EE540 Advance Electromagnetic Theory & Antennas

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Hertz dipole (revisited)

- Near fields (magnetic) of Hertz dipole
- $H_{\phi nf} = \frac{I_0 dle^{+j\omega t}e^{-j\beta r}sin\theta}{4\pi r^2}$
- *Analogy with Biot Savart's law*
- $\vec{B}_{BSL} = \frac{\mu_0 I_0 d\vec{l}\times\hat{r}}{4\pi r^2} = \frac{\mu_0 I_0 dl\hat{z}\times\hat{r}}{4\pi r^2}$
- Note that the current element is directed along z-axis
- We do analysis in spherical coordinates, hence
- $\hat{z} = cos\theta\hat{r} - sin\theta\hat{\theta}$
- Therefore $\hat{z} \times \hat{r} = sin\theta\hat{\phi}$

Hertz dipole (revisited)

- Hence Biot Savart's law becomes
- $\vec{B}_{BSL} = \frac{\mu_0 I_0 d\vec{l} \times \hat{r}}{4\pi r^2} = \frac{\mu_0 I_0 dl sin\theta \hat{\phi}}{4\pi r^2}$
- Therefore, $\vec{H}_{BSL} = \frac{I_0 dl sin\theta \hat{\phi}}{4\pi r^2}$
- Near fields (magnetic) of Hertz dipole for $\omega \to 0, \beta \to 0$
- $H_{\phi nf} \cong \frac{I_0 dl sin\theta}{4\pi r^2} = H_{BSL}$
- Magnetic field of a Hertz dipole is like the steady induction magnetic field produced by a current carrying element of length dl

Hertz dipole (revisited)

- Near fields (electric) of Hertz dipole
- $\vec{E}_{nf} = \frac{2I_0 dl e^{+j\omega t} e^{-j\beta r} cos\theta}{j\omega\varepsilon 4\pi r^3}\hat{r} + \frac{I_0 dl e^{+j\omega t} e^{-j\beta r} sin\theta}{j\omega\varepsilon 4\pi \ ^3}\hat{\theta}$
- *The potential due to an electric dipole*
- $V_{dipole} = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r_1} - \frac{1}{r_2}\right) = \frac{Q}{4\pi\varepsilon_0}\left(\frac{r_2 - r_1}{r_1 r_2}\right)$
- Note that for sufficiently far away distance
- $r_1 = r_2 \cong r$ and $r_2 - r_1 \cong dl\ cos\theta$
- Hence$V_{dipole} \cong \frac{Q}{4\pi\varepsilon_0}\left(\frac{dl\ cos\theta}{r^2}\right)$
- Therefore,
- $\vec{E}_{dipole} = -\nabla V_{dipole} = -\left(\frac{\partial V_{dipole}}{\partial r}\hat{r} + \frac{\partial V_{dipole}}{r\partial\theta}\hat{\theta} + \frac{\partial V_{dipole}}{rsin\theta\partial\phi}\hat{\phi}\right)$

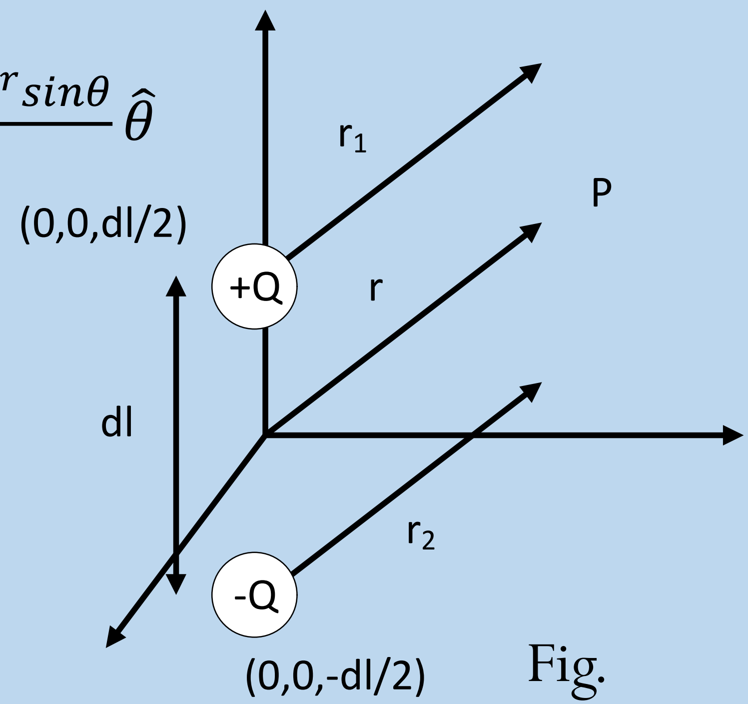
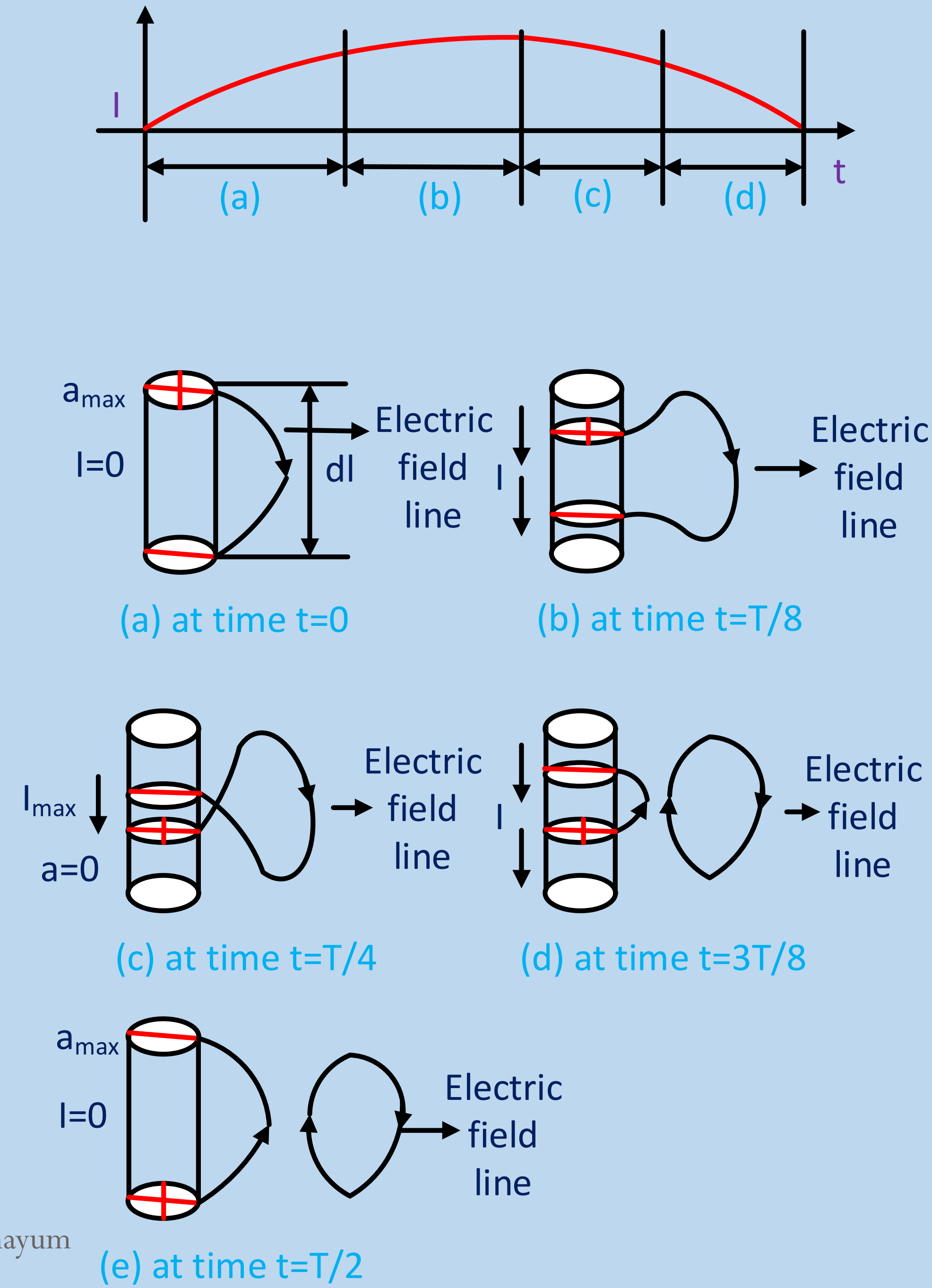


Fig. Electric dipole

Hertz dipole (revisited)

- Near fields (electric) of Hertz dipole
- $\vec{E}_{nf} = \frac{2I_0 dl e^{+j\omega t} e^{-j\beta r} cos\theta}{j\omega\varepsilon 4\pi r^3}\hat{r} + \frac{I_0 dl e^{+j\omega t} e^{-j\beta r} sin\theta}{j\omega\varepsilon 4\pi r^3}\hat{\theta}$
- The electric field due to an electric dipole
- $\vec{E}_{dipole} = \frac{2Qdlcos\theta}{4\pi\varepsilon_0 r^3}\hat{r} + \frac{Qdlsin\theta}{4\pi\varepsilon_0 r^3}\hat{\theta}$
- Note that $\frac{1}{j\omega}\left(I_0 e^{+j\omega t}\right) = Q = \int I_0 e^{+j\omega t} dt$
- Therefore,
- $\vec{E}_{nf} \cong \vec{E}_{dipole}$
- The electric field of a Hertz dipole in near field is like that of a electric dipole of the same length dl

Fig. Oscillating dipole



Small antennas

- Small antennas are a necessity for portable devices
- For example,
- We need small antennas for mobile devices
- Smaller antenna is an attractive feature
- But the fundamental question remains
- Can we keep on reducing antenna size?
  - Is there some limit beyond which we can reduce the antenna size?
- Consider a smallest sphere which enclose a small antenna as shown in next slide

Small antennas

Smallest sphere enclosing the antenna

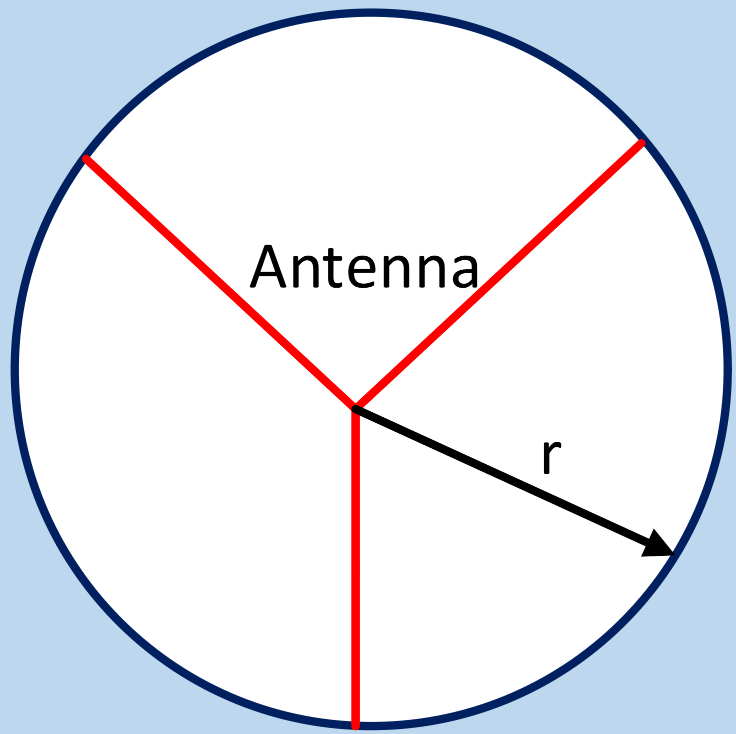


Fig. Smallest sphere which can enclose a small antenna

Small antennas

- There is a very important concept on designing electrically small antennas
- When kr<1 (electrically small antennas),
- the quality factor Q of a small antenna can found from the J. L. Chu's relation

$$Q = \frac{1 + 2(kr)^2}{(kr)^3 \left\{ 1 + (kr)^2 \right\}} \bullet e_{rad}$$

- where k is the wave number and
- r is the radius of the smallest sphere enclosing the antenna

Small antennas

- The above relation gives the relationship between the antenna size, efficiency and quality factor.
- This expression can be reduced further for smallest Q for a LP very small antenna (kr<<1) as follows:

$$Q_{\min} = \frac{1}{(kr)^3} + \frac{1}{kr}$$

- Harrington gave also a practical upper limit to the gain of a small antenna for a reasonable BW as

$$G_{\max} = (kr)^2 + 2(kr)$$

Small antennas

- CASE STUDY of a small antenna
- for a Hertz dipole of
  - very small length 0.01λ,
  - Q_{min} =32283
- It has very high Q and
  - hence a very narrow FBW (0.000031) (*very narrow*)
- Gmax=0.0638 or -12dB (*very small no practical use*)
- Note that in the above calculations r=0.005λ has been used

Small antennas

- Very small antenna like Hertz dipole has no practical use
- For practical use, we need to have a sufficiently large antenna
  - Antenna size,
  - quality factor,
  - bandwidth and
  - radiation efficiency
    - is interlinked
- There is no complete freedom to optimize each one of them independently
- Employ different technologies such as metamaterials, EBG, SSPP, etc. to design antennas with better performance