EE540 Advance Electromagnetic Theory & Antennas

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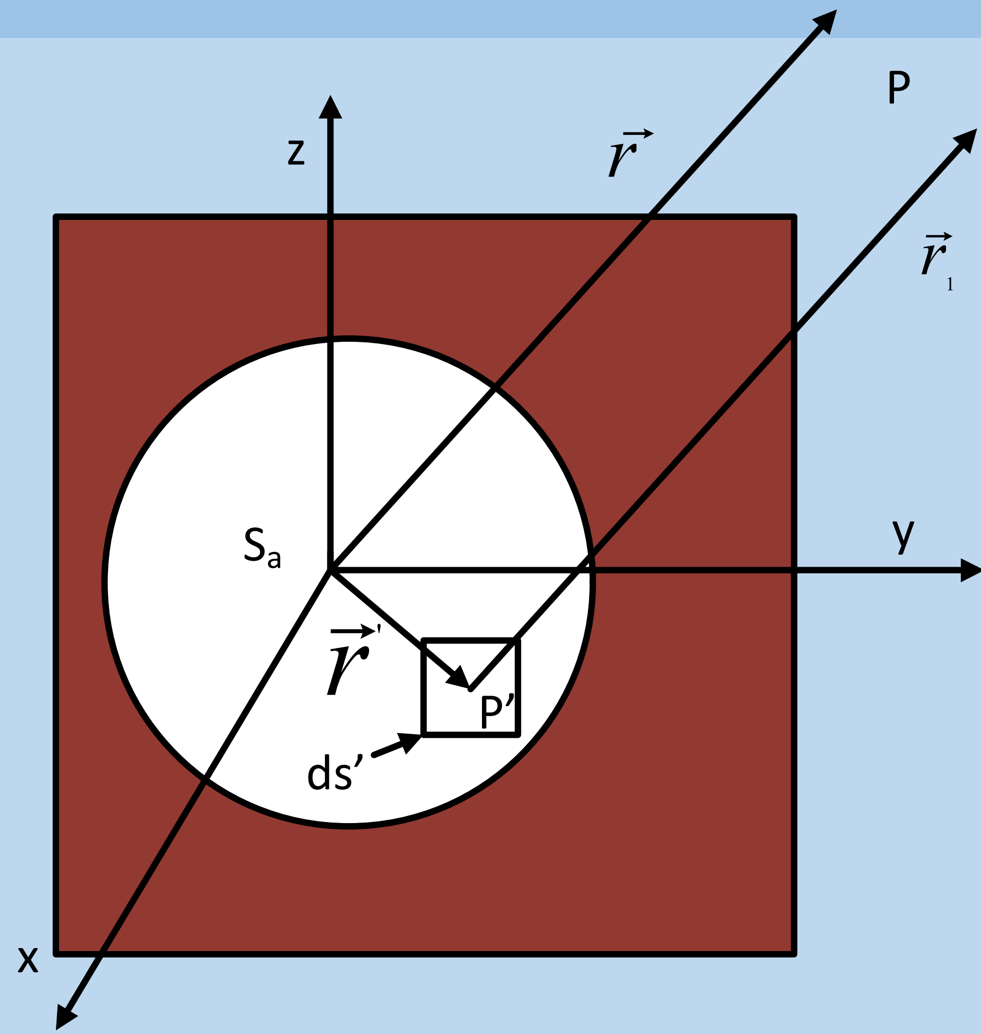
Aperture antennas

- Two types of antennas:
- Type I (actual source) antennas
  - such as Dipole, Loop antennas
- Type II (equivalent source) antennas
  - such as aperture antennas
- Antenna analysis for Type I antennas:
  - $(\vec{J}_S, \vec{M}_s) \to (\vec{A}, \vec{F}) \to (\vec{E}, \vec{H})$
- Antenna analysis for Type II antennas:
  - $(\vec{E}_a, \vec{H}_a) \to (\vec{J}_S, \vec{M}_s) \to (\vec{A}, \vec{F}) \to (\vec{E}, \vec{H})$

Aperture antennas

- Fig. Radiation from Type II antenna (an aperture S_a positioned in x-y plane)



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Aperture antennas

- In type II antennas,
  - fields in the aperture $\left(\vec{E}_a, \vec{H}_a\right)$ is always known
- For example,
  - in open-ended waveguide,
  - we know the field propagating inside the waveguide
- Hence we can find equivalent currents from the aperture fields as
  - $\vec{J}_S = \hat{n} \times \vec{H}_a, \vec{M}_S = -\hat{n} \times \vec{E}_a$
- So we can find
- $\vec{A} = \frac{\mu}{4\pi} \iint_{S_a} \vec{J}_S \frac{e^{-j\beta|\vec{r}-\vec{r}'|}}{|\vec{r}-\vec{r}'|} ds'$

Aperture antennas

- For our case,
  - $\vec{J}_S = \hat{n} \times \vec{H}_a = \hat{z} \times \left(H_{ax}\hat{x} + H_{ay}\hat{y}\right) = -H_{ay}\hat{x} + H_{ax}\hat{y}$
- So we can find
- $\vec{A} = \frac{\mu}{4\pi} \iint_{S_a} \left(-H_{ay}\hat{x} + H_{ax}\hat{y}\right) \frac{e^{-j\beta|\vec{r}-\vec{r}'|}}{|\vec{r}-\vec{r}'|} ds'$
- In far field,
- $|\vec{r} - \vec{r}'| = r$ for amplitude term
- For phase term,
- $|\vec{r} - \vec{r}'| = [(x - x')^2 +(y - y')^2 +(z)^2]^{\frac{1}{2}}$

$$= [r^2 - 2xx' - 2yy' + (x')^2 + +(y')^2]^{\frac{1}{2}}$$

Aperture antennas

- For phase term,
- $|\vec{r}-\vec{r}'| = [r^2 - 2xx' - 2yy' + (x')^2 + (y')^2]^{\frac{1}{2}}$

$$= r\left[1 - \frac{2xx'}{r^2} - \frac{2yy'}{r^2} + \frac{(x')^2 + (y')^2}{r^2}\right]^{\frac{1}{2}}$$

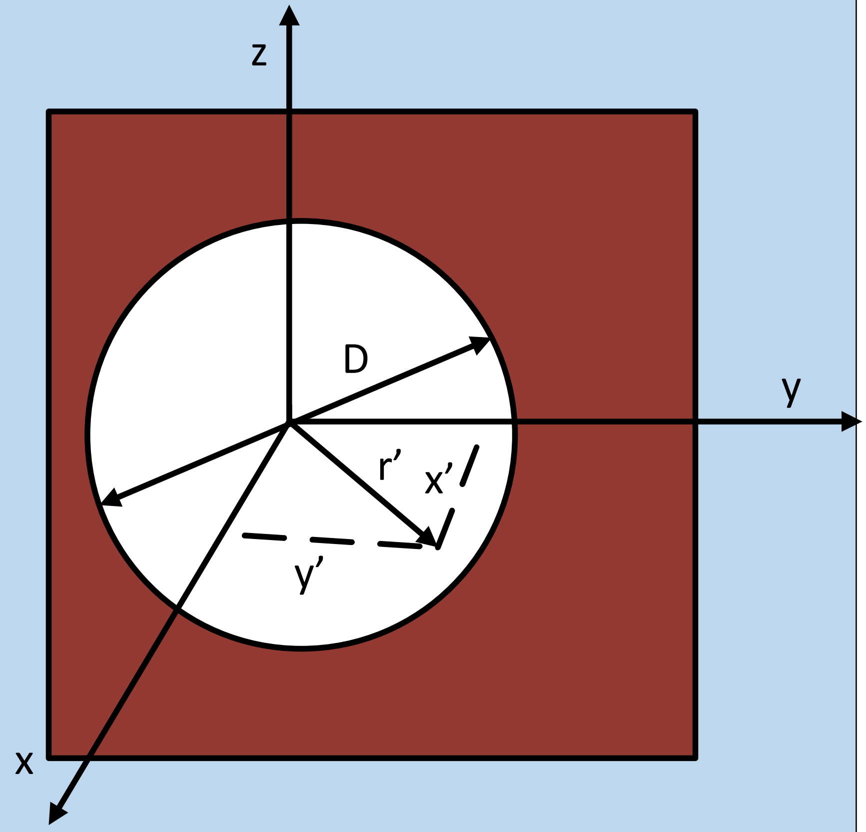
- Doing binomial expansion,
- $|\vec{r}-\vec{r}'| = r - \frac{xx'}{r} - \frac{yy'}{r} + \frac{(x')^2 + (y')^2}{2r} + \cdots$
- For far fields, last term can be neglected

$$|\vec{r}-\vec{r}'| \cong r - \frac{xx'}{r} - \frac{yy'}{r}$$

Aperture antennas

- How can we neglect the last term for far fields?
- Note that for far fields, neglecting the last term will give an error in phase approximation as
- $\beta \frac{(x')^2+(y')^2}{2r_{ff}} \leq \frac{\pi}{8}$
- Or, $\frac{(x')^2+(y')^2}{2r_{ff}} \leq \frac{\lambda}{16}$
- Since $r' = \sqrt{(x')^2 + (y')^2} \leq \frac{D}{2}$
- where D is the largest diameter/dimension
- of the antenna, therefore,
- $\frac{(D)^2}{8r_{ff}} \leq \frac{\lambda}{16}$
- Hence, $\frac{2(D)^2}{\lambda} \leq r_{ff}$

Fig. Largest dimension of the antenna



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- Hence,
- $\frac{e^{-j\beta|\vec{r}-\vec{r}'|}}{|\vec{r}-\vec{r}'|} \cong \frac{e^{-j\beta\left(r-\frac{xx'}{r}-\frac{yy'}{r}\right)}}{r}$
- Also note that
- $\frac{x}{r} = \frac{\beta_x}{\beta} = sin\theta cos\phi, \frac{y}{r} = \frac{\beta_y}{\beta} = sin\theta sin\phi$
- Hence
- $\frac{e^{-j\beta|\vec{r}-\vec{r}'|}}{|\vec{r}-\vec{r}'|} \cong \frac{e^{(-j\beta r+j\beta_x x'+j\beta_y y')}}{r}$

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- Therefore,
- $\vec{A} = \frac{\mu e^{(-j\beta r)}}{4\pi r} \iint_{S_a} \left(-H_{ay}\hat{x} + H_{ax}\hat{y}\right) e^{\left(j\beta_x x' + j\beta_y y'\right)} dx' dy'$
- It can be written as
- $\vec{A} = \frac{\mu e^{(-j\beta r)}}{4\pi r} \left(-\widetilde{H}_{ay}\hat{x} + \widetilde{H}_{ax}\hat{y}\right)$
- where $\widetilde{H}_{ay}$ and $\widetilde{H}_{ax}$ are the 2-D FT of H_{ay} and H_{ax} respectively
- *This is profound and useful result*
  - *which shows that FT plays a significant role in finding fields of type II antenna such as aperture antennas*

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- Similarly,
- $\vec{F} = \frac{\varepsilon}{4\pi} \iint_{S_a} \vec{M}_S \frac{e^{-j\beta|\vec{r}-\vec{r}'|}}{|\vec{r}-\vec{r}'|} ds'$
- $\vec{F} = -\frac{\varepsilon e^{(-j\beta r)}}{4\pi r}\left(-\tilde{E}_{ay}\hat{x} + \tilde{E}_{ax}\hat{y}\right)$
- where $\tilde{E}_{ay}$ and $\tilde{E}_{ax}$ are the 2-D FT of E_{ay} and E_{ax} respectively
- The negative sign is due to $\vec{M}_S = -\hat{n} \times \vec{E}_a$
- But we always find fields in spherical coordinates
- So we need to do a coordinate transformation

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- Coordinate transformation from Cartesian → Spherical
- $\begin{pmatrix} A_r/F_r \\ A_\theta/F_\theta \\ A_\phi/F_\phi \end{pmatrix} = \begin{pmatrix} sin\theta cos\phi & sin\theta sin\phi & cos\theta \\ cos\theta cos\phi & cos\theta sin\phi & -sin\theta \\ -sin\phi & cos\phi & 0 \end{pmatrix} \begin{pmatrix} A_x/F_x \\ A_y/F_y \\ A_z/F_z \end{pmatrix}$
- For far fields, $A_r/F_r \cong 0$
- and for equivalent source in x-y plane, $A_z = F_z$=0
- $A_\theta = \frac{\mu e^{(-j\beta r)}}{4\pi r}\left(-\tilde{H}_{ay} cos\theta cos\phi + \tilde{H}_{ax} cos\theta sin\phi\right)$
- $A_\phi = \frac{\mu e^{(-j\beta r)}}{4\pi r}\left(\tilde{H}_{ay} sin\phi + \tilde{H}_{ax} cos\phi\right)$

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- Similarly,
- $F_\theta = -\frac{\varepsilon e^{(-j\beta r)}}{4\pi r}\left(-\tilde{E}_{ay}cos\theta cos\phi + \tilde{E}_{ax}cos\theta sin\phi\right)$
- $F_\phi = -\frac{\varepsilon e^{(-j\beta r)}}{4\pi r}\left(\tilde{E}_{ay}sin\phi + \tilde{E}_{ax}cos\phi\right)$
- Electric field in the far field will have contribution from $\vec{A}$ and $\vec{F}$
- Hence,
- $\vec{E}_{ff} = \vec{E}_{ff}^{A} + \vec{E}_{ff}^{F}$
- where $\vec{E}_{ff}^{A} \cong -j\omega A_\theta\hat{\theta} - j\omega A_\phi\hat{\phi}$
- and $\vec{E}_{ff}^{F} = \eta\vec{H}_{ff}^{F} \times \hat{r}$

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- where $\vec{H}_{ff}^{F} \cong -j\omega F_{\theta}\hat{\theta} - j\omega F_{\phi}\hat{\phi}$
- and $\vec{E}_{ff}^{F} = -j\omega\eta\left(F_{\theta}\hat{\theta} + F_{\phi}\hat{\phi}\right) \times \hat{r} = -j\omega\eta\left(F_{\phi}\hat{\theta} - F_{\theta}\hat{\phi}\right)$
- Therefore,
- $E_{ff\theta} \cong -j\omega A_{\theta} - j\omega\eta F_{\phi}$
- $E_{ff\phi} \cong -j\omega A_{\phi} + j\omega\eta F_{\theta}$
- Hence,
- $E_{ff\theta} \cong -\frac{j\omega\mu e^{(-j\beta r)}}{4\pi r}\left(-\tilde{H}_{ay}cos\theta cos\phi + \tilde{H}_{ax}cos\theta sin\phi\right) + \frac{j\omega\eta\varepsilon e^{(-j\beta r)}}{4\pi r}\left(\tilde{E}_{ay}sin\phi + \tilde{E}_{ax}cos\phi\right)$

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- Note that
- $-\frac{j\omega\mu e^{(-j\beta r)}}{4\pi r} = -\frac{j\omega\mu\beta e^{(-j\beta r)}}{\beta 4\pi r} = -\frac{j\eta\beta e^{(-j\beta r)}}{4\pi r}$
- $\frac{j\omega\eta\varepsilon e^{(-j\beta r)}}{4\pi r} = \frac{j\omega\omega\mu\varepsilon e^{(-j\beta r)}}{\beta 4\pi r} = \frac{j\beta^2 e^{(-j\beta r)}}{\beta 4\pi r} = \frac{j\beta e^{(-j\beta r)}}{4\pi r}$
- Finally, $E_{ff\theta} \cong \frac{j\beta e^{(-j\beta r)}}{4\pi r}\left[\eta\left(\widetilde{H}_{ay}cos\theta cos\phi - \widetilde{H}_{ax}cos\theta sin\phi\right) + \left(\tilde{E}_{ay}sin\phi + \tilde{E}_{ax}cos\phi\right)\right]$ ---(24.1)
- Similarly, $E_{ff\phi} \cong \frac{j\beta e^{(-j\beta r)}}{4\pi r}\left[-\eta\left(\widetilde{H}_{ay}sin\phi + \widetilde{H}_{ax}cos\phi\right) + \left(\tilde{E}_{ay}cos\theta cos\phi - \tilde{E}_{ax}cos\theta sin\phi\right)\right]$ ---(24.2)