EE540 Advance Electromagnetic Theory & Antennas

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Time Harmonic Electromagnetic Fields

- For time harmonic fields:
- Separate the space and time dependence as $\vec{F}(x,y,z,t)=\vec{F}(x,y,z)e^{j\omega t}$
  - where ω is the angular frequency of the time varying field
- Time derivative time harmonic fields

$$\frac{\partial \vec{F}(x,y,z,t)}{\partial t}=\vec{F}(x,y,z)\frac{\partial e^{j\omega t}}{\partial t}=\vec{F}(x,y,z)j\omega e^{j\omega t}=j\omega\vec{F}(x,y,z)e^{j\omega t}=j\omega\vec{F}(x,y,z,t)$$

- Integration of time harmonic field w.r.t. time

$$\int \vec{F}(x,y,z,t)dt=\int \vec{F}(x,y,z)e^{j\omega t}dt=\vec{F}(x,y,z)\int e^{j\omega t}dt=\vec{F}(x,y,z)\frac{e^{j\omega t}}{j\omega}+c_1=\frac{\vec{F}(x,y,z,t)}{j\omega}+c_1$$

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Time Harmonic Electromagnetic Fields

- where c_1 is a constant of integration
- Since we are interested only in time varying quantities, we can take, $c_1=0$
- So we can define

$$\frac{\partial}{\partial t} \equiv j\omega; \frac{\partial^2}{\partial t^2} = j\omega \times j\omega = -\omega^2$$

- For time harmonic fields, we can modify Maxwell's equations
  - By suppressing time dependence completely
  - It is called *Maxwell's equations in phasor form*

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Time Harmonic Electromagnetic Fields

- Maxwell's equations in phasor form
  - all time dependence has been suppressed and replace $\frac{\partial}{\partial t} \equiv j\omega$

$$\nabla \bullet \tilde{D} = \rho_\upsilon$$

$$\nabla \bullet \tilde{B} = 0$$

$$\nabla \times \tilde{H} = j\omega\tilde{D} + \tilde{J}$$

$$\nabla \times \tilde{E} = -j\omega\tilde{B}$$

Vector Potentials

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- Usually for antennas, we find fields from the current density
- Observation: difficult to find fields directly from current density
  - calculations are highly complex and tedious
- We know that it is easier to
  - find electric field from electric potential than
  - directly finding electric field (Why easier?) $\vec{E} = -\nabla V$
- A major simplification is possible when
  - we find the magnetic vector potential first from current density (Why easier?)
    - and find the fields from it

$$\vec{A}(\vec{r}) = \frac{\mu}{4\pi} \int_V \vec{J}(\vec{r}^{\,'}) \frac{e^{-j\beta|\vec{r}-\vec{r}^{\,'}|}}{|\vec{r}-\vec{r}^{\,'}|} dv'$$

Vector Potentials

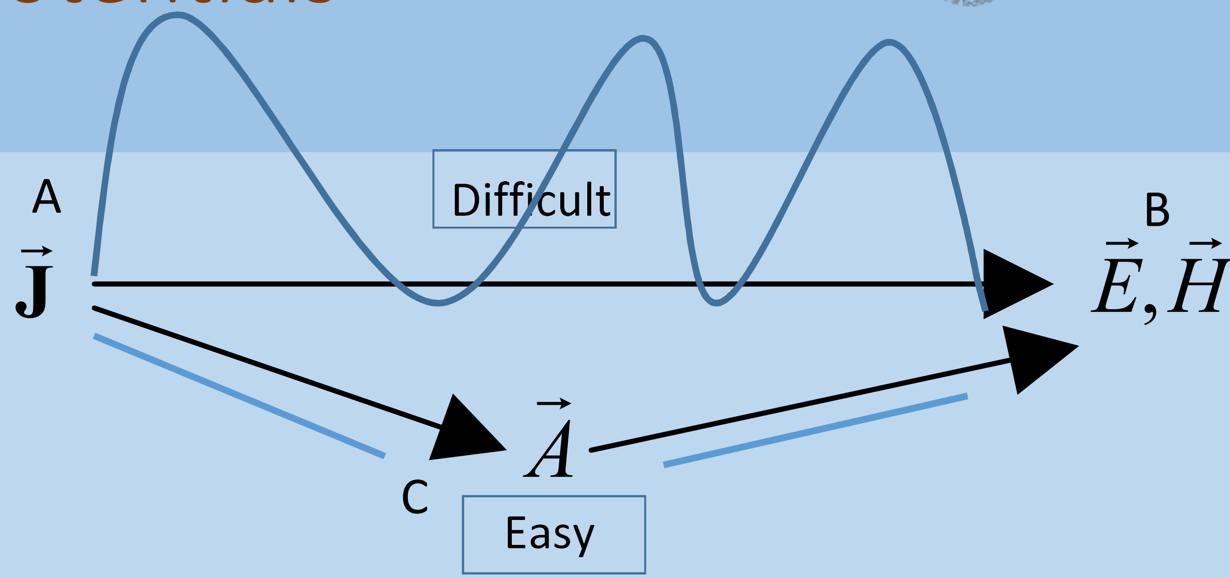

Fig. Two ways to find fields

- How do we find magnetic and electric field from magnetic vector potential?

Vector Potentials

- One of the Maxwell's divergence equation $\nabla \bullet \vec{B} = 0$
- Hence, we can write

$$\because \nabla \bullet \vec{B} = 0, \nabla \bullet \left(\nabla \times \vec{A}\right) = 0 \therefore \vec{B} = \nabla \times \vec{A}$$

- *It means that we can find magnetic flux density*
  - *from the curl of magnetic vector potential*
- Putting this in the following Maxwell's curl equation $\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$

$$\Rightarrow \nabla \times \vec{E} = -\frac{\partial\left(\nabla \times \vec{A}\right)}{\partial t}$$

Vector Potentials

- which can be rewritten as $$\Rightarrow \nabla \times \left(\vec{E} + \frac{\partial \vec{A}}{\partial t} \right) = 0$$
- From vector analysis $$\nabla \times \left(\vec{E} + \frac{\partial \vec{A}}{\partial t} \right) = \nabla \times \left(\nabla V \right) = 0 \Rightarrow \nabla \times \left(-\nabla V \right) = 0$$
- For time varying fields, $$\vec{E} = -\nabla V - \frac{\partial \vec{A}}{\partial t}$$
- For electrostatics, $$\vec{E} = -\nabla V$$
- The additional term for time varying fields is $-\frac{\partial \vec{A}}{\partial t}$
- *It means that we can find electric field*
  - *from the negative of the gradient of electric potential*
  - *and the negative of the time derivative of magnetic vector potential*