EE540 Advance Electromagnetic Theory & Antennas

Prof. Rakhesh Singh Kshetrimayum

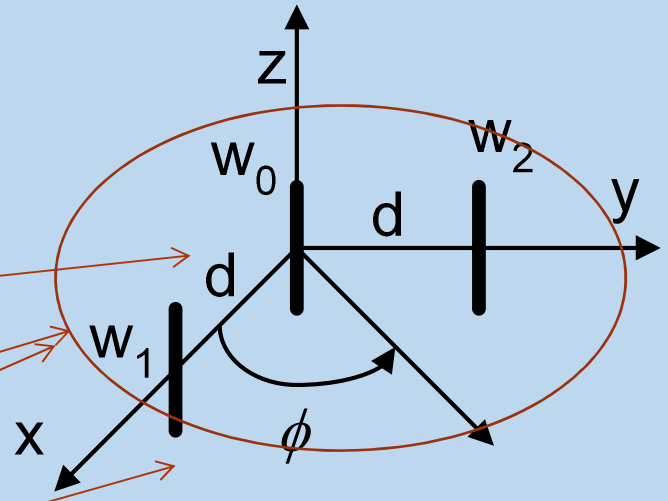
Dept. of EEE, IIT Guwahati, India

Antenna Arrays

- One of the disadvantages of single antenna is that
  - it has fixed radiation pattern
- That means once we have designed and constructed an antenna,
  - the beam or radiation pattern is fixed
- If we want to tune the radiation pattern,
  - we need to apply the technique of antenna arrays
- Antenna array is a
  - configuration of multiple antennas (elements) arranged
  - to achieve a given radiation pattern

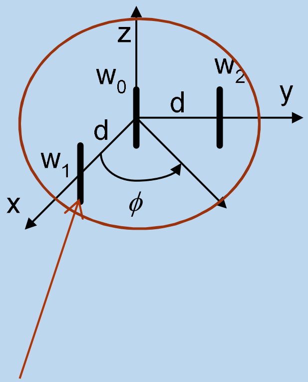
Antenna Arrays

- There are several array design variables
  - which can be changed to achieve the overall array pattern design
- Some of the array design variables are:
- (a) array shape
  - linear,
  - circular,
  - planar, etc.
- (b) element spacing
- (c) element excitation amplitude
- (d) element excitation phase
- (e) patterns of array elements



Antenna Arrays

- Given an antenna array of identical elements,
  - the radiation pattern of the antenna array may be found according to the
    - ***pattern multiplication principle***
- It basically means that array pattern is equal to
  - the product of the
    - **pattern of the individual array element into**
    - **array factor, a function dependent only on**
      - **the geometry of the array**
      - **the excitation amplitude and phase of the elements**



Antenna Arrays

Two element array

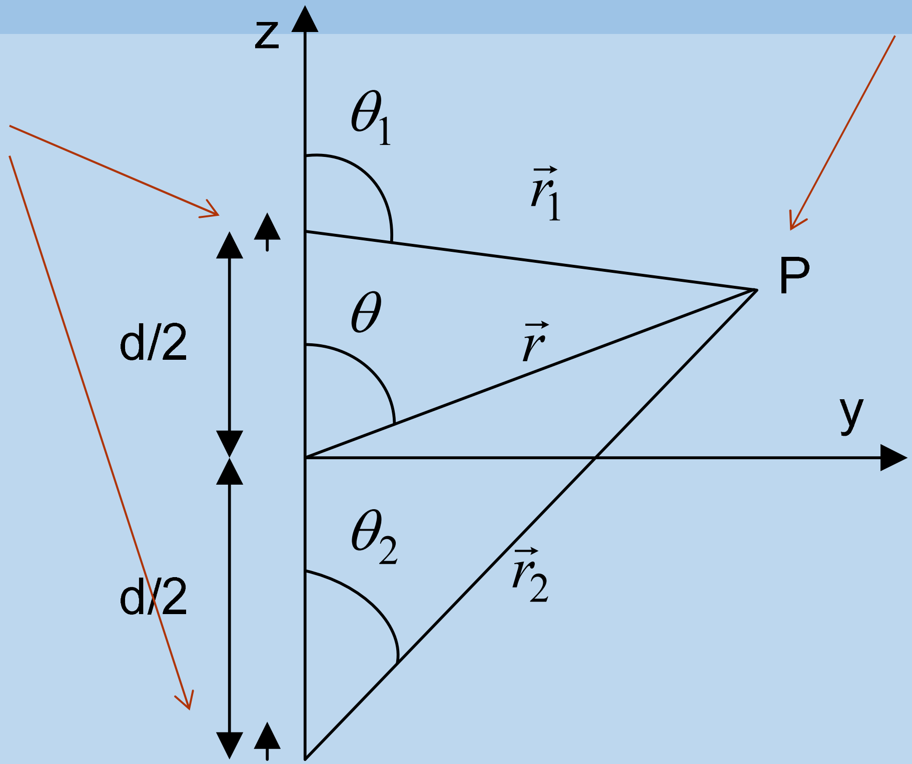
- Let us investigate an array of
  - two infinitesimal dipoles positioned along the z axis as shown in Fig. (a)
- The field radiated by the two elements,
  - assuming no coupling between the elements
- is equal to the sum of the two fields

$$\vec{E}_{total} = \vec{E}_1 + \vec{E}_2 = \hat{\theta}\frac{jI_1 dl \sin\theta e^{-j\beta r_1}\beta^2 e^{j\delta_1}}{4\pi\varepsilon\omega r_1} + \hat{\theta}\frac{jI_2 dl \sin\theta e^{-j\beta r_2}\beta^2 e^{j\delta_2}}{4\pi\varepsilon\omega r_2}$$

- where the two antennas are excited with current

$$I_1 < \delta_1 \quad and \quad I_2 < \delta_2$$

Antenna Arrays



- Fig. (a) Two Hertz dipoles

Antenna Arrays

- For $I_1 = I_2 = I_o$

$$\delta_1 = \frac{\alpha}{2}, \delta_2 = -\frac{\alpha}{2}$$

$$\vec{E}_{total} = \hat{\theta}\frac{jI_0 dl \sin\theta\beta^2 e^{-j\beta r}}{4\pi\varepsilon\omega r}\left(e^{j\left(\frac{\beta d}{2}\cos\theta+\frac{\alpha}{2}\right)} + e^{-j\left(\frac{\beta d}{2}\cos\theta+\frac{\alpha}{2}\right)}\right)$$

$$= \hat{\theta}\frac{jI_0 dl \sin\theta\beta^2 e^{-j\beta r}}{4\pi\varepsilon\omega r} 2\cos\left(\frac{\beta d}{2}\cos\theta + \frac{\alpha}{2}\right)$$

Antenna Arrays

- Hence the total field of the array is equal to
- the field of single element positioned at the origin
- multiplied by a factor which is called as the array factor
- Array factor is given

$$AF = 2\cos\left[\frac{1}{2}\left(\beta d\cos\theta + \alpha\right)\right]$$

- Normalized array factor is

$$AF_2 = \cos\left[\frac{1}{2}\left(\beta d\cos\theta + \alpha\right)\right]$$

Antenna Arrays

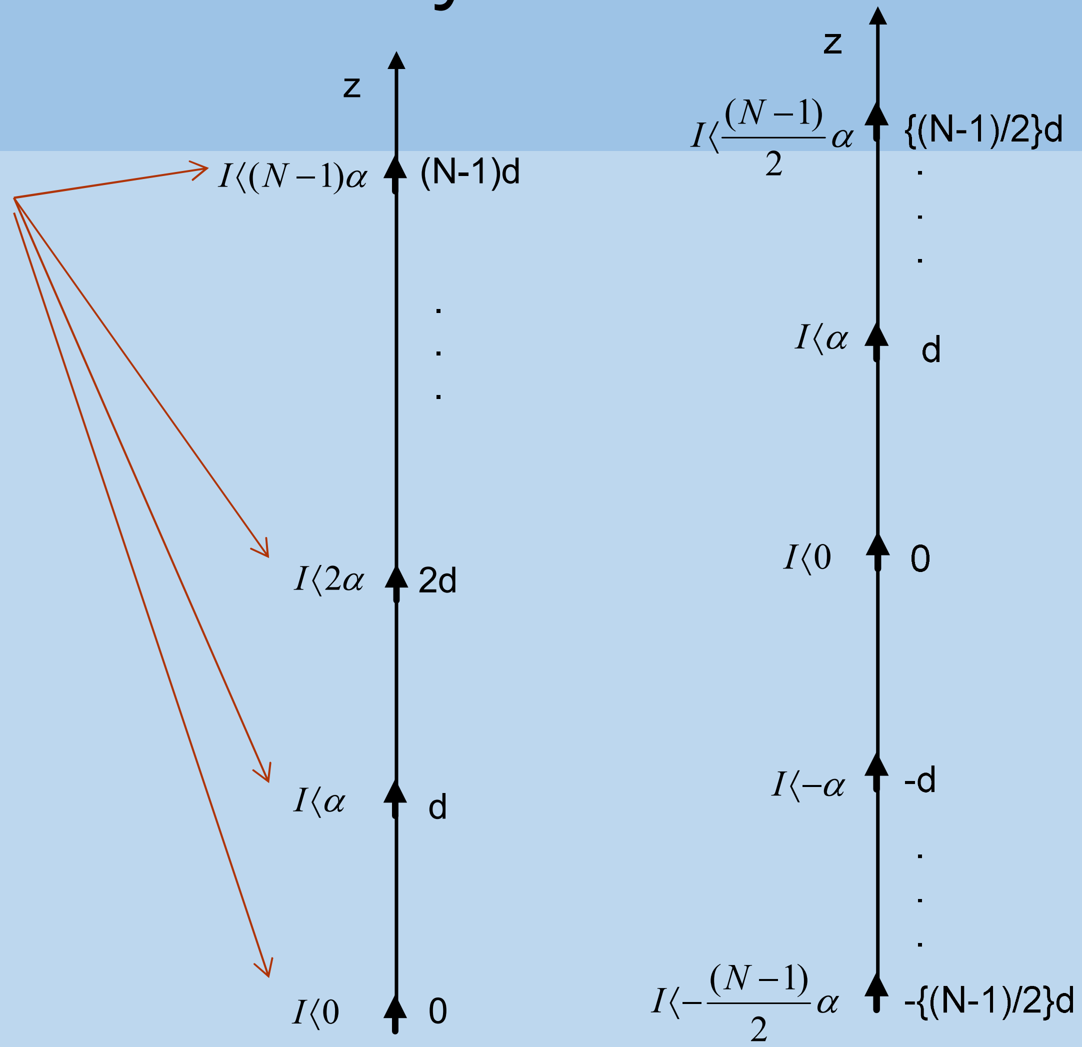
N element uniform linear array (ULA)

- This idea of two element array can be extended
  - to N element array of uniform amplitude and spacing
- Let us assume that N Hertz dipoles are placed along a straight line along z-axis at positions
  - 0,
  - d,
  - 2d, …,
  - (N-2) d and
  - (N-1) d
  - respectively

Antenna Arrays

- Current of equal amplitudes
- but with phase difference of
  - 0,
  - α,
  - 2α, … ,
  - (N-2) α and
  - (N-1) α
- are excited to the corresponding dipoles at
  - 0,
  - d,
  - 2d, …,
  - (N-2) d and
  - (N-1) d
  - respectively

Antenna Arrays



- Fig. (b) ULA 1 (c) ULA 2 (assume N is an odd number)

Antenna Arrays

- Then the array factor for the N element ULA of Fig. (b) will become

$$\therefore AF = 1 + e^{j(\beta d\cos\theta+\alpha)} + e^{2j(\beta d\cos\theta+\alpha)} + ... + e^{j(N-1)(\beta d\cos\theta+\alpha)}$$

$$\Rightarrow AF = \sum_{n=1}^{N} e^{j(n-1)(\beta d\cos\theta+\alpha)} \Rightarrow AF_N = \frac{1-e^{jN\psi}}{1-e^{j\psi}}; \psi = \beta d\cos\theta + \alpha$$

$$= \frac{e^{jN\psi}-1}{e^{j\psi}-1} = e^{j(\frac{N-1}{2})\psi}\left[\frac{e^{j(\frac{N}{2})\psi}-e^{-j(\frac{N}{2})\psi}}{e^{j(\frac{1}{2})\psi}-e^{-j(\frac{1}{2})\psi}}\right] = e^{j(\frac{N-1}{2})\psi}\frac{\sin(\frac{N}{2})\psi}{\sin(\frac{\psi}{2})}$$

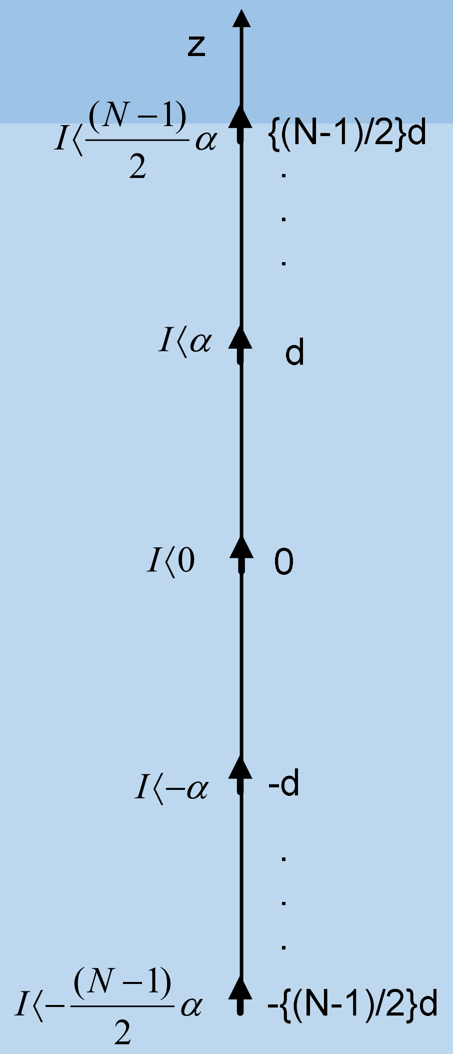
Antenna Arrays

- If the reference point is at the physical
- center of the array as depicted in
- Fig. (c), the array factor is

$$(AF)_N = \frac{\sin(\frac{N}{2})\psi}{\sin(\frac{\psi}{2})}$$

- For small values of Ψ,

$$(AF)_N = \frac{\sin(\frac{N}{2})\psi}{\frac{\psi}{2}}$$



Antenna Arrays

- The maximum value of AF is for $\psi = 0$ and its value is N
- Apply L' Hospital rule since it is of the form $\frac{\sin 0}{0}$
- To normalize the array factor so that the maximum value is equal to unity, we get,

$$(AF)_N = \frac{1}{N}\left[\frac{\sin(\frac{N}{2})\psi}{\sin\frac{1}{2}\psi}\right] \cong \frac{\sin(\frac{N}{2})\psi}{\frac{N}{2}\psi}$$

Antenna Arrays

- This is the normalized array factor for ULA
- As N increases, the main lobe narrows
- The number of lobes is equal to N
  - one main lobe and
  - other N-1 side lobes
- in one period of the AF
- The side lobes widths are of $2\pi/N$ and
- main lobes are two times wider than the side lobes
- The SLL decreases with increasing N
- This can be verified from Fig. 8.11 (see textbook)

Antenna Arrays

$$(AF)_N = \frac{1}{N}\left[\frac{\sin(\frac{N}{2})\psi}{\sin\frac{1}{2}\psi}\right]$$

- *Null of the array*
- To find the null of the array,

$$\sin(\frac{N}{2})\psi = 0 \quad \because \psi = (\beta\cos\theta)d + \alpha$$

$$\Rightarrow \frac{N}{2}\psi = \pm n\pi \Rightarrow \frac{N}{2}\{(\beta\cos\theta)d + \alpha\} = \pm n\pi \Rightarrow \beta d\cos\theta = -\alpha \pm \frac{2n\pi}{N}$$

$$\Rightarrow \boxed{\theta_n = \cos^{-1}\left[\frac{1}{\beta d}\left(-\alpha \pm \frac{2n\pi}{N}\right)\right]} \qquad n = 1, 2, 3, \ldots\ldots$$

Antenna Arrays

$$\left(AF\right)_N = \frac{1}{N}\left[\frac{\sin(\frac{N}{2})\psi}{\sin\frac{1}{2}\psi}\right]$$

- *Maximum values*
- It attains the maximum values for $\psi = 0$

$$\frac{\psi}{2} = \frac{1}{2}\left(\beta d\cos\theta + \alpha\right)\Big|_{\theta=\theta_m} = 0$$

$$\Rightarrow \boxed{\theta_m = \cos^{-1}\left[-\frac{\alpha}{\beta d}\right]}$$

Broadside array

- We know that when

Antenna Arrays

$$\psi = \beta d \cos\theta + \alpha = 0$$

- the maximum radiation occurs
- It is desired that maximum occurs at $\theta = 90^\circ$

$$\psi = \beta d \cos\theta + \alpha \Big|_{\theta=90^0} = 0 \Rightarrow \boxed{\alpha = 0^0}$$

Endfire array

- We know that when $\psi = \beta d \cos\theta + \alpha = 0$
- the maximum radiation occurs

Antenna Arrays

- It is desired that maximum occurs at $\theta = 0^\circ$,

$$\psi = \beta d \cos\theta + \alpha \Big|_{\theta=0^0} = 0 \Rightarrow \boxed{\alpha = -\beta d}$$

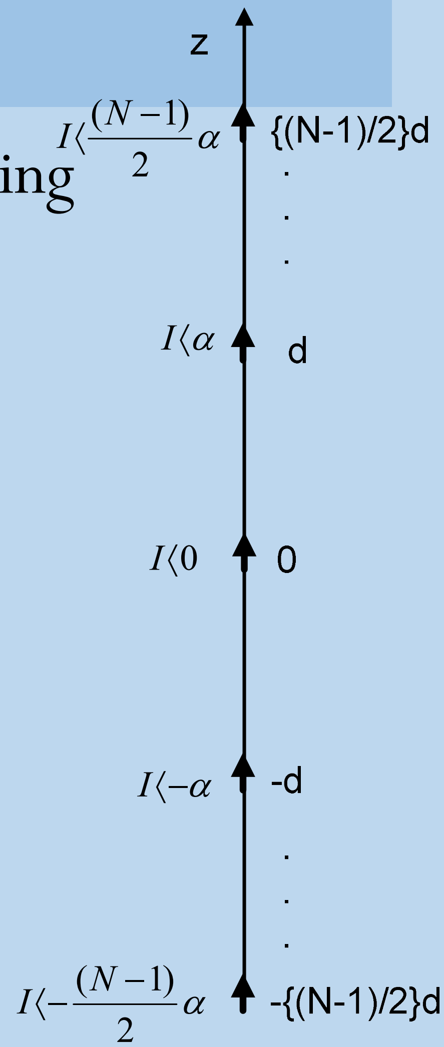
Phase scanning array

- We know that when $\psi = \beta d \cos\theta + \alpha = 0$
- the maximum radiation occurs
- It is desired that maximum occurs at $\theta = \theta_0$

$$\psi = \beta d \cos\theta + \alpha \Big|_{\theta=\theta_0} = 0 \Rightarrow \boxed{\alpha = -\beta d \cos\theta_0}$$

Antenna Arrays

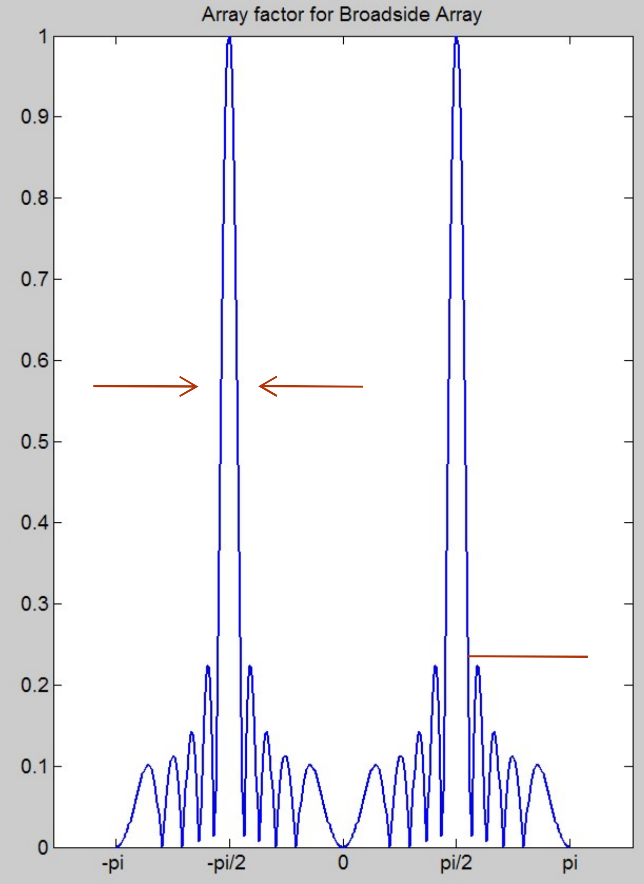
- Draw the polar plot of radiation pattern for the following
- uniform linear array (ULA)
- of N isotropic radiating antennas spaced $\lambda/2$
- apart for the following cases:
  - (a) Broadside array (Maximum field is at $\theta=90^\circ$)
  - (b) End fire array (Maximum field is at $\theta=0^\circ$)
  - (c) Maximum field is at $\theta=60^\circ$ and
  - (d) Null at $\theta=60^\circ$

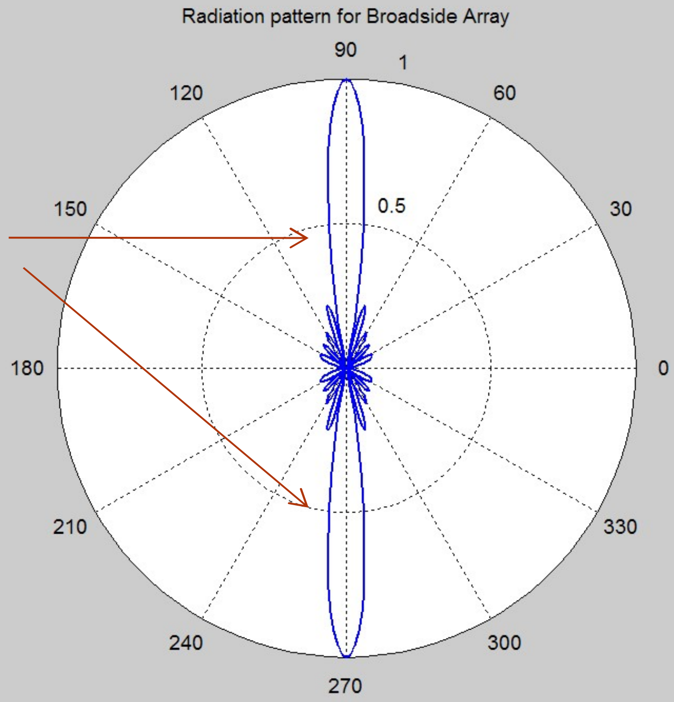


$$(AF)_N = \frac{1}{N}\left[\frac{\sin(\frac{N}{2})\psi}{\sin\frac{1}{2}\psi}\right]$$

Antenna Arrays

$$\psi = \beta d \cos\theta + \alpha \Big|_{\theta=90^0} = 0 \Rightarrow \alpha = 0^0$$

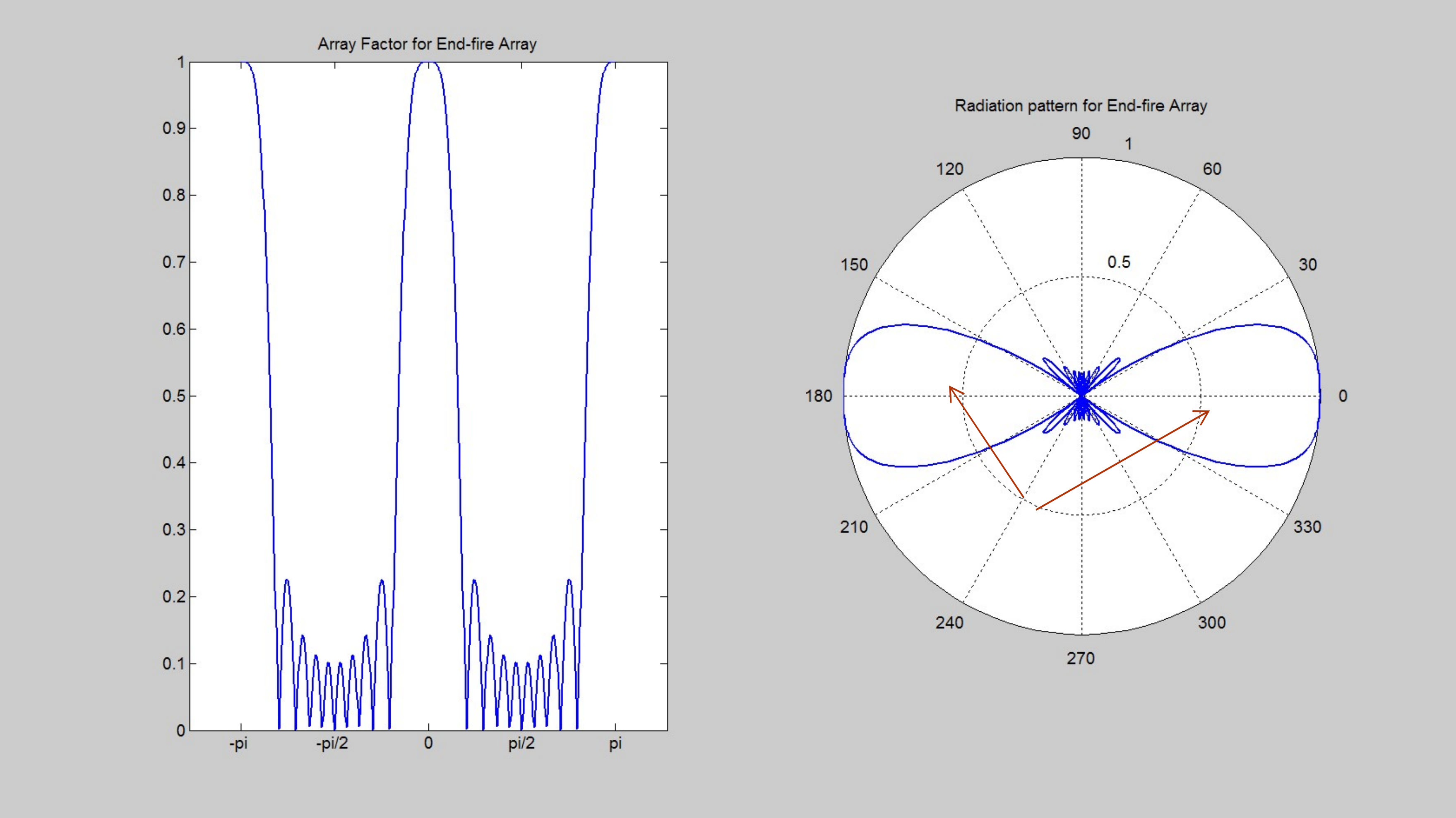




- (a) Broadside array (Maximum field is at θ=90°)

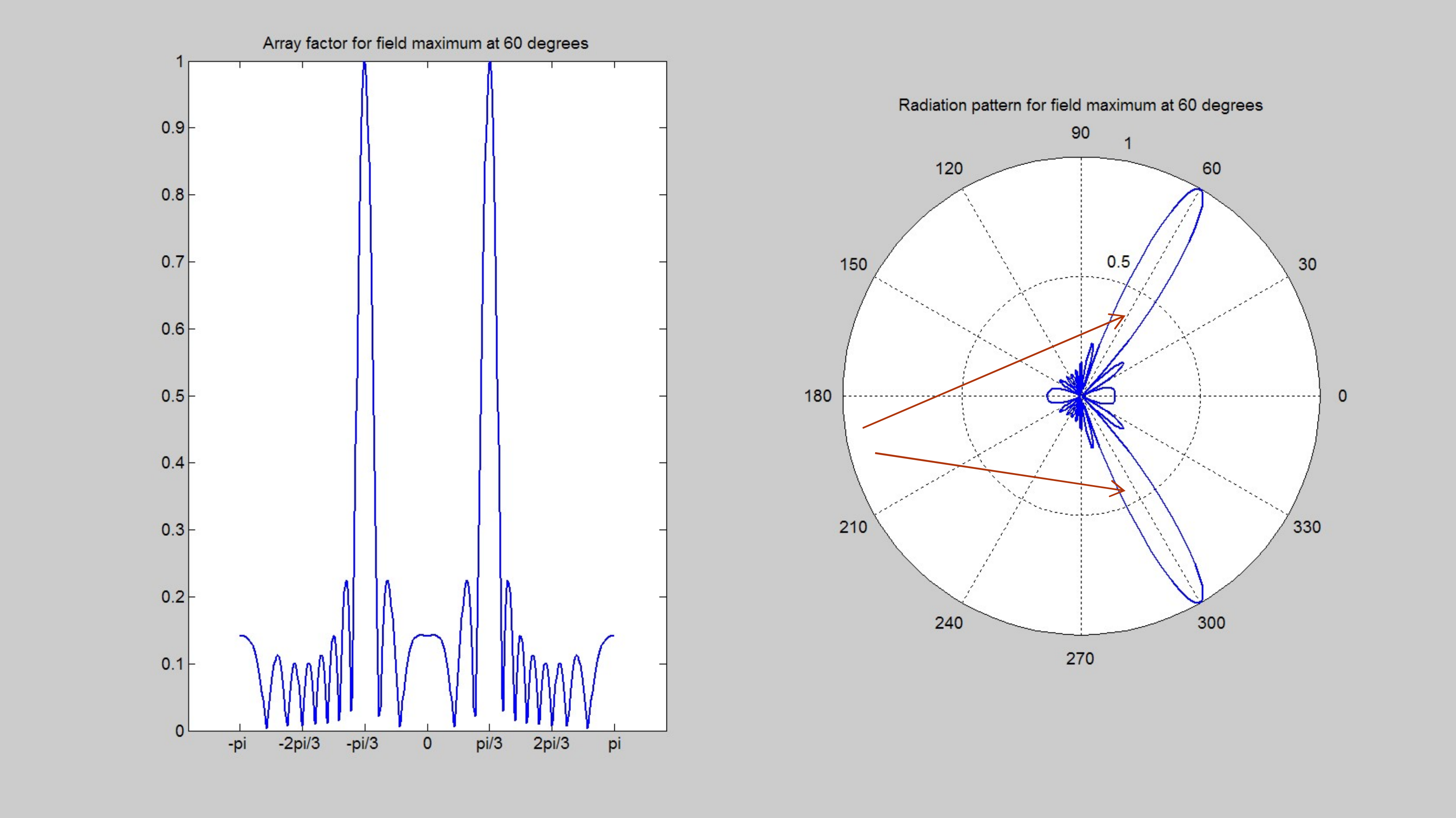
Antenna Arrays

$$\psi = \beta d \cos\theta + \alpha\Big|_{\theta=0^0} = 0 \Rightarrow \alpha = -\beta d$$



- (b) End fire array (Maximum field is at θ=0°)

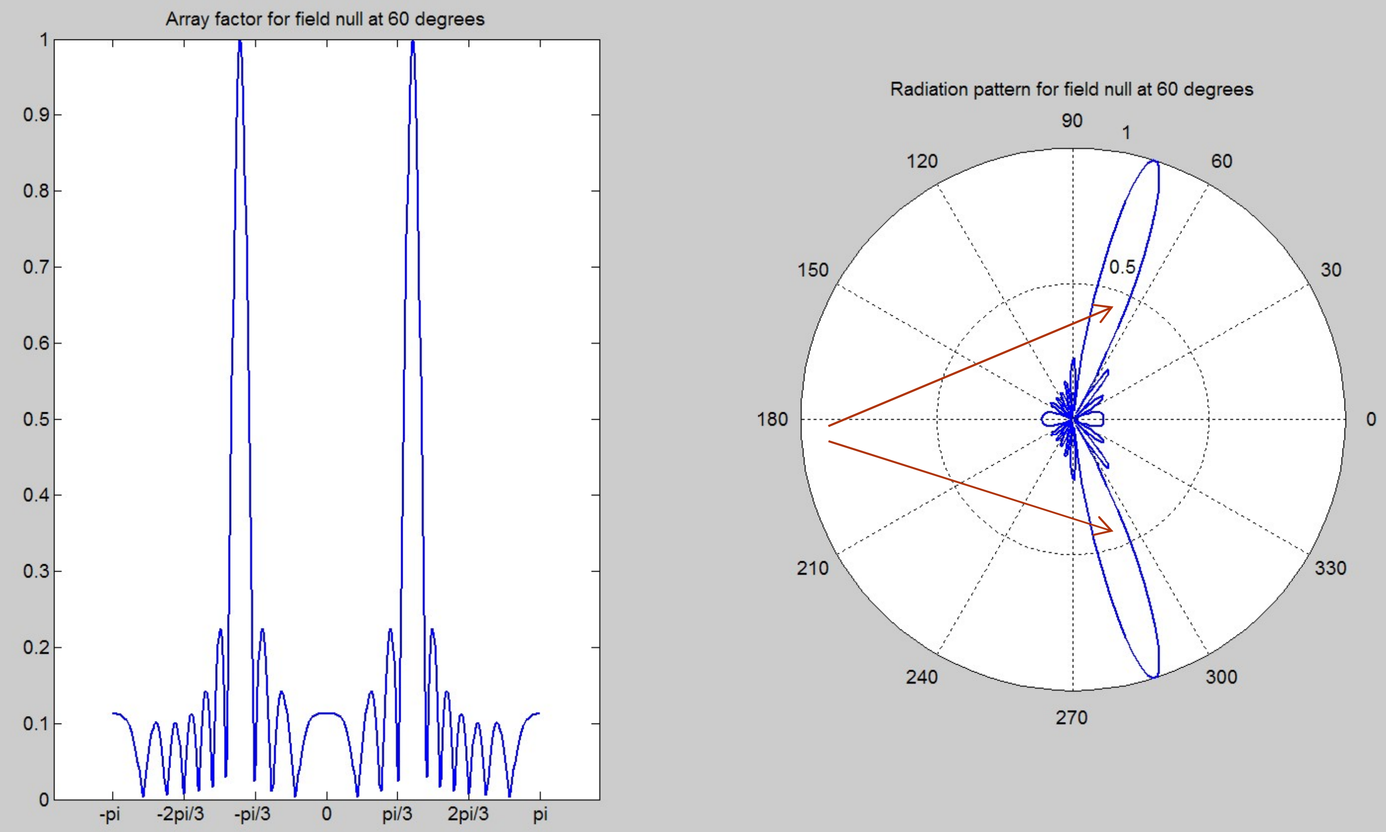
Antenna Arrays



- (c) Maximum field is at $\theta = 60^\circ$

$$\psi = \beta d \cos\theta + \alpha\Big|_{\theta=\theta_0} = 0 \Rightarrow \alpha = -\beta d \cos\theta_0$$

Antenna Arrays

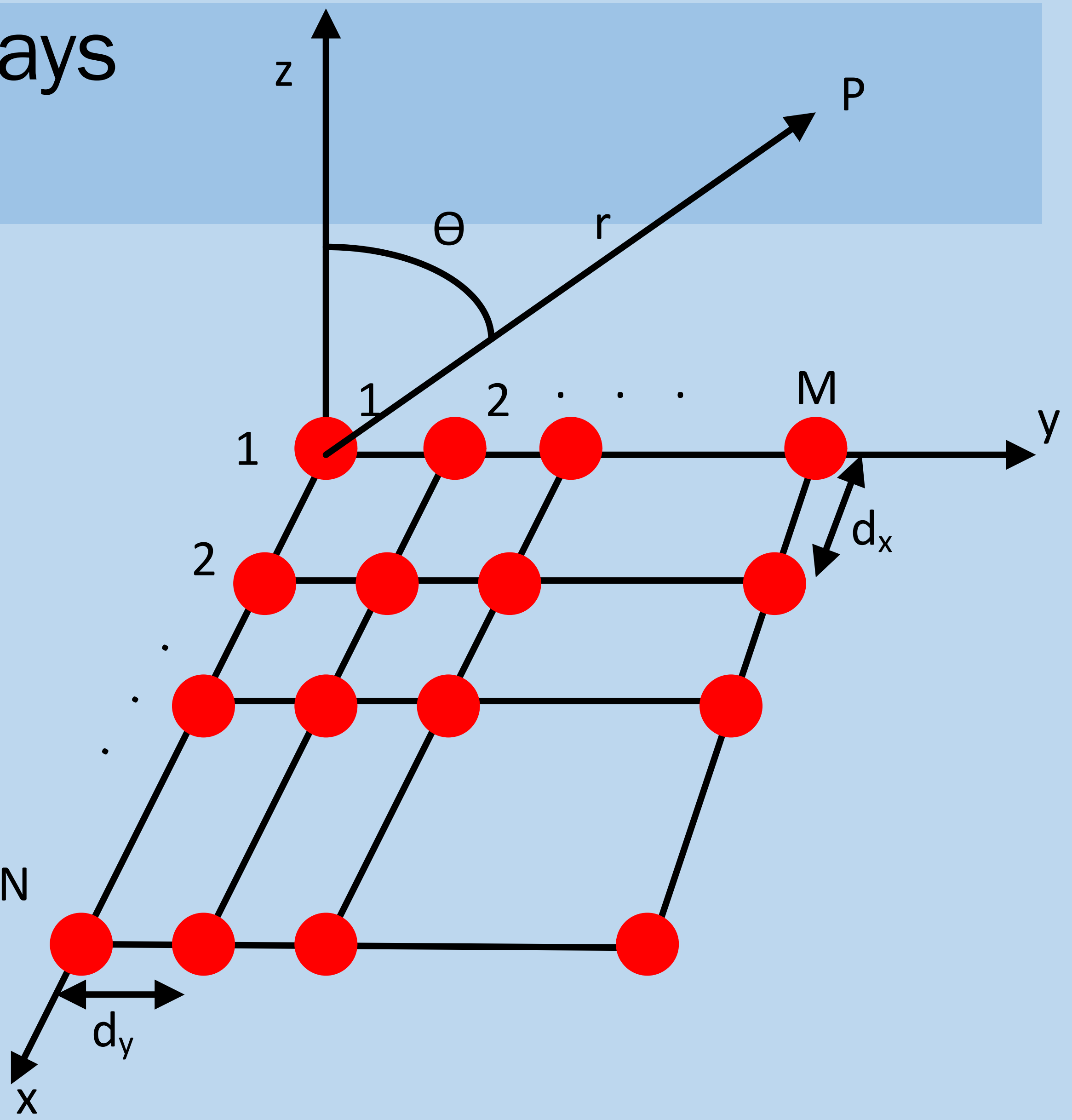


- (d) Null at $\theta=60^{\circ}$

$$\alpha = \frac{2n\pi}{N} - \beta d \cos\theta_{null}, n = 1,2,3,\cdots$$

Antenna Arrays

- **Planar arrays:**
- Fig. M×N planar antenna array



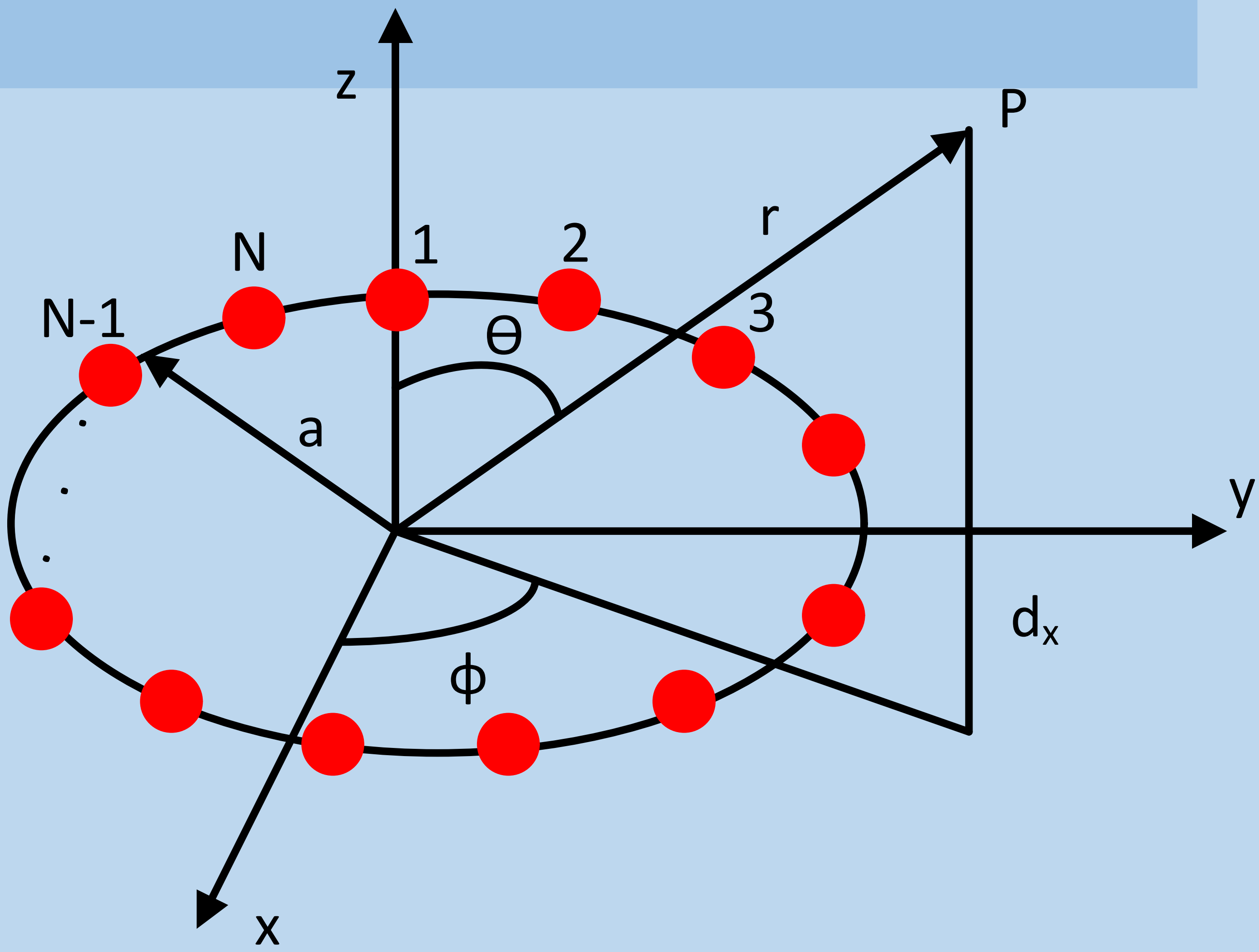
Prof. Rakhesh Singh Kshetrimayum

Antenna Arrays

- Planar arrays:
- Linear arrays can scan main beam in one polar plane (θ or ϕ)
- Planar arrays can scan main beam along both θ and ϕ
- It offers more gain and lower side lobes
- $AF_{planar} = (AF_x)(AF_y)$
- Or, $AF = \left[\frac{1}{N}\left(\frac{sin\left(\frac{N}{2}\right)\Psi_x}{sin\left(\frac{1}{2}\right)\Psi_x}\right)\right]\left[\frac{1}{M}\left(\frac{sin\left(\frac{M}{2}\right)\Psi_y}{sin\left(\frac{1}{2}\right)\Psi_y}\right)\right]$
- where $\Psi_x = \beta d_x sin\theta cos\phi + \alpha_x$ and $\Psi_y = \beta d_y sin\theta cos\phi + \alpha_y$

Antenna Arrays

- Fig. N element circular antenna array



Antenna Arrays

- **Circular arrays:**
- $AF(\theta,\phi) = \sum_{n=1}^{N} I_n e^{j\beta a[sin\theta cos(\phi-\phi_n)-sin\theta_0 cos(\phi_0-\phi_n)]}$
- where
- ϕ_0 and θ_0 are the angles of the main beam
- *a* is the radius of the circular array
- I_n Is the excitation coefficients of the elements

Antenna Arrays

- **Nonuniform arrays:**
- Either amplitude or phase may vary
- **Binomial array:**
- $AF = \left(1 + e^{j\psi}\right)^{N-1} = 1 + (N-1)e^{j\psi} + \frac{(N-1)(N-2)}{2!}e^{j2\psi} + \frac{(N-1)(N-2)(N-3)}{3!}e^{j3\psi} + \cdots$
- It is like a binomial series with N elements (amplitude of current excitation vary according to binomial coefficients)
- Side lobes equal to zero for $d = \frac{\lambda}{2}$
- But has larger beamwidth and lower efficiencies

Modern Antenna Arrays

- Applications:
  - MIMO applications
  - Full duplex radios
  - Ambient Back Scattering
  - Energy harvesting
  - Reconfigurable

Modern Antenna Arrays

Diversity gain in MIMO systems

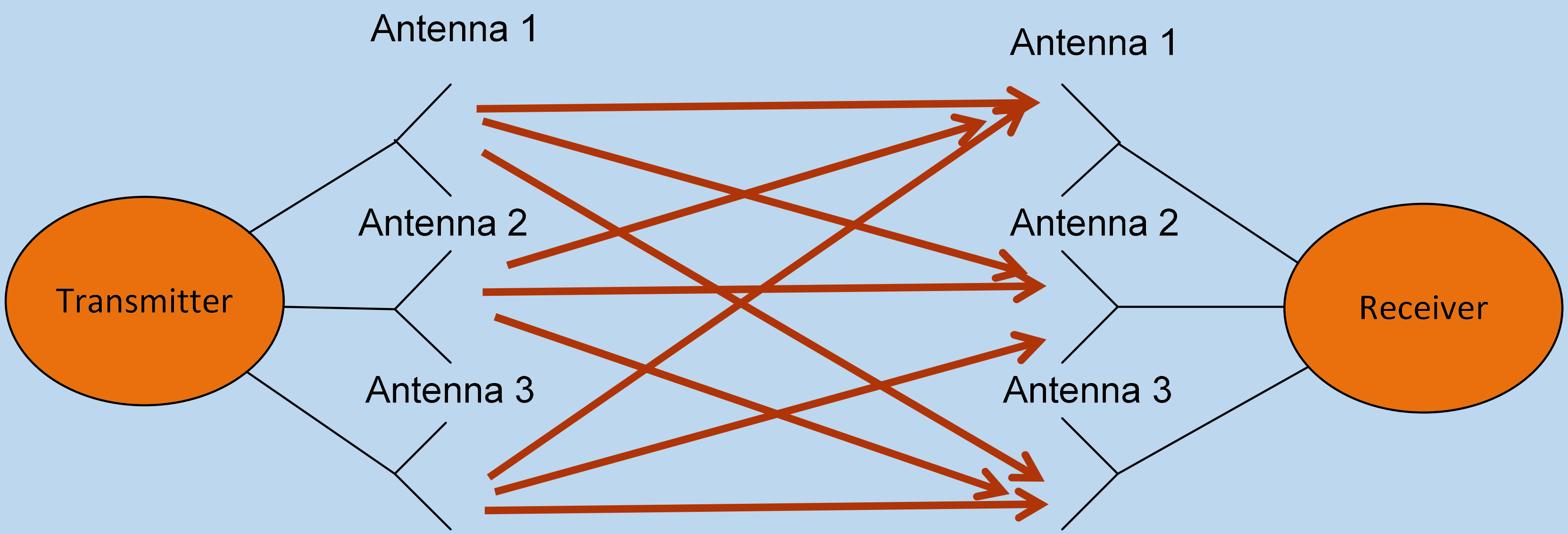


Fig. 3×3 MIMO system[1]

1. Rakhesh Singh Kshetrimayum, Fundamentals of MIMO Wireless Communications, Cambridge University Press, 2017

Modern Antenna Arrays

Multiplexing gain in MIMO systems

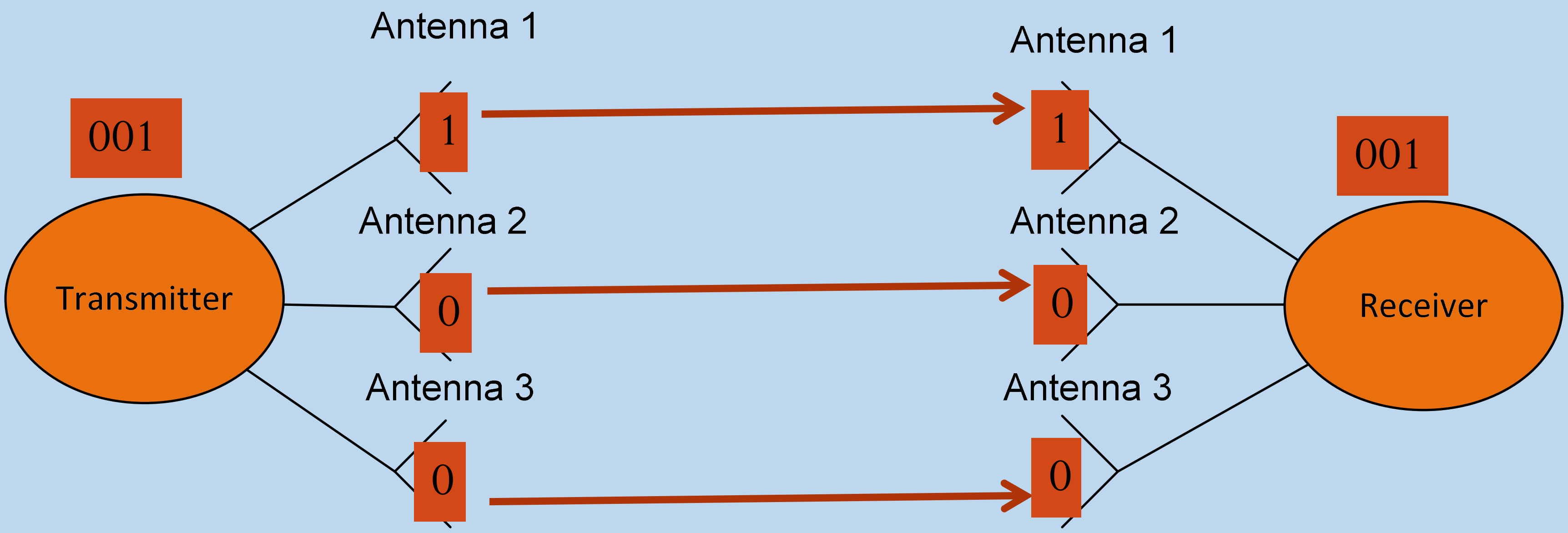


Fig. 3×3 MIMO system

Modern Antenna Arrays

Advantages of Diversity gain in MIMO systems

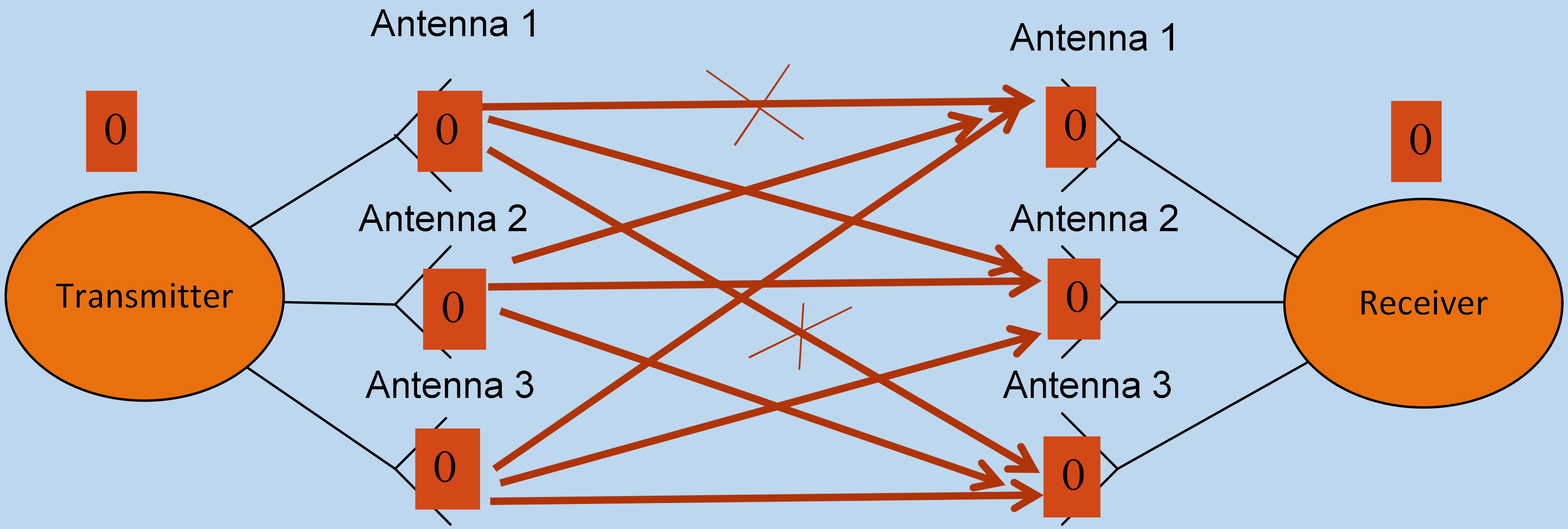


Fig. 3×3 MIMO system

Prof. Rakhesh Singh Kshetrimayum

Modern Antenna Arrays

- MIMO antenna arrays
  - It is no longer single port it is multiport
  - For example, 3×3 MIMO system, Tx antenna array will have 3 ports
  - Aim is to have independent channels for all antennas
    - Antennas should minimum correlations
  - New Parameters for MIMO antenna arrays:
  - envelope correlation coefficient (ECC) among all the antenna ports
  - $\rho_{ij} = \frac{|\iint [\vec{F}_i(\theta,\phi) * \vec{F}_j(\theta,\phi)\, d\Omega|^2}{(\iint |\vec{F}_i(\theta,\phi)|^2 d\Omega)(\iint |\vec{F}_j(\theta,\phi)|^2 d\Omega)}$

Modern Antenna Arrays

- **Mutual coupling between antennas in arrays**
- Mutual coupling between i[th] and j[th] antennas in an array
- $S_{ij} = \left.\frac{b_i}{a_j}\right|_{a_k=0, k\neq j}$
- Drive port j with an incident wave of amplitude a_j
- Measure the reflected wave of amplitude b_i at the port I
- For $k \neq j$, we terminate all ports with matched load to avoid reflections
- Interested in the Inter-antenna interaction in MIMO wireless, explore some of my recent invited talks:
- https://www.youtube.com/watch?v=brdNB9rjSn8&t=1654s
- https://www.youtube.com/watch?v=Aoxevvci9D8&t=1025s