EE540 Advance Electromagnetic Theory & Antennas

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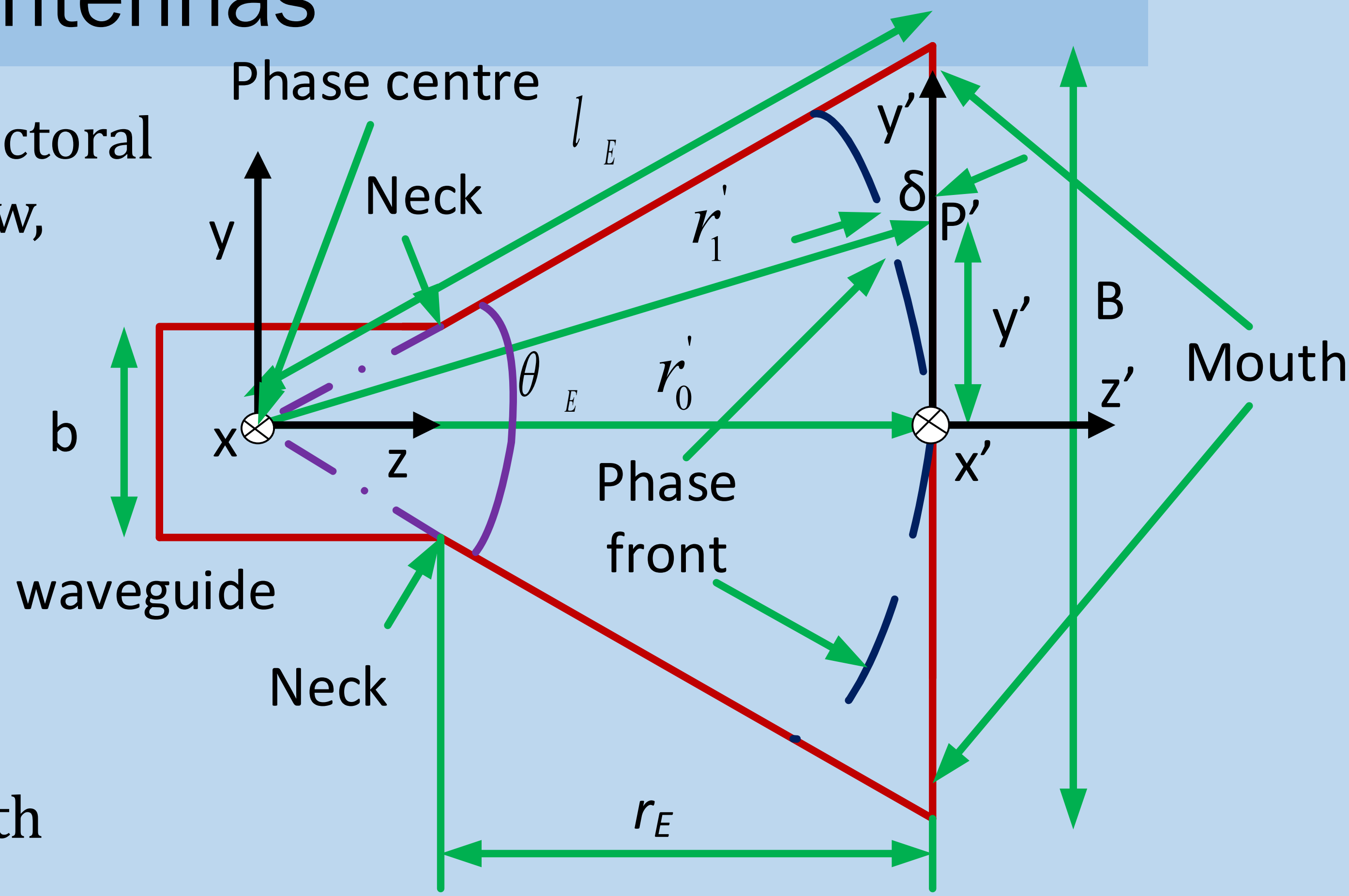
Aperture antennas

- Fig. E-plane sectoral horn (side view,
- y-z cut)



E-plane sectoral horn antenna

- l_E flaring length
- δ path difference
- θ_E flaring angle
- r_0' distance of phase centre and centre of mouth of horn (*this value may be different for E-plane and H-plane sectoral horn*)

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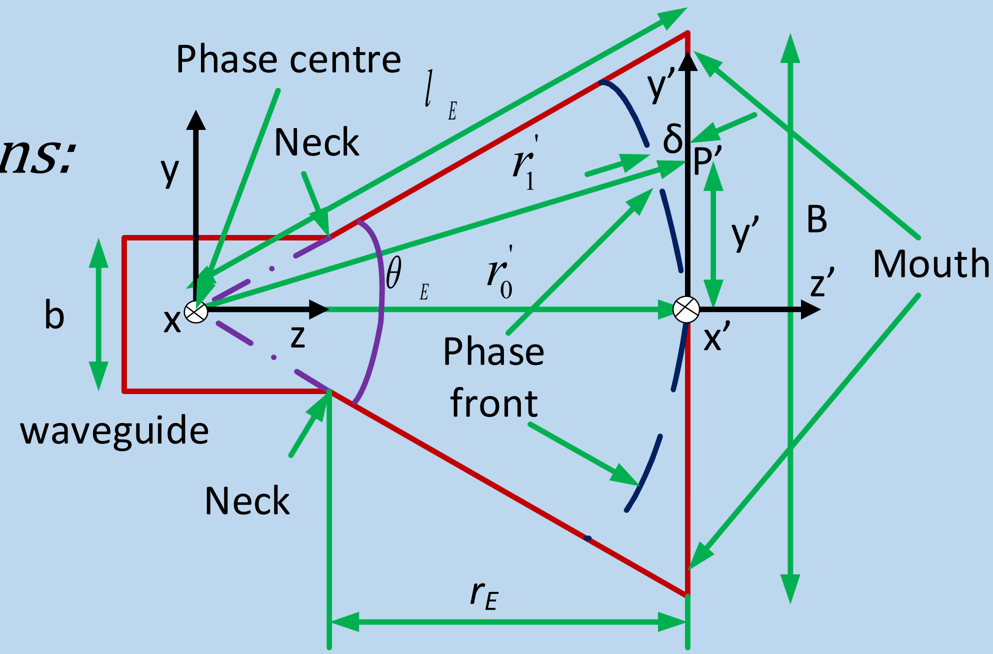
- Two dimensions required for construction of horn antenna
- A and r_E
- *Simple geometrical relations:*
- Flare angle
- $\theta_E = 2tan^{-1}\left(\frac{B}{2r_0'}\right)$
- Flaring length
- $l_E^2 = \frac{B^2}{4} + (r_0')^2$
- Distance between neck and mouth r_E
- $r_E = (B-b)\sqrt{\left(\frac{l_E}{B}\right)^2 - \frac{1}{4}}$



E-plane sectoral
horn antenna

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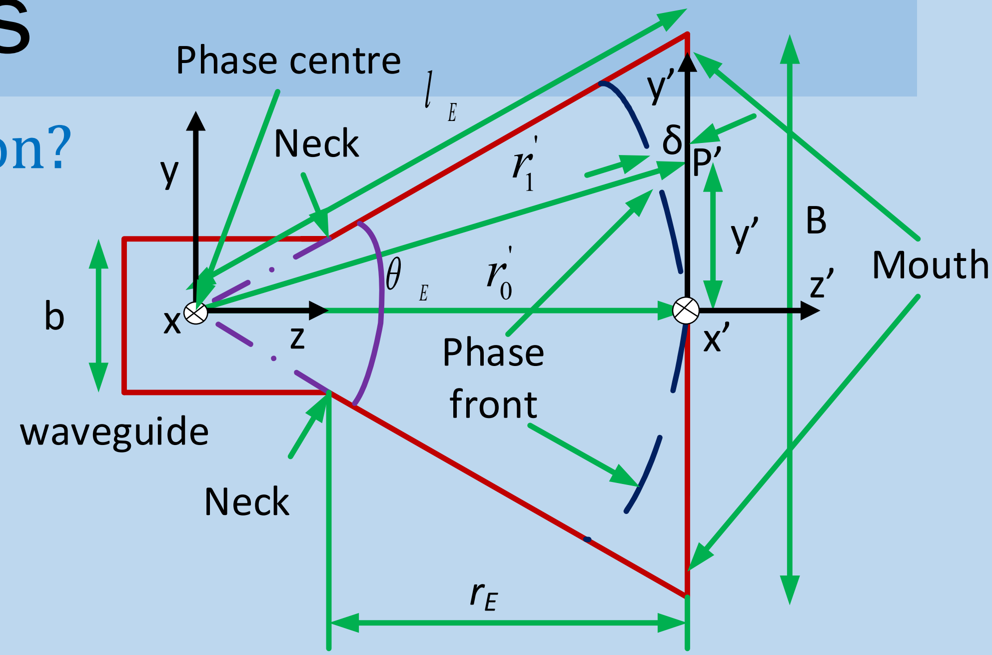
- How do you get last equation?
- Note that
- $tan\left(\frac{\theta_E}{2}\right) = \frac{B}{2r_0'} = \frac{b}{2(r_0' - r_E)}$
- Inverting and cancelling 2
- $\frac{r_0'}{B} = \frac{(r_0' - r_E)}{b}$
- Cross multiplying
- $br_0' = Br_0' - Br_E \Rightarrow Br_E = (B-b)r_0' \Rightarrow r_E = (B-b)\frac{r_0'}{B}$
- $\Rightarrow r_E = (B-b)\frac{\sqrt{l_E^2 - \frac{B^2}{4}}}{B} = (B-b)\sqrt{\left(\frac{l_E}{B}\right)^2 - \frac{1}{4}}$



E-plane sectoral horn antenna

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- Note that at any point P',
  - since the distance r_0' and r_1' are not same
- Hence there will be some phase change
- *How much is the phase change?*
- Let us find the difference in the distance $r_1' - r_0'$ first
- Applying the Pythagoras theorem,
- $r_1' = \sqrt{(r_0')^2 + (y')^2} = r_0'\left(1 + \left(\frac{y'}{r_0'}\right)^2\right)^{1/2} \cong r_0'\left(1 + \frac{1}{2}\left(\frac{y'}{r_0'}\right)^2\right)$
- Hence, $r_1' - r_0' \cong \frac{1}{2r_0'}(y')^2$

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- $\delta = r_1' - r_0' \cong \frac{1}{2r_0'}(y')^2$
- We can observe that $r_1' - r_0'$ has quadratic variation with y'
- Hence phase has quadratic variation
- *First step:* so we can write the aperture field variation as
- $E_{ay} = E_0 cos\left(\frac{\pi x'}{a}\right) e^{-j\beta\frac{1}{2r_0'}(y')^2}$ inside the aperture and zero elsewhere
- $E_0 cos\left(\frac{\pi x'}{a}\right)$ is the amplitude variation and
- $e^{-j\beta\frac{1}{2r_0'}(y')^2}$ is the phase variation

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Table: Amplitude and phase variation of E-sectoral horn antenna

Aperture field parameter	Expression	$x' = 0$ (amplitude) $y' = 0$ (phase)	$x' = -a/2$ (amplitude) $y' = -B/2$ (phase)	$x' = a/2$ (amplitude) $y' = B/2$ (phase)
Amplitude	$E_0 cos\left(\frac{\pi x'}{a}\right)$	E_0	0	0
Phase	$e^{-j\beta\frac{1}{2r_0'}(y')^2}$	1	$e^{-j\beta\frac{1}{8r_0'}(B)^2}$ (maximum)	$e^{-j\beta\frac{1}{8r_0'}(B)^2}$ (maximum)

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- *Second step:*
- Take the 2-D FT of aperture field
- $\tilde{E}_{ay} = \int_{-a/2}^{a/2} E_0 cos\left(\frac{\pi x'}{a}\right) e^{j\beta_x x'} dx' \int_{-B/2}^{B/2} e^{-j\beta \frac{1}{2r_0'}\left(y'\right)^2} e^{j\beta_y y'} dy'$
- $= E_0 \left\{2\pi a \frac{cos\left(\frac{\beta_x a}{2}\right)}{\pi^2 - (\beta_x a)^2}\right\} \left\{\sqrt{\frac{\pi r_0'}{\beta}} e^{\frac{j\beta r_0'(v)^2}{2}} I_1(\theta, \phi)\right\}$
- where $v = sin\theta sin\phi$
- $I_1(\theta, \phi) = \left(f(s_4') - f(s_3')\right)$
- $f(\xi') = C(\xi') - jS(\xi'),$
- $C(\xi') = \int_0^{\xi'} cos\left(\frac{\pi}{2}\tau^2\right) d\tau, S(\xi') = \int_0^{\xi'} sin\left(\frac{\pi}{2}\tau^2\right) d\tau$

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- $s_3' = \sqrt{\frac{\beta}{\pi r_0'}}\left(-\frac{B}{2} - r_0'\nu\right)$
- $s_4' = \sqrt{\frac{\beta}{\pi r_0'}}\left(+\frac{B}{2} - r_0'\nu\right)$
- *Third step:*
- Use equations (27.1) and (27.2) to find the FF electric fields
- $E_{ff\theta} \cong \frac{j\beta e^{(-j\beta r)}}{4\pi r}(1 + cos\theta)\left[\tilde{E}_{ay} sin\phi\right]$ ---(27.1)
- $E_{ff\phi} \cong \frac{j\beta e^{(-j\beta r)}}{4\pi r}(1 + cos\theta)\left[\tilde{E}_{ay} cos\phi\right]$ ---(27.2)

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- E-plane patterns ($\phi = \frac{\pi}{2}$):
- $E_{ff\theta} \cong \frac{j\beta e^{(-j\beta r)}}{4\pi r}(1 + cos\theta)\left[\tilde{E}_{ay}\right]$ and $E_{ff\phi} \cong 0$
- where $\tilde{E}_{ay} = E_0\left\{2\pi a \frac{cos\left(\frac{\beta_x a}{2}\right)}{\pi^2-(\beta_x a)^2}\right\}\left\{\sqrt{\frac{\pi r_0'}{\beta}}e^{\frac{j\beta r_0'(v)^2}{2}} I_1(\theta,\phi)\right\}$
- where $v = sin\theta sin\phi = sin\theta$ and $\beta_x = \beta sin\theta cos\phi = 0$
- $I_1(\theta,\phi) = \left(f(s_4') - f(s_3')\right)$ and $e^{\frac{j\beta r_0'(v)^2}{2}} = e^{\frac{j\beta r_0'(sin\theta)^2}{2}}$
- $s_3' = \sqrt{\frac{\beta}{\pi r_0'}}\left(-\frac{B}{2} - r_0' sin\theta\right)$
- $s_4' = \sqrt{\frac{\beta}{\pi r_0'}}\left(+\frac{B}{2} - r_0' sin\theta\right)$

Aperture antennas

- $E_{ff\theta} \cong$
- $j\beta a E_0 \sqrt{\frac{\pi r_0'}{\beta}} \frac{e^{(-j\beta r)}}{2r\pi^2} e^{\frac{j\beta r_0'(v)^2}{2}} (1 + cos\theta)\left(f(s_4') - f(s_3')\right)$
- where
- $s_3' = \sqrt{\frac{\beta}{\pi r_0'}} \left(-\frac{B}{2} - r_0' sin\theta\right)$
- $s_4' = \sqrt{\frac{\beta}{\pi r_0'}} \left(+\frac{B}{2} - r_0' sin\theta\right)$
- H-plane patterns ($\phi = 0$):
- $E_{ff\theta} \cong 0$ and $E_{ff\phi} \cong \frac{j\beta e^{(-j\beta r)}}{4\pi r}(1 + cos\theta)\left[\tilde{E}_{ay}\right]$

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- $\tilde{E}_{ay} = E_0 \left\{ 2\pi a \frac{cos\left(\frac{\beta_x a}{2}\right)}{\pi^2-(\beta_x a)^2} \right\} \left\{ \sqrt{\frac{\pi r_0'}{\beta}} e^{\frac{j\beta r_0'(v)^2}{2}} I_1(\theta,\phi) \right\}$
- $I_1(\theta,\phi) = \left(f(s_4') - f(s_3')\right), f(\xi') = C(\xi') - jS(\xi'),$
- $\because \phi = 0 \therefore v = sin\theta sin\phi = 0$ and $e^{\frac{j\beta r_0'(v)^2}{2}} = 1$
- $s_3' = \sqrt{\frac{\beta}{\pi r_0'}}\left(-\frac{B}{2} - r_0' v\right) = \sqrt{\frac{\beta}{\pi r_0'}}\left(-\frac{B}{2}\right)$
- $s_4' = \sqrt{\frac{\beta}{\pi r_0'}}\left(+\frac{B}{2} - r_0' v\right) = \sqrt{\frac{\beta}{\pi r_0'}}\left(+\frac{B}{2}\right) = -s_3'$
- $\therefore I_1(\theta,\phi) = 2f(s_4')$

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- $\tilde{E}_{ay} = 4\pi a E_0 \sqrt{\frac{\pi r_0'}{\beta}} \left\{ \frac{cos\left(\frac{\beta_x a}{2}\right)}{\pi^2 - (\beta_x a)^2} \right\} \{f(s_4')\}$
- Hence
- $E_{ff\phi} \cong \frac{j\beta a E_0 e^{(-j\beta r)}}{r} \sqrt{\frac{\pi r_0'}{\beta}} \left\{ f\left(\frac{B}{2} \sqrt{\frac{\beta}{\pi r_0'}} \right) \right\} (1 + cos\theta) \left\{ \frac{cos\left(\frac{\beta_x a}{2}\right)}{\pi^2 - (\beta_x a)^2} \right\}$
- Directivity:
- $D_E = \frac{4\pi}{\lambda^2} \epsilon_t \epsilon_{ph}^E a B$

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- $\epsilon_t = \frac{8}{\pi^2} \cong 0.81$
- $\epsilon_{ph}^{E} = \frac{C^2(q)+S^2(q)}{q^2}, q = 2\sqrt{s}$

How do we find s for maximum phase error?

- Maximum phase error is for $y' = \frac{B}{2}$,
- hence $\beta \frac{1}{2r_0'}(y')^2 = \beta \frac{1}{8r_0'}(B)^2 = 2\pi \frac{B^2}{8\lambda r_0'}$
- Let us equate the maximum phase error to $2\pi s$
- So, $\frac{B^2}{8\lambda r_0'} = s$
-

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- *Is directivity dependent on B?*
- Yes and optimal value is for $B = \sqrt{2\lambda r_0'}$
- *What is the value of s for this?*
- $s = \frac{B^2}{8\lambda r_0'} = \frac{2\lambda r_0'}{8\lambda r_0'} = \frac{2}{8} = \frac{1}{4}$
- $D_E = \frac{4\pi}{\lambda^2} \epsilon_t \epsilon_{ph}^E aB$