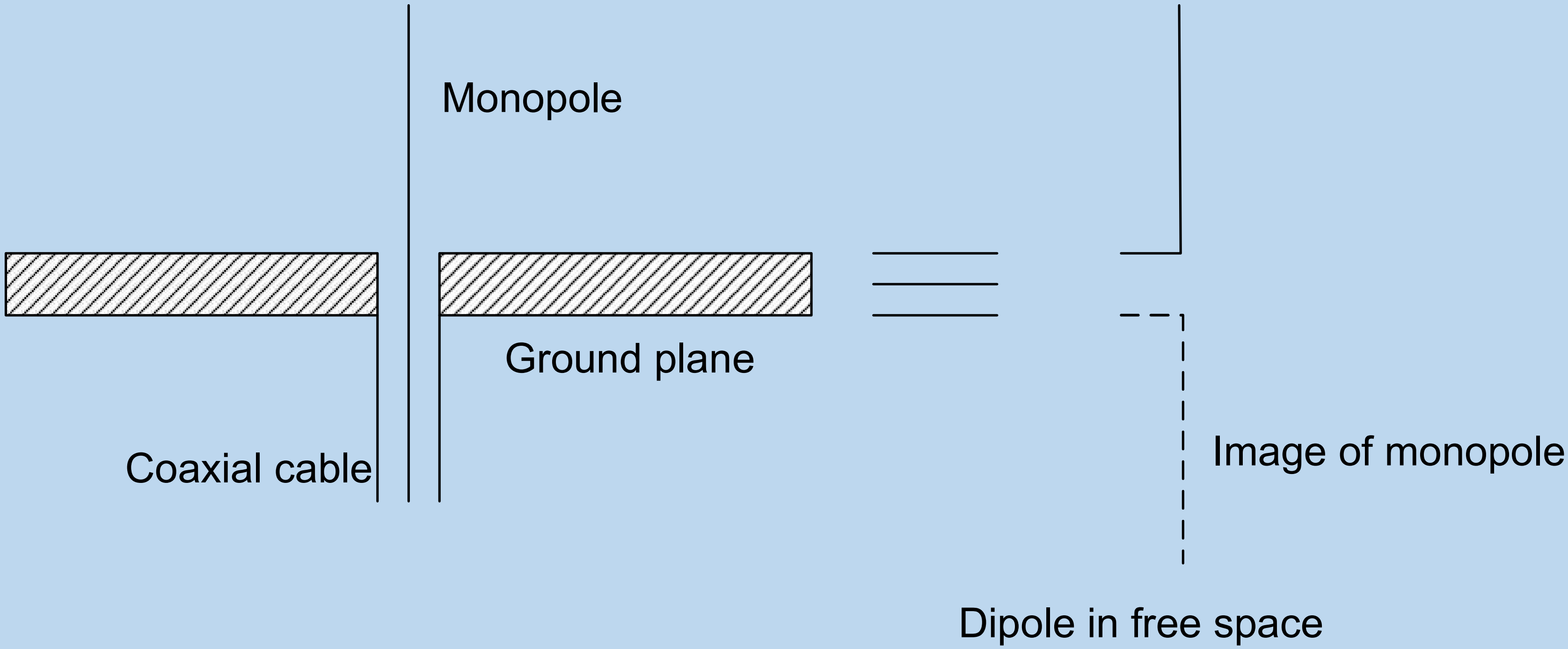
$$P_{rad}=\frac{30}{\pi}I_0^2\int_{\theta=0}^{\pi}\frac{\cos^2\left(\frac{\pi}{2}\cos\theta\right)}{\sin^2\theta}d\theta$$

Kinds of antennas

Monopole antennas:

- A monopole is a dipole that has been divided in half at its center feed point and
  - fed against a ground plane
- Monopole is usually fed from a coaxial cable (see Fig. (b))
- A monopole of length L placed above a perfectly conducting and infinite ground plane
  - will have the same field distribution to that of a dipole of length 2L without the ground plane

Kinds of antennas



- (b) Monopole of length L over a ground plane

Kinds of antennas

- This is because an image of the monopole will be formed inside the ground plane
  - (similar to the method of images)
- The monopole looks like a dipole in free space (see Fig. (b))
- Since this monopole is of length L only,
  - it will radiate only half of the total radiated power of a dipole of length 2L
- Hence, the radiation resistance of a monopole is half that of a dipole

Kinds of antennas

- Similarly, directivity of the monopole is twice that of a dipole
- Since the field distributions are the same for a monopole and dipole,
  - the maximum radiation intensity will be also same for both cases
- But for monopole,
  - the total radiated power is half that of a dipole
- Hence, the directivity of a monopole above a conducting ground plane is
  - twice that of dipole in free space

Kinds of antennas

Loop antenna

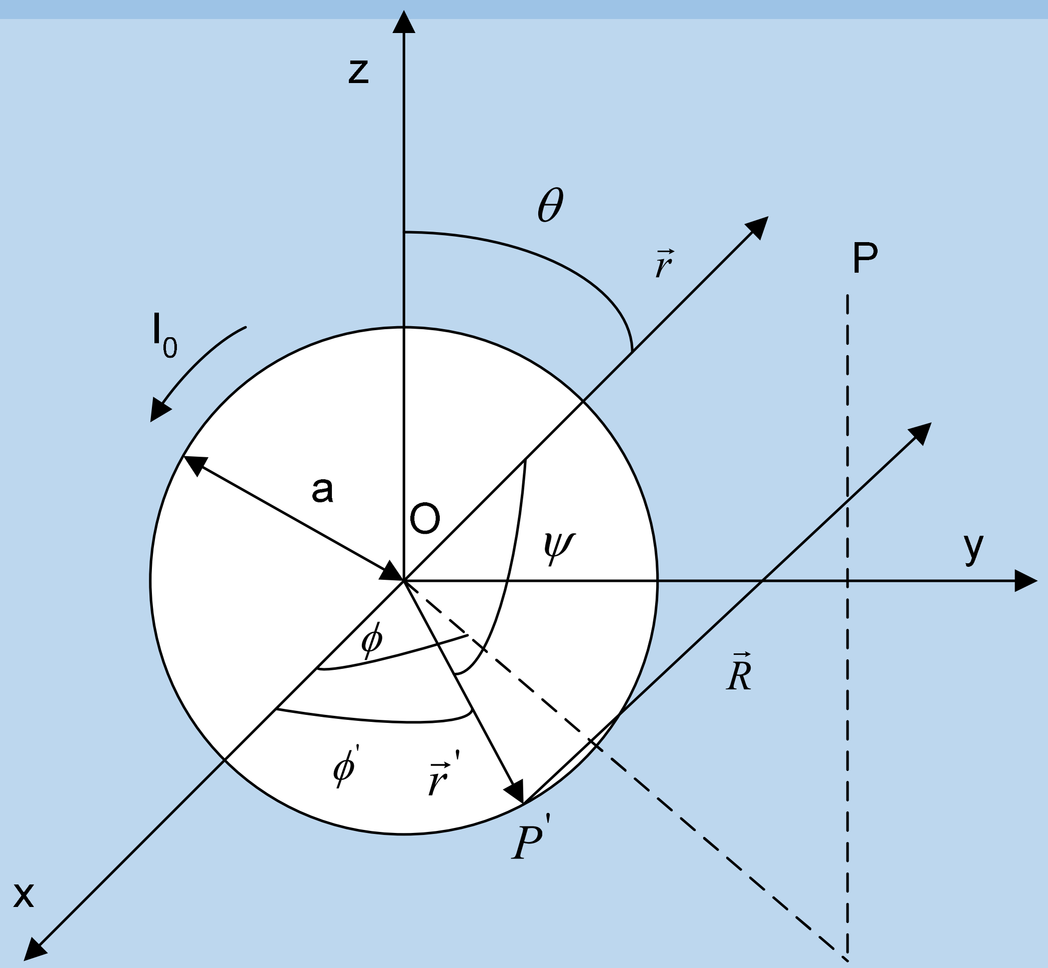
- Loop antennas could be of various shapes:
  - circular,
  - triangular,
  - square,
  - elliptical, etc.
- They are widely used in applications up to 3GHz
- Loop antennas can be classified into two:
  - electrically small (circumference < 0.1 λ) and
  - electrically large (circumference approximately equals to λ)

Kinds of antennas

- Electrically small loop antennas have
  - very small radiation resistance
- They have
  - very low radiation and
  - are practically useless
- Electrically small loop antennas could be analyzed assuming that
  - it is equivalently represented as a Hertz dipole
- Let us consider electrically large circular loop of constant current

Kinds of antennas

- Loop antenna



Kinds of antennas

- We can express magnetic vector potential (see textbook) as

$$A_\phi(\theta) = \frac{\mu I_0 a e^{-j\beta r}}{4\pi r}\pi j\left\{J_1(\beta a\sin\theta) - J_1(-\beta a\sin\theta)\right\}$$

$$\because J_n\left(z\right) = z^n\sum_{m=0}^{\infty}\frac{\left(-1\right)^m z^{2m}}{2^{2m+n}m!\left(n+m\right)!}\therefore J_n(-z) = (-1)^n J_n(z) \Rightarrow J_1(-\beta a\sin\theta) = -J_1(\beta a\sin\theta)$$

$$\therefore A_\phi(\theta) = \frac{j\mu I_0 a e^{-j\beta r} J_1(\beta a\sin\theta)}{2r}$$

- We can express electric field as

$$\therefore \vec{E} = -j\frac{\nabla\left(\nabla\bullet\vec{A}\right)}{\omega\mu\varepsilon} - j\omega\vec{A}$$

Kinds of antennas

- Note that magnetic vector potential has only ϕ component which is a function of θ variable only

$$\therefore \nabla \bullet \vec{A} = 0;$$

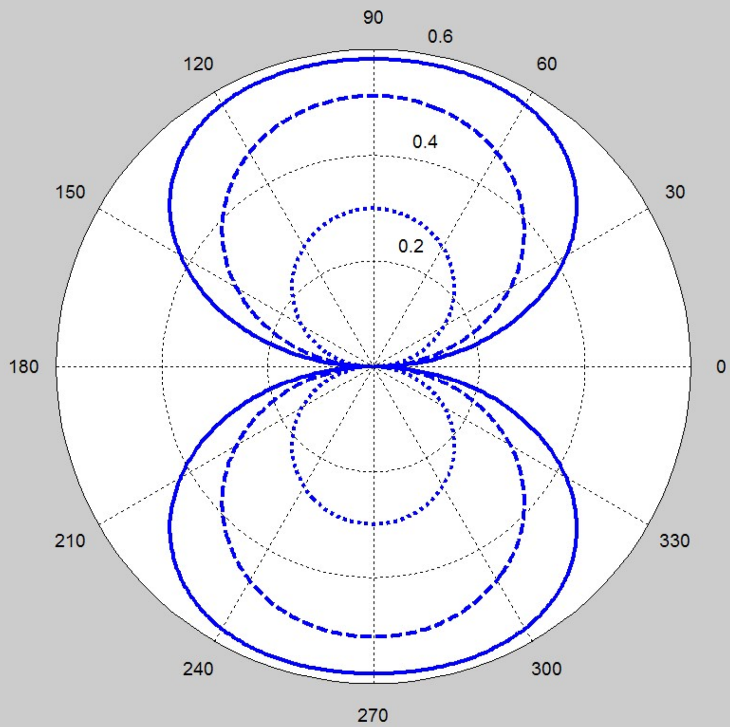
$$\because A_\theta = A_r = 0 \Rightarrow E_\theta = -j\omega A_\theta = 0; E_r = -j\omega A_r = 0 \therefore E_\phi = -j\omega A_\phi = \frac{\omega\mu I_0 a e^{-j\beta r} J_1(\beta a \sin\theta)}{2r}$$

$$H_\theta = -\frac{E_\phi}{\eta} = \frac{-\omega\mu I_0 a e^{-j\beta r}}{2\eta r} J_1\left(\beta a \sin\theta\right); H_r = H_\phi = 0$$

Kinds of antennas

- Poynting vector for a wave radiating in radially outward direction
  - should have direction along positive radial direction
- Therefore H_θ must be negative
- Fig. shows the far-field radiation pattern of the loop antenna
- It can be observed that the radiation field has higher magnitude
  - with the larger radius of the loop antenna
- For larger radiation power we need a loop antenna of larger radius

Kinds of antennas



- Fig. Plot of for various values of angle θ (far-field radiation patterns) (dotted line → a=0.1λ, dashed line → a=0.2λ, solid line → a=0.3λ)

Kinds of antennas

- For small loops

$$\beta a < \frac{1}{3}; J_1(\beta a \sin\theta) = \frac{1}{2}\beta a \sin\theta - \frac{1}{16}(\beta a \sin\theta)^3 + \ldots \cong \frac{1}{2}\beta a \sin\theta$$

$$\therefore E_\phi = \frac{\omega\mu I_0 a}{2}\frac{e^{-j\beta r}}{r}\frac{1}{2}(\beta a \sin\theta); H_\theta = -\frac{\omega\mu I_0 a}{\eta}\frac{e^{-j\beta r}}{2r}\frac{1}{2}(\beta a \sin\theta)$$

- Note that for the dipole polarization was along $\hat{\theta}$ direction
- But for small loop antennas it is along the $\hat{\phi}$ direction