EE540 Advance Electromagnetic Theory & Antennas

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Radiation

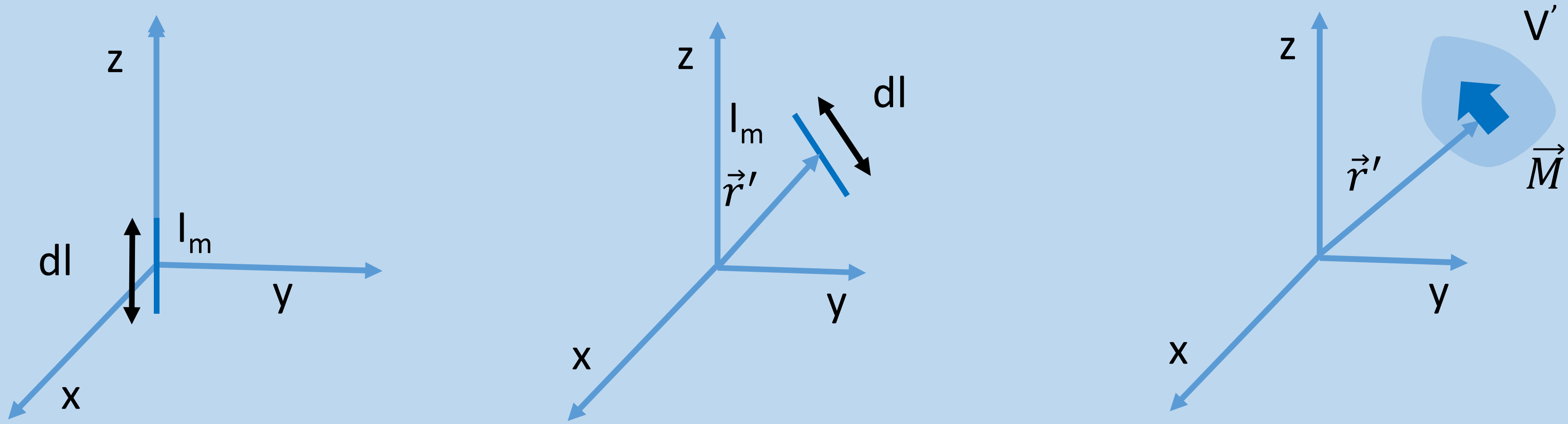

Fig. 1st case: A z-directed current carrying element carrying magnetic current I_m of length dl at the origin

Fig. 2nd case: An arbitrary oriented current carrying element carrying magnetic current I_m of length dl at the position $\vec{r}'$

Fig. 3rd case: An arbitrary oriented magnetic current density $\vec{M}$ is flowing in a volume V′ positioned at $\vec{r}'$

Radiation

- *Procedure to find the radiated fields:*
- It involves three steps
  - *Step 1:* Solve $\vec{F}$ from $\vec{M}$ as

$$\vec{F}(\vec{r}) = \frac{\varepsilon}{4\pi} \int_{V'} \vec{M}(\vec{r}\,') \frac{e^{-j\beta|\vec{r}-\vec{r}\,'|}}{|\vec{r}-\vec{r}\,'|} dv'$$

  - *Step 2:* Find electric field ($\vec{E}_F$) as

$$\vec{E}_F = -\frac{1}{\epsilon}(\nabla \times \vec{F})$$

  - *Step 3:* Find magnetic field ($\vec{H}_F$) as

$$\vec{H}_F = \frac{\nabla \times \nabla \times \vec{F}}{j\omega\mu\varepsilon} = -\frac{1}{j\omega\mu} \nabla \times -\left(\frac{\nabla \times \vec{F}}{\varepsilon}\right) = -\frac{1}{j\omega\mu}(\nabla \times \vec{E}_F)$$

Radiation

Table: *Procedure to find the radiated fields*	Electric Sources ($\vec{J} \neq 0,\ \vec{M} = 0$)	Magnetic Sources ($\vec{J} = 0,\ \vec{M} \neq 0$)
	$\vec{A}(\vec{r}) = \frac{\mu}{4\pi} \int_{V'} \vec{J}(\vec{r}') \frac{e^{-j\beta\lvert\vec{r}-\vec{r}'\rvert}}{\lvert\vec{r}-\vec{r}'\rvert} dv'$	$\vec{F}(\vec{r}) = \frac{\varepsilon}{4\pi} \int_{V'} \vec{M}(\vec{r}') \frac{e^{-j\beta\lvert\vec{r}-\vec{r}'\rvert}}{\lvert\vec{r}-\vec{r}'\rvert} dv'$
	$\vec{H}_A = \frac{1}{\mu}(\nabla \times \vec{A})$	$\vec{E}_F = -\frac{1}{\epsilon}(\nabla \times \vec{F})$
	$\vec{E}_A = \frac{\nabla \times \nabla \times \vec{A}}{j\omega\mu\varepsilon}$	$\vec{H}_F = \frac{\nabla \times \nabla \times \vec{F}}{j\omega\mu\varepsilon}$

Radiation

- When two equations that describe the behavior of two different variables
  - are of the same mathematic form,
    - their solutions will also be identical
- The variables in the two equations that
  - occupy identical positions are known as dual quantities and
    - a solution of one can be obtained by a
      - systematic interchange of symbols to the other
- This concept is known as the *duality theorem*

Radiation

Table: Dual quantities for electric ($\vec{J}$) and magnetic ($\vec{M}$) current sources

Electric Sources ($\vec{J} \neq 0, \vec{M} = 0$)	Magnetic Sources ($\vec{J} = 0, \vec{M} \neq 0$)
$\vec{H}_A$ →	$-\vec{E}_F$
$\vec{E}_A$ →	$\vec{H}_F$
$\vec{J}$ →	$\vec{M}$
$\vec{A}$ →	$\vec{F}$
ϵ →	μ
μ →	ϵ
β →	β
$\eta = \sqrt{\frac{\mu}{\varepsilon}}$ →	$\frac{1}{\eta} = \sqrt{\frac{\varepsilon}{\mu}}$
$\frac{1}{\eta} = \sqrt{\frac{\varepsilon}{\mu}}$ →	$\eta = \sqrt{\frac{\mu}{\varepsilon}}$

Radiation

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- *Far fields of VMD:*
  - $\hat{z}$ directed magnetic current carrying element
  - of infinitesimally small length dl and
  - carrying magnetic current I_m
  - Apply duality theorem

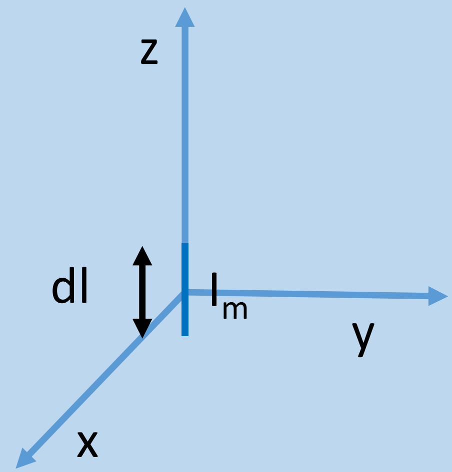


Fig. A z-directed current carrying element carrying magnetic current I_m of length dl at the origin

VED	$\vec{H} = \frac{j\beta I_0 dl e^{-j\beta r}}{4\pi r} sin\theta\hat{\varphi}$	$\vec{E} = \eta\frac{j\beta I_0 dl e^{-j\beta r}}{4\pi r} sin\theta\hat{\theta}$
VMD	$\vec{E} = -\frac{j\beta I_m dl e^{-j\beta r}}{4\pi r} sin\theta\hat{\varphi}$	$\vec{H} = \frac{1}{\eta}\frac{j\beta I_m dl e^{-j\beta r}}{4\pi r} sin\theta\hat{\theta}$

- *Far fields of HMD:*
- $\hat{y}$ directed magnetic current carrying element of
- infinitesimally small length dl
- and carrying magnetic current Im
- Apply duality theorem

HED	$\vec{H} = -\frac{j\beta I_0 dl e^{-j\beta r}}{4\pi r}\{\cos\theta\sin\varphi\hat{\varphi} - \cos\varphi\hat{\theta}\}$	$\vec{E} = -\eta\frac{j\beta I_0 dl e^{-j\beta r}}{4\pi r}\{\cos\theta\sin\varphi\hat{\theta} + \cos\varphi\hat{\varphi}\}$
HMD	$\vec{E} = \frac{j\beta I_m dl e^{-j\beta r}}{4\pi r}\{\cos\theta\sin\varphi\hat{\varphi} - \cos\varphi\hat{\theta}\}$	$\vec{H} = -\frac{1}{\eta}\frac{j\beta I_m dl e^{-j\beta r}}{4\pi r}\{\cos\theta\sin\varphi\hat{\theta} + \cos\varphi\hat{\varphi}\}$

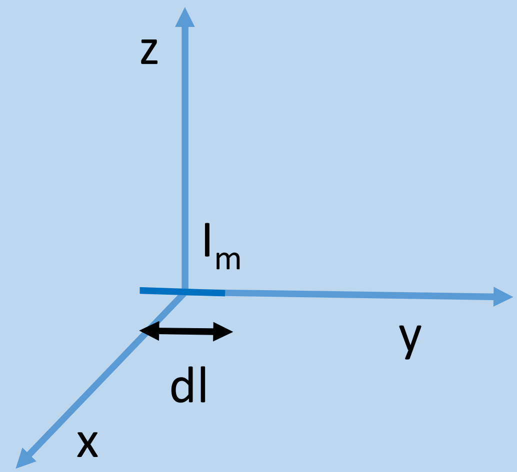


Fig. A y-directed current carrying element carrying magnetic current I_m of length dl at the origin

Electromagnetic Theorems and Concepts

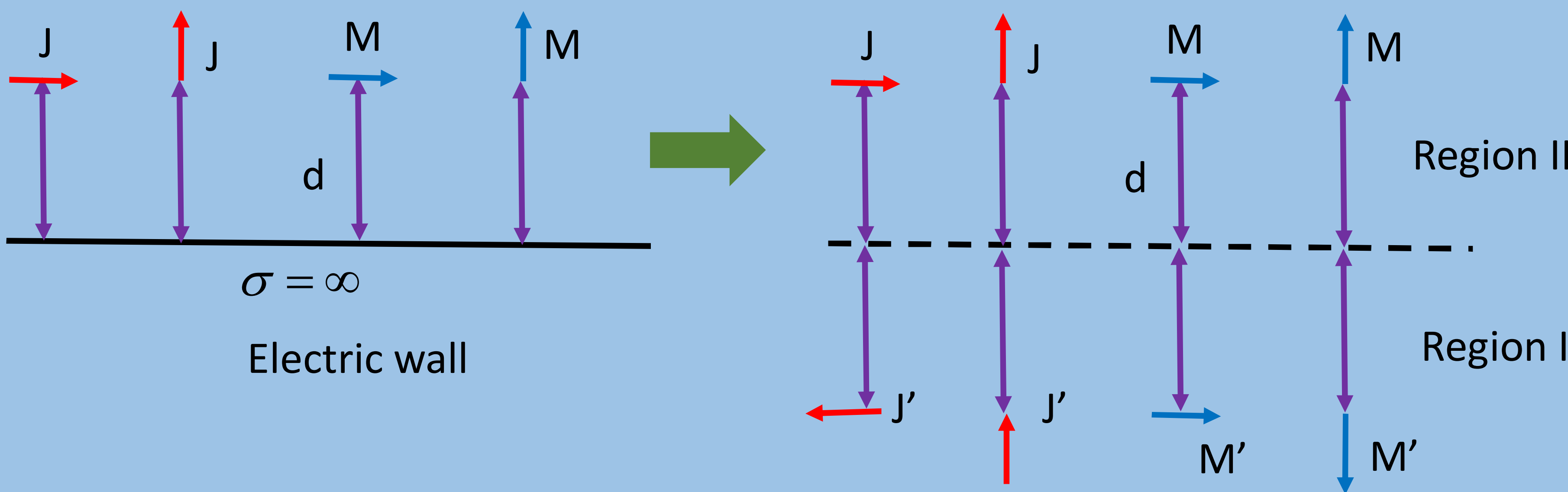
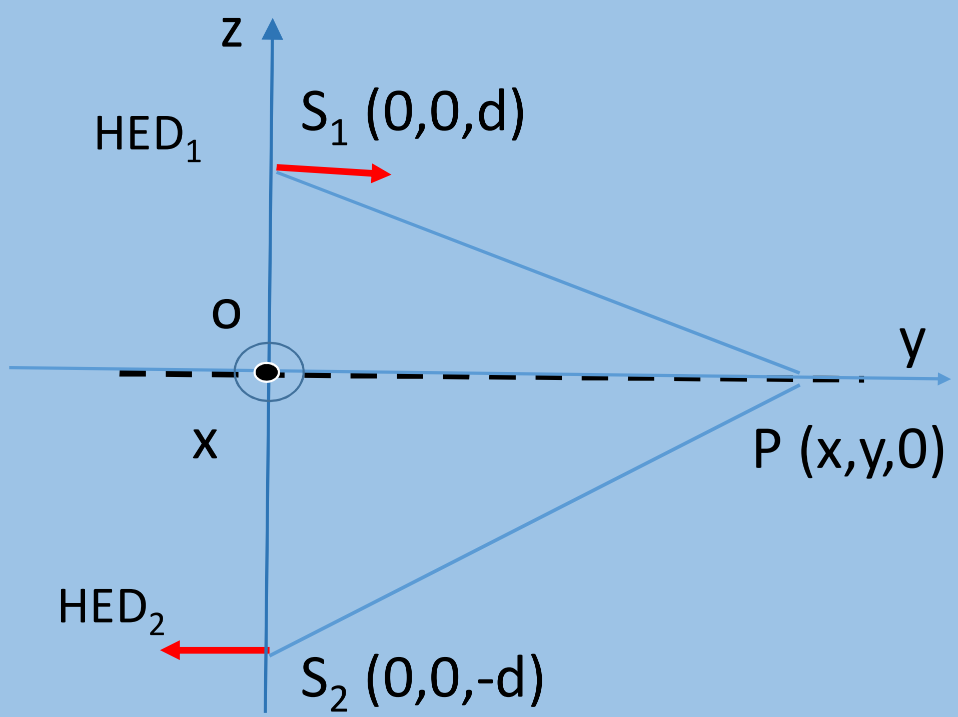


Fig. Classic image current problem for electric wall

Electromagnetic Theorems and Concepts



- Interface is at z=0
- For HED$_1$, $S_1P = r_d = \sqrt{x^2 + y^2 + d^2}$
- $\hat{r}_d = \frac{x\hat{x}+y\hat{y}-d\hat{z}}{r_d}$
- For HED$_2$, $S_2P = r_d = \sqrt{x^2 + y^2 + d^2}$
- $\hat{r}_d = \frac{x\hat{x}+y\hat{y}+d\hat{z}}{r_d}$
- Note that r_d is equal for both HED$_s$
- but $\hat{r}_d$ is not the same for both HED$_s$

Fig. Classic image current problem for electric wall

Electromagnetic Theorems and Concepts

- $\vec{E} = \frac{1}{j4\pi\omega\varepsilon}\iiint_{V'}\left\{\frac{\beta^2}{r_d}e^{-j\beta r_d}\left(-1-\frac{3}{j\beta r_d}+\frac{3}{\beta^2 {r_d}^2}\right)\hat{r}_d\times\hat{r}_d\times\vec{J}(\vec{r}') - 2e^{-j\beta r_d}\left(-\frac{1}{{r_d}^3}-\frac{j\beta}{{r_d}^2}\right)\vec{J}(\vec{r}')\right\}dv'$
- For both HEDs, 2nd term has y-component
- But they cancel each other (due to opposite direction)
- For both HEDs, we have $\hat{r}_d\times\hat{r}_d\times\vec{J}(\vec{r}')$ in 1st term
- For HED_1,
- $\hat{r}_d\times\hat{r}_d\times\vec{J}(\vec{r}') = \frac{x\hat{x}+y\hat{y}-d\hat{z}}{r_d}\times\left(\frac{x\hat{x}+y\hat{y}-d\hat{z}}{r_d}\times I_0\hat{y}\right) = \frac{I_0}{r_d^2}(x\hat{x}+y\hat{y}-d\hat{z})\times(x\hat{z}+d\hat{x}) = \frac{I_0}{r_d^2}(-x^2\hat{y}+xy\hat{x}-yd\hat{z}-d^2\hat{y})$

Electromagnetic Theorems and Concepts

- $\vec{E}_{tan}^{HED_1} = \frac{1}{j4\pi\omega\varepsilon}\left\{\frac{\beta^2}{r_d}e^{-j\beta r_d}\left(-1 - \frac{3}{j\beta\ \ d} + \frac{3}{\beta^2 {r_d}^2}\right)\frac{I_0}{r_d^2}\left(-x^2\hat{y} + xy\hat{x} - d^2\hat{y}\right)\right\}dl$
- For HED$_2$,
- $\hat{r}_d \times \hat{r}_d \times \vec{J}(\vec{r}') = \frac{x\hat{x}+y\hat{y}+d\hat{z}}{r_d} \times \left(\frac{x\hat{x}+y\hat{y}+d\hat{z}}{r_d} \times -I_0\hat{y}\right) = \frac{I_0}{r_d^2}(x\hat{x} + y\hat{y} + d\hat{z}) \times (-x\hat{z} + d\hat{x}) = \frac{I_0}{r_d^2}(x^2\hat{y} - xy\hat{x} - yd\hat{z} + d^2\hat{y})$
- $\vec{E}_{tan}^{HED_2} = \frac{1}{j4\pi\omega\varepsilon}\left\{\frac{\beta^2}{r_d}e^{-j\beta r_d}\left(-1 - \frac{3}{j\beta r_d} + \frac{3}{\beta^2 {r_d}^2}\right)\frac{I_0}{r_d^2}\left(x^2\hat{y} - xy\hat{x} + d^2\hat{y}\right)\right\}dl$
- Therefore $\vec{E}_{tan}^{HED_1} + \vec{E}_{tan}^{HED_2} = 0$

Electromagnetic Theorems and Concepts

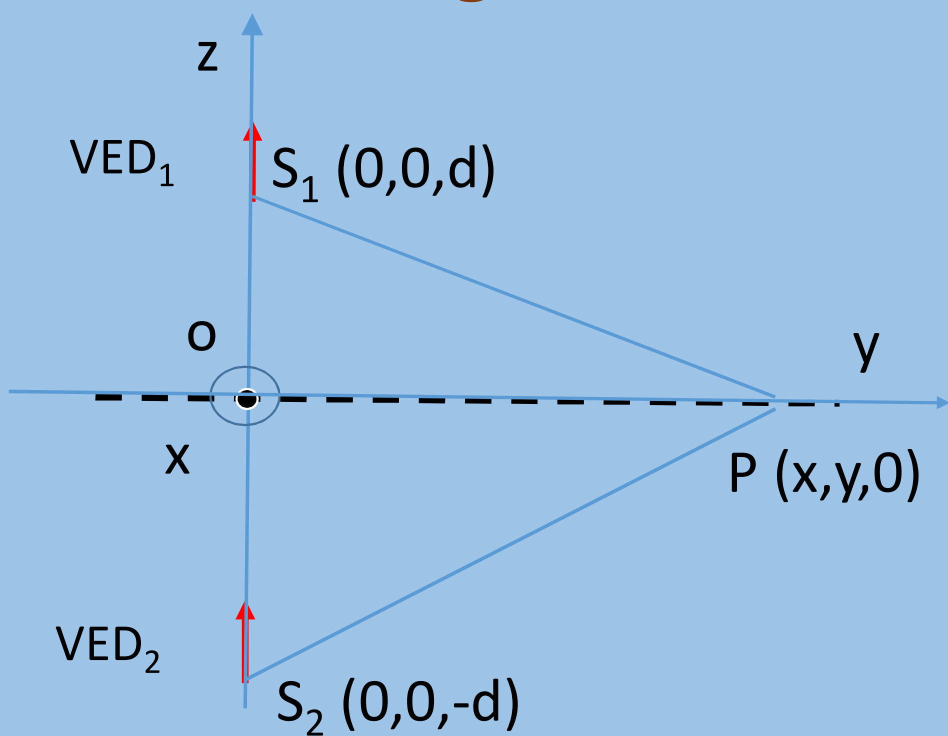


Fig. Classic image current problem

- Interface is at z=0
- For VED$_1$, $S_1P = r_d = \sqrt{x^2 + y^2 + d^2}$
- $\hat{r}_d = \frac{x\hat{x}+y\hat{y}-d\hat{z}}{r_d}$
- For VED$_2$, $S_2P = r_d = \sqrt{x^2 + y^2 + d^2}$
- $\hat{r}_d = \frac{x\hat{x}+y\hat{y}+d\hat{z}}{r_d}$
- Note that r_d is equal for both VED$_s$ but $\hat{r}_d$ is not the same for both VED$_s$

Electromagnetic Theorems and Concepts

- $\vec{E} = \frac{1}{j4\pi\omega\varepsilon}\iiint_{V'}\left\{\frac{\beta^2}{r_d}e^{-j\beta r_d}\left(-1-\frac{3}{j\beta r_d}+\frac{3}{\beta^2 {r_d}^2}\right)\hat{r}_d\times\hat{r}_d\times\vec{J}(\vec{r}') - 2e^{-j\beta r_d}\left(-\frac{1}{{r_d}^3}-\frac{j\beta}{{r_d}^2}\right)\vec{J}(\vec{r}')\right\}dv'$
- For both VED$_s$, 2nd term has only z-component
- But we are interested in tangential components
- For both VED$_s$, we have $\hat{r}_d\times\hat{r}_d\times\vec{J}(\vec{r}')$ in the first term
- For VED$_1$,
- $\hat{r}_d\times\hat{r}_d\times\vec{J}(\vec{r}') = \frac{x\hat{x}+y\hat{y}-d\hat{z}}{r_d}\times\left(\frac{x\hat{x}+y\hat{y}-d\hat{z}}{r_d}\times I_0\hat{z}\right) = \frac{I_0}{r_d^2}(x\hat{x}+y\hat{y}-d\hat{z})\times(y\hat{x}-x\hat{y}) = \frac{I_0}{r_d^2}(-x^2\hat{z}-y^2\hat{z}-yd\hat{y}-xd\hat{x})$

Electromagnetic Theorems and Concepts

- $\vec{E}_{tan}^{VED_1} = \frac{1}{j4\pi\omega\varepsilon}\left\{\frac{\beta^2}{r_d}e^{-j\beta r_d}\left(-1-\frac{3}{j\beta r_d}+\frac{3}{\beta^2 {r_d}^2}\right)\frac{I_0}{r_d^2}(-yd\hat{y}-xd\hat{x})\right\}dl$
- For VED_2,
- $\hat{r}_d \times \hat{r}_d \times \vec{J}(\vec{r}') = \frac{x\hat{x}+y\hat{y}+d\hat{z}}{r_d} \times \left(\frac{x\hat{x}+y\hat{y}+d\hat{z}}{r_d} \times I_0\hat{z}\right) = \frac{I_0}{r_d^2}(x\hat{x}+y\hat{y}+d\hat{z}) \times (y\hat{x}-x\hat{y}) = \frac{I_0}{r_d^2}(-x^2\hat{z}-y^2\hat{z}+yd\hat{y}+xd\hat{x})$
- $\vec{E}_{tan}^{VED_2} = \frac{1}{j4\pi\omega\varepsilon}\left\{\frac{\beta^2}{r_d}e^{-j\beta r_d}\left(-1-\frac{3}{j\beta\ \ d}+\frac{3}{\beta^2 {r_d}^2}\right)\frac{I_0}{r_d^2}(yd\hat{y}+xd\hat{x})\right\}dl$
- Therefore $\vec{E}_{tan}^{VED_1} + \vec{E}_{tan}^{VED_2} = 0$