EE540 Advance Electromagnetic Theory & Antennas

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Radiation

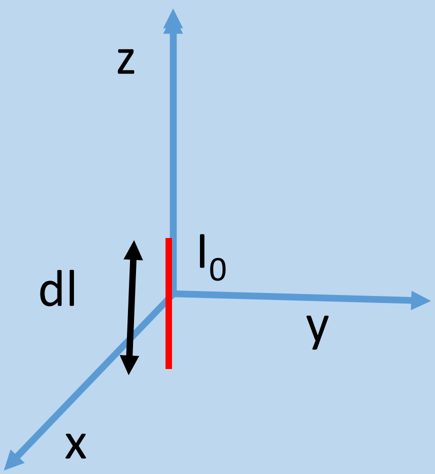

Fig. 1st case: A z-directed current carrying element carrying current I_0 of length dl at the origin

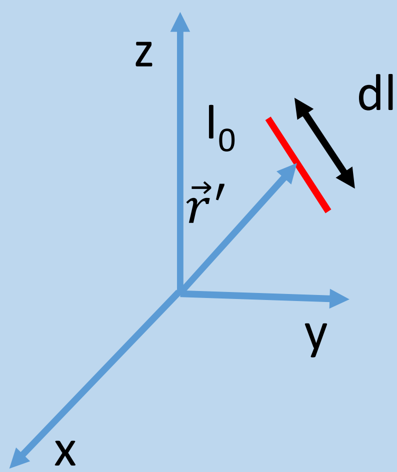

Fig. 2nd case: An arbitrary oriented current carrying element carrying current I_0 of length dl at the position $\vec{r}'$

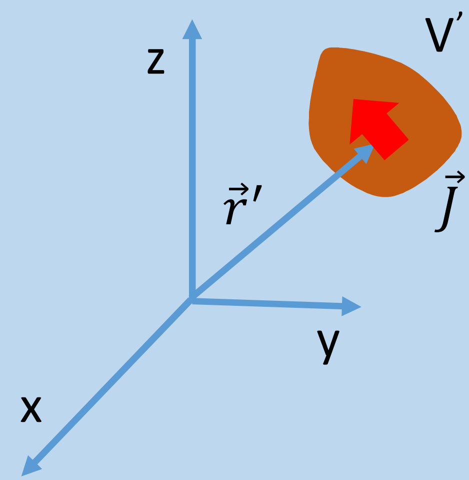

Fig. 3rd case: An arbitrary oriented current density $\vec{J}$ is flowing in a volume V' positioned at $\vec{r}'$

Radiation

- *Procedure to find the radiated fields:*
- It involves three steps
  - *Step 1:* Solve $\vec{A}$ from $\vec{J}$ as

$$\vec{A}(\vec{r}) = \frac{\mu}{4\pi}\int_{V'} \vec{J}(\vec{r}')\frac{e^{-j\beta|\vec{r}-\vec{r}'|}}{|\vec{r}-\vec{r}'|}dv' = \int_{V'} \vec{J}(\vec{r}')G(\vec{r},\vec{r}')dv'$$

  - *Step 2:* Find magnetic field ($\vec{H}_A$) as

$$\vec{H}_A = \frac{1}{\mu}(\nabla \times \vec{A})$$

  - *Step 3:* Find electric field ($\vec{E}_A$) as

$$\vec{E}_A = \frac{\nabla\times\nabla\times\vec{A}}{j\omega\mu\varepsilon} = \frac{1}{j\omega\varepsilon}\nabla\times\left(\frac{\nabla\times\vec{A}}{\mu}\right) = \frac{1}{j\omega\varepsilon}(\nabla\times\vec{H})$$

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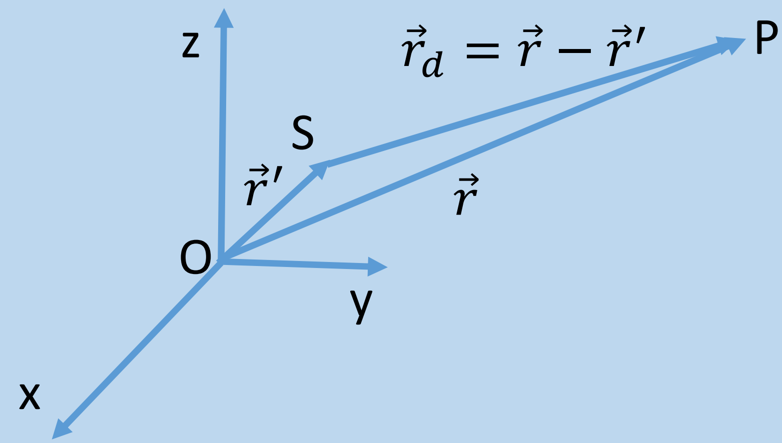

- *Points to be noted:*
  - There are two coordinates:
    - Source coordinates (x', y', z')
    - Observation coordinates (x, y, z)
  - They are independent of each other
  - Note that ∇ is w.r.t. observation coordinates

Fig. Radiation from an arbitrary current density $\vec{J}$ at point S

Observation point P: $\vec{r} = (x\hat{x} + y\hat{y} + z\hat{z})$,

Source Point: $\vec{r}' = (x'\hat{x} + y'\hat{y} + z'\hat{z})$,

Difference between source and observation point:

$$\vec{r}_d = \vec{r} - \vec{r}' = \{(x - x')\hat{x} + (y - y')\hat{y} + (z - z')\hat{z}\}$$

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- *Step 1:* Solve $\vec{A}$ from $\vec{J}$ as
- Similar to convolution of $-\mu\vec{J}\,(\vec{r}\,')$
  - and $G(r) = -\frac{1}{4\pi r}\left(e^{-j\beta r}\right)^*$

$$\vec{A}(\vec{r}) = \frac{\mu}{4\pi}\int_{V'}\vec{J}(\vec{r}\,')\frac{e^{-j\beta|\vec{r}-\vec{r}\,'|}}{|\vec{r}-\vec{r}\,'|}dv' = \int_{V'}\vec{J}(\vec{r}\,')G(\vec{r},\vec{r}\,')dv'$$

- *Step 2:* Find magnetic field from $\vec{A}$ as
  - $\vec{H} = \frac{1}{\mu}\left(\nabla\times\vec{A}\right) = \frac{1}{\mu}\iiint_{V'}\nabla\times\left\{\vec{J}(\vec{r}\,')G(\vec{r},\vec{r}\,')\right\}dv'$
- Integrand $\nabla\times\left\{\vec{J}(\vec{r}\,')G(\vec{r},\vec{r}\,')\right\} = G(\vec{r},\vec{r}\,')\left\{\nabla\times\vec{J}(\vec{r}\,')\right\} + \left\{\nabla G(\vec{r},\vec{r}\,')\right\}\times\vec{J}(\vec{r}\,')$
  - 1st term of $\nabla\times\left\{\vec{J}(\vec{r}\,')G(\vec{r},\vec{r}\,')\right\}$: $\left\{\nabla\times\vec{J}(\vec{r}\,')\right\} = 0$

** minus sign is because $-\mu\vec{J}\,(\vec{r}\,')$ on the RHS of the wave equation was equated to $\delta\,(\vec{r}\,')$ to get the Green's function from the wave equation*

Radiation

- *2nd term of* $\nabla \times \left\{\vec{J}(\vec{r}')G(\vec{r},\vec{r}')\right\}$*:*
- *Let us find* $\nabla G(\vec{r},\vec{r}')$
  - Note that $G(\vec{r},\vec{r}') = \frac{\mu}{4\pi} e^{-j\ \ |\vec{r}-\vec{r}'|} \frac{1}{|\vec{r}-\vec{r}'|}$
  - Put $r_d = |\vec{r} - \vec{r}'|$
  - Then $G(\vec{r},\vec{r}') = \frac{\mu}{4\pi}\left(e^{-j\beta r_d}\right)\left(\frac{1}{r_d}\right)$
- We have
  - $\nabla = \frac{\partial}{\partial x}\hat{x} + \frac{\partial}{\partial y}\hat{y} + \frac{\partial}{\partial z}\hat{z}$ and $r_d = \sqrt{(x-x')^2 + (y-y')^2 + (z-z')^2}$
- Therefore,
  - $\nabla\left(\frac{1}{r_d}\right) = -\frac{1}{{r_d}^2}\hat{r}_d$ and $\nabla\left(e^{-j\beta r_d}\right) = -j\beta e^{-j\beta r_d}\hat{r}_d$

    where $\hat{r}_d = \frac{\vec{r}-\vec{r}'}{|\vec{r}-\vec{r}'|}$ and $\vec{r}_d = r_d\hat{r}_d$

- Therefore,
  - $\nabla G(\vec{r},\vec{r}') = \frac{\mu}{4\pi} e^{-j\beta r_d} \left(-\frac{1}{{r_d}^2} - \frac{j\beta}{r_d}\right) \hat{r}_d = \frac{\mu}{4\pi} e^{-j\beta r_d} \left(-\frac{1}{{r_d}^3} - \frac{j\beta}{{r_d}^2}\right) r_d \hat{r}_d$
  - which further simplifies to $\nabla G(\vec{r},\vec{r}') = \frac{\mu}{4\pi} e^{-j\beta r_d} \left(-\frac{1}{{r_d}^3} - \frac{j\beta}{{r_d}^2}\right) \vec{r}_d$
  - So 2nd term of *of* $\nabla \times \{\vec{J}(\vec{r}')G(\vec{r},\vec{r}')\}$:
  - $\nabla G(\vec{r},\vec{r}') \times \vec{J}(\vec{r}') = \frac{\mu}{4\pi} e^{-j\beta r_d} \left(-\frac{1}{{r_d}^3} - \frac{j\beta}{{r_d}^2}\right) \vec{r}_d \times \vec{J}(\vec{r}')$
- Hence $\vec{H} = \frac{1}{\mu} \iiint_{V'} \nabla \times \{\vec{J}(\vec{r}')G(\vec{r},\vec{r}')\} dv'$
  - becomes
  - $\vec{H} = \frac{1}{4\pi} \iiint_{V'} e^{-j\beta r_d} \left(-\frac{1}{{r_d}^3} - \frac{j\beta}{{r_d}^2}\right) \vec{r}_d \times \vec{J}(\vec{r}') dv'$
- Put $A(\vec{r},\vec{r}') = e^{-j\beta r_d} \left(-\frac{1}{{r_d}^3} - \frac{j\beta}{{r_d}^2}\right)$ and $\vec{B}(\vec{r},\vec{r}') = \vec{r}_d \times \vec{J}(\vec{r}')$
  - $\vec{H} = \frac{1}{4\pi} \iiint_{V'} A(\vec{r},\vec{r}') \vec{B}(\vec{r},\vec{r}') dv'$

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- *Step 3:*
- Find the electric field from magnetic field
- Then electric field simplifies to
  - $\vec{E} = \frac{1}{j\omega\varepsilon}(\nabla \times \vec{H}) = \frac{1}{j4\pi}\iiint_{V'} \nabla \times \{A(\vec{r},\vec{r}')\vec{B}(\vec{r},\vec{r}')\}dv'$
- Note that Integrand $\nabla \times (A\vec{B}) = A(\nabla \times \vec{B}) + \nabla A \times \vec{B}$
- 2nd term of $\nabla \times (A\vec{B})$:
- $\because \nabla A = \nabla\left\{e^{-j\beta r_d}\left(-\frac{1}{{r_d}^3} - \frac{j\beta}{{r_d}^2}\right)\right\} = -j\beta e^{-j\beta r_d}\left(-\frac{1}{{r_d}^3} - \frac{j\beta}{{r_d}^2}\right)\hat{r}_d + e^{-j\beta r_d}\left(\frac{3}{{r_d}^4} + \frac{2j\beta}{{r_d}^3}\right)\hat{r}_d$

$$= \frac{\beta^2}{{r_d}^2}e^{-j\beta r_d}\left(-1 - \frac{3}{j\beta r_d} + \frac{3}{\beta^2 {r_d}^2}\right)\hat{r}_d$$

- $\therefore \nabla A \times \vec{B} = \frac{\beta^2}{{r_d}^2}e^{-j\beta r_d}\left(-1 - \frac{3}{j\beta r_d} + \frac{3}{\beta^2 {r_d}^2}\right)\hat{r}_d \times \vec{r}_d \times \vec{J}(\vec{r}')$

- $\therefore \nabla A \times \vec{B} = \frac{\beta^2}{{r_d}^2} e^{-j\beta r_d}\left(-1 - \frac{3}{j\beta r_d} + \frac{3}{\beta^2 {r_d}^2}\right)\hat{r}_d \times \vec{r}_d \times \vec{J}(\vec{r}')$

$$= \frac{\beta^2}{{r_d}^2} e^{-j\beta r_d}\left(-1 - \frac{3}{j\beta r_d} + \frac{3}{\beta^2 {r_d}^2}\right)\hat{r}_d \times r_d \hat{r}_d \times \vec{J}(\vec{r}')$$

$$= \frac{\beta^2}{r_d} e^{-j\beta r_d}\left(-1 - \frac{3}{j\beta r_d} + \frac{3}{\beta^2 {r_d}^2}\right)\hat{r}_d \times \hat{r}_d \times \vec{J}(\vec{r}')$$

- 1st term of $\nabla \times (A\vec{B})$:
- $\because \nabla \times \vec{B} = \nabla \times \{\vec{r}_d \times \vec{J}(\vec{r}')\}$

$$= \{\vec{J}(\vec{r}') \cdot \nabla\}\vec{r}_d - \vec{J}(\vec{r}')(\nabla \cdot \vec{r}_d) - \{\vec{r}_d \cdot \nabla\}\vec{J}(\vec{r}') + \vec{r}_d\left(\nabla \cdot \vec{J}(\vec{r}')\right)$$

- 1st term of $\nabla \times \vec{B}$:
- $\{\vec{J}(\vec{r}') \cdot \nabla\}\vec{r}_d = \left(J_x \frac{\partial}{\partial x} + J_y \frac{\partial}{\partial y} + J_z \frac{\partial}{\partial z}\right)\{(x - x')\hat{x} + (y - y')\hat{y} + (z - z')\hat{z}\} = \vec{J}(\vec{r}')$

- 2[nd] term of $\nabla \times \vec{B}$:
- $-\vec{J}(\vec{r}')(\nabla \cdot \vec{r}_d) = -\vec{J}(\vec{r}')\left(\hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}\right) \cdot \{(x - x')\hat{x} + (y - y')\hat{y} + (z - z')\hat{z}\} =$ $-\, 3\vec{J}(\vec{r}')$
- 3[rd] term of $\nabla \times \vec{B}$:
- $-\{\vec{r}_d \cdot \nabla\}\vec{J}(\vec{r}') = -\left\{(x - x')\frac{\partial}{\partial x} + (y - y')\frac{\partial}{\partial y} + (z - z')\frac{\partial}{\partial z}\right\}\vec{J}(\vec{r}') = 0$
- 4[th] term of $\nabla \times \vec{B}$:
- $\vec{r}_d\left(\nabla \cdot \vec{J}(\vec{r}')\right) = 0$
- $\because \nabla \times \vec{B} = \vec{J}(\vec{r}') - 3\vec{J}(\vec{r}') = -2\vec{J}(\vec{r}')$
- $\therefore A\left(\nabla \times \vec{B}\right) = -2e^{-j\beta r_d}\left(-\frac{1}{{r_d}^3} - \frac{j\beta}{{r_d}^2}\right)\vec{J}(\vec{r}')$

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- Hence,
- $\nabla \times \left(A\vec{B}\right) = A\left(\nabla \times \vec{B}\right) + \nabla A \times \vec{B}$

$$= \frac{\beta^2}{r_d} e^{-j\beta r_d}\left(-1 - \frac{3}{j\beta r_d} + \frac{3}{\beta^2 {r_d}^2}\right)\hat{r}_d \times \hat{r}_d \times \vec{J}(\vec{r}') - 2e^{-j\beta r_d}\left(-\frac{1}{{r_d}^3} - \frac{j\beta}{{r_d}^2}\right)\vec{J}(\vec{r}')$$

- Hence,
  - $\vec{E} = \frac{1}{j4\pi\omega}\iiint_{V'} \nabla \times \left\{A(\vec{r},\vec{r}')\vec{B}(\vec{r},\vec{r}')\right\}dv'$

$$= \frac{1}{j4\pi\omega\varepsilon}\iiint_{V'} \left\{\frac{\beta^2}{r_d} e^{-j\beta r_d}\left(-1 - \frac{3}{j\beta r_d} + \frac{3}{\beta^2 {r_d}^2}\right)\hat{r}_d \times \hat{r}_d \times \vec{J}(\vec{r}') - 2e^{-j\beta r_d}\left(-\frac{1}{{r_d}^3} - \frac{j\beta}{{r_d}^2}\right)\vec{J}(\vec{r}')\right\} dv'$$

- *For far fields:*
  - We need to consider $\frac{1}{r_d}$ term only,
    - higher order terms can be neglected
- $\vec{H} = \frac{1}{4\pi}\iiint_{V'} e^{-j\beta r_d}\left(-\frac{1}{{r_d}^3} - \frac{j\beta}{{r_d}^2}\right)\vec{r}_d \times \vec{J}(\vec{r}')dv'$

$$= \frac{1}{4\pi}\iiint_{V'} e^{-j\beta r_d}\left(-\frac{1}{{r_d}^2} - \frac{j\beta}{r_d}\right)\hat{r}_d \times \vec{J}(\vec{r}')dv'$$

$$= \frac{1}{4\pi}\iiint_{V'} e^{-j\beta r_d}\left(-\frac{j\beta}{r_d}\right)\hat{r}_d \times \vec{J}(\vec{r}')dv'$$

- $\vec{E} = \frac{1}{j4\pi}\iiint_{V'}\left\{\frac{\beta^2}{r_d}e^{-j\beta r_d}\left(-1 - \frac{3}{j\beta r_d} + \frac{3}{\beta^2 {r_d}^2}\right)\hat{r}_d \times \hat{r}_d \times \vec{J}(\vec{r}') - 2e^{-j\beta r_d}\left(-\frac{1}{{r_d}^3} - \frac{j\beta}{{r_d}^2}\right)\vec{J}(\vec{r}')\right\}dv'$

$$= -\frac{1}{j4\pi\omega\varepsilon}\iiint_{V'}\left\{\frac{\beta^2}{r_d}e^{-j\beta r_d}\hat{r}_d \times \hat{r}_d \times \vec{J}(\vec{r}')\right\}dv'$$

Radiation

- Unit vector: $\hat{r}_d = \hat{r}$
- Let us simplify term $\frac{1}{r_d} e^{-j\beta r_d}$
  - Amplitude: $\frac{1}{r_d} \cong \frac{1}{r}$ (range of values of r is 0 to ∞)
  - Phase: $e^{-j\beta r_d} \cong e^{-j\ \ (r - \vec{r}' \cdot \hat{r})}$

  (phase can change from 0 to 2π only)
- *Magnetic field (far field):*
- $\vec{H} \cong -\frac{j\beta e^{-j\beta r}}{4\pi r} \hat{r} \times \iiint_{V'} \left(\vec{J}(\vec{r}') e^{j\beta(\vec{r}' \cdot \hat{r})} \right) dv'$
- *Electric field (far field):*
- $\vec{E} = -\frac{\beta^2}{j\omega\varepsilon} \frac{e^{-j\beta r}}{4\pi r} \hat{r} \times \hat{r} \times \iiint_{V'} \left\{ \vec{J}(\vec{r}') e^{j\beta(\vec{r}' \cdot \hat{r})} \right\} dv'$

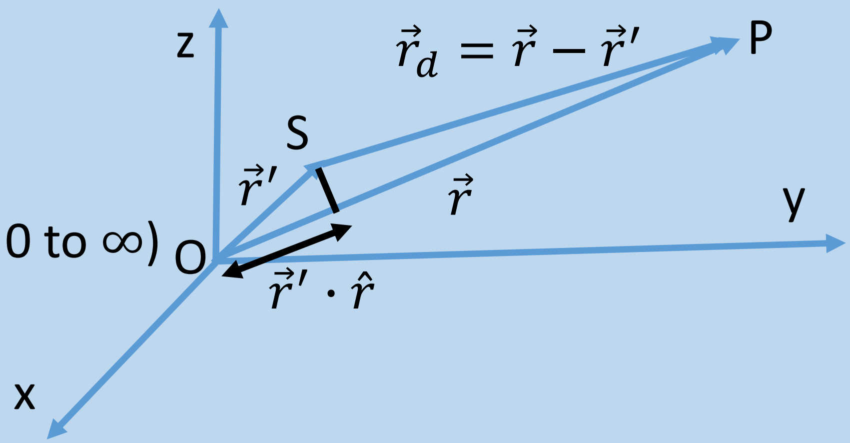

Fig. Radiation from an arbitrary current density $\vec{J}$ at point S in the far field region

Radiation

- *Points to be noted:*
  - Magnetic field is
    - perpendicular to the direction of wave propagation $\hat{r}$ since it has $\hat{r} \times (another\ vector)$
  - Relation between electric and magnetic field
    - $\vec{E} = -\frac{\beta}{\omega\varepsilon}\frac{e^{-j\beta}}{4\pi r}\hat{r} \times \vec{H} = \sqrt{\frac{\mu}{\varepsilon}}\vec{H} \times \hat{r} = \eta\vec{H} \times \hat{r}$
    - Free space wave impedance (intrinsic impedance of free space) is approximately $120\,\pi\Omega \cong 377\Omega$
  - Electric field is also
    - perpendicular to the direction of wave propagation $\hat{r}$ and
    - magnetic field
  - EM waves propagate in radial direction
    - away from the source

Radiation

- *Far fields of an arbitrary oriented current carrying element:*
  - $\hat{\mathrm{d}}$ directed current carrying element (arbitrary direction)
  - of infinitesimally small length dl and
  - carrying current I_0
- $\vec{H} \cong -\frac{j\beta e^{-j\beta r}}{4\pi r}\hat{r} \times \iiint_{V'}\left(\vec{J}(\vec{r}')e^{j\beta(\vec{r}'\cdot\hat{r})}\right)dv'$
- Assume that it is positioned at origin $e^{j\beta(\vec{r}'\cdot\hat{r})} = 1$
  - $\left(\vec{J}(\vec{r}')e^{j\beta(\vec{r}'\cdot\hat{r})}\right)dv' = I_0 dl\hat{d}$
- Magnetic field:
  - $\vec{H} \cong -\frac{j\beta I_0 dl\ ^{-j\beta}}{4\pi r}\left(\hat{r} \times \hat{d}\right)$
- Electric field:
- $\vec{E} = \eta\vec{H} \times \hat{r} = -\eta\frac{j\beta\ _0 dle^{-j\beta r}}{4\pi r}\left(\hat{r} \times \hat{d}\right) \times \hat{r}$

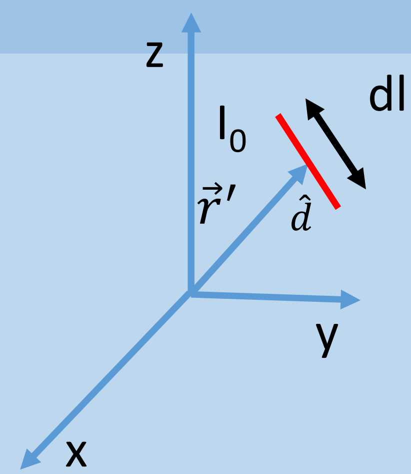

Fig. An arbitrary oriented current carrying element carrying current I_0 of length dl at the position $\vec{r}'$

Radiation

- *Coordinate transformations:*
- $\hat{r} = sin\theta cos\varphi\hat{x} + sin\theta sin\varphi\hat{y} + cos\theta\hat{z}$
- For $\hat{\theta}$, we replace $\theta = \left(\theta + \frac{\pi}{2}\right)$ in the above expression of $\hat{r}$
- $\hat{\theta} = sin\left(\theta + \frac{\pi}{2}\right) cos\varphi\hat{x} + sin\left(\theta + \frac{\pi}{2}\right) sin\varphi\hat{y} + cos\left(\theta + \frac{\pi}{2}\right)\hat{z}$
  $= cos\theta cos\varphi\hat{x} + cos\theta sin\varphi\hat{y} - sin\theta\hat{z}$
- For $\hat{\varphi}$, we replace $\theta = \frac{\pi}{2}$ and $\varphi = \left(\varphi + \frac{\pi}{2}\right)$ in the above expression of $\hat{r}$
- $\hat{\varphi} = sin\left(\frac{\pi}{2}\right) cos\left(\varphi + \frac{\pi}{2}\right)\hat{x} + sin\left(\frac{\pi}{2}\right) sin\left(\varphi + \frac{\pi}{2}\right)\hat{y} + cos\left(\frac{\pi}{2}\right)\hat{z}$
  $= -sin\varphi\hat{x} + cos\varphi\hat{y}$

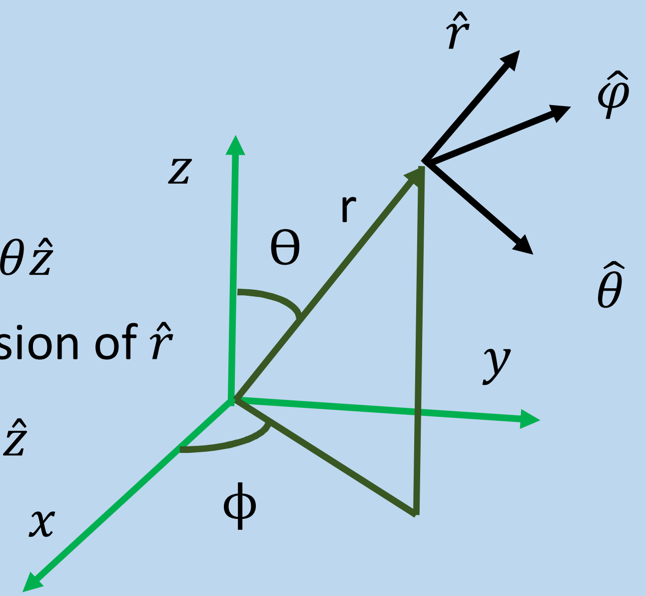

Fig. Relation between spherical and Cartesian coordinates

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- *Coordinate transformations:*
- In matrix form (Cartesian → Spherical),
- $$\begin{bmatrix} \hat{r} \\ \hat{\theta} \\ \hat{\varphi} \end{bmatrix} = \begin{bmatrix} sin\theta cos\varphi & sin\theta sin\varphi & cos\theta \\ cos\theta cos\varphi & cos\theta sin\varphi & -sin\theta \\ -sin\varphi & cos\varphi & 0 \end{bmatrix} \begin{bmatrix} \hat{x} \\ \hat{y} \\ \hat{z} \end{bmatrix}$$
- In matrix form (Spherical → Cartesian),
- $$\begin{bmatrix} \hat{x} \\ \hat{y} \\ \hat{z} \end{bmatrix} = \begin{bmatrix} sin\theta cos\varphi & sin\theta sin\varphi & cos\theta \\ cos\theta cos\varphi & cos\theta sin\varphi & -sin\theta \\ -sin\varphi & cos\varphi & 0 \end{bmatrix}^{-1} \begin{bmatrix} \hat{r} \\ \hat{\theta} \\ \hat{\varphi} \end{bmatrix}$$
- For unitary matrix, $UU^T = I \Rightarrow U^T = U^{-1}$, therefore,
- $$\begin{bmatrix} \hat{x} \\ \hat{y} \\ \hat{z} \end{bmatrix} = \begin{bmatrix} sin\theta cos\varphi & cos\theta cos\varphi & -sin\varphi \\ sin\theta sin\varphi & cos\theta sin\varphi & cos\varphi \\ cos\theta & -sin\theta & 0 \end{bmatrix} \begin{bmatrix} \hat{r} \\ \hat{\theta} \\ \hat{\varphi} \end{bmatrix}$$

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- *Far fields of VED:*
  - $\hat{z}$ directed current carrying element
  - of infinitesimally small length dl and
  - carrying current I_0
- Magnetic field: replace $\hat{d} = \hat{z}$
  - $\vec{H} \cong -\frac{j\beta I_0 dl e^{-j\beta r}}{4\pi r}(\hat{r} \times \hat{z})$

$$= -\frac{j\beta I_0 dl e^{-j\beta r}}{4\pi r}\{\hat{r} \times (cos\theta\hat{r} - sin\theta\hat{\theta})\}$$

$$= \frac{j\beta I_0 dl e^{-j\beta r}}{4\pi r} sin\theta\hat{\varphi}$$

- Electric field:
- $\vec{E} = \eta\vec{H} \times \hat{r} = \eta \frac{j\beta I_0 dl e^{-j\beta r}}{4\pi r} sin\theta\hat{\varphi} \times \hat{r} = \eta \frac{j\beta I_0 dl e^{-j\beta r}}{4\pi r} sin\theta\hat{\theta}$

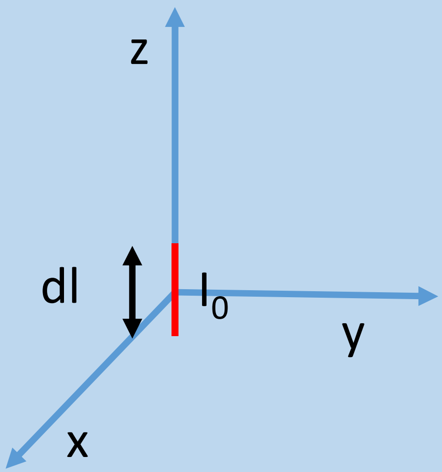

Fig. A z-directed current carrying element carrying current I_0 of length dl at the origin

Radiation

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- *Far fields of VED:*
- Because of $\sin\theta$ term
- in electric field
- Polar radiation pattern
- $(\varphi = \text{constant})$
- will look line 8
- with maximum
- radiation at $\theta = \frac{\pi}{2}$
- null at $\theta = 0$

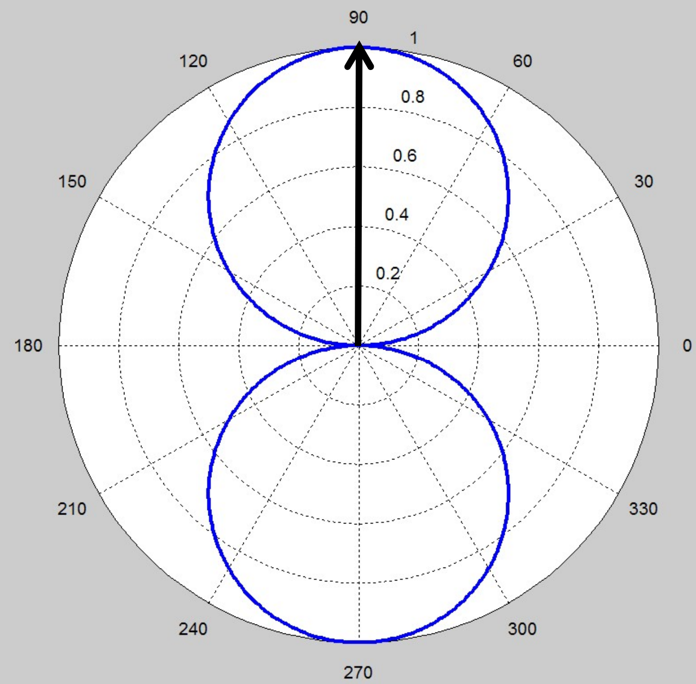

Fig. Polar radiation pattern of a Hertz dipole (VED) in far field ($\varphi = constant$)

Radiation

- *Far fields of HED:*
  - $\hat{y}$ directed current carrying element of
  - infinitesimally small length dl
  - and carrying current I_0
- Magnetic field: replace $\hat{d} = \hat{y}$
  - $\vec{H} \cong -\frac{j\beta I_0 dl}{4\pi r}^{-j\beta r}(\hat{r} \times \hat{y})$

$$= -\frac{j\beta I_0 dl e^{-j\beta r}}{4\pi r}\left\{\hat{r} \times \left(sin\theta sin\varphi\hat{r} + cos\theta sin\varphi\hat{\theta} + cos\varphi\hat{\varphi}\right)\right\}$$

$$= -\frac{j\beta I_0 dl e^{-j\beta r}}{4\pi r}\left\{cos\theta sin\varphi\hat{\varphi} - cos\varphi\hat{\theta}\right\}$$

- Electric field:
- $\vec{E} = \eta\vec{H} \times \hat{r} = -\eta\frac{j\beta I_0 dl e^{-j\beta r}}{4\pi r}\left\{cos\theta sin\varphi\hat{\theta} + cos\varphi\hat{\varphi}\right\}$

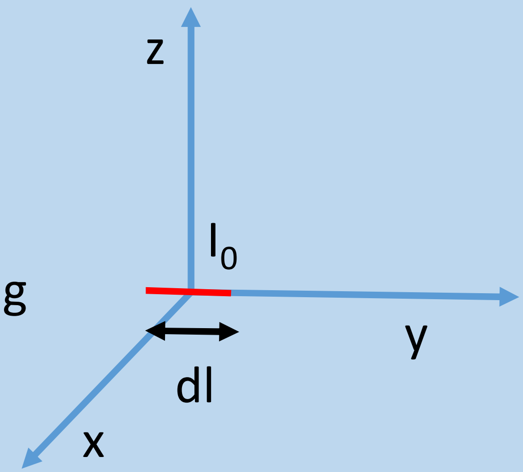

Fig. A y-directed current carrying element carrying current I_0 of length dl at the origin