EE540 Advance Electromagnetic Theory & Antennas

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Aperture antennas

- ***Geometric relations of Parabolic reflectors:***
- (a) *In spherical coordinates,*
- All waves emerging from the source and
  - ending at the dashed line must have traveled
  - equal distance to be in phase at the dashed line
  - It means
  - OP+PQ=2f=constant
  - $\Rightarrow r' + r'\cos\theta' = 2f$
  - $\Rightarrow r' = \frac{2f}{1+\cos\theta'}$
  - $\Rightarrow r' = f\sec^2\frac{\theta'}{2}, \theta' \leq \theta'_m$

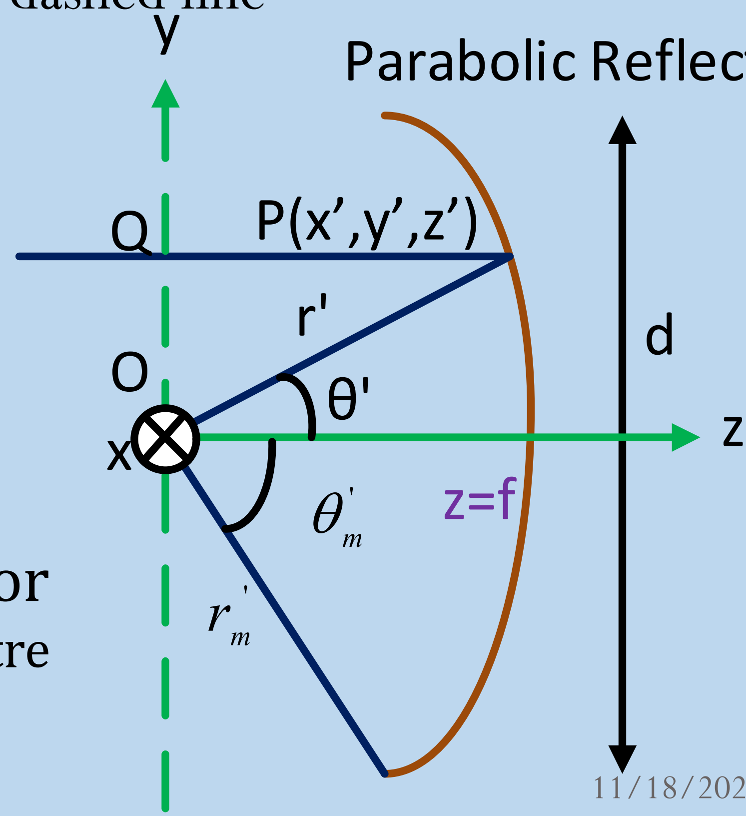

Fig. Geometry of Parabolic reflector
(f: distance between focal point and centre of the reflector)

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- The above relation is in spherical coordinates
- *(b) In Cartesian coordinates,*
- $r' + r'\cos\theta' = 2f$
- $\Rightarrow \sqrt{(x')^2 + (y')^2 + (z')^2} + z' = 2f$
- $\Rightarrow \sqrt{(x')^2 + (y')^2 + (z')^2} = 2f - z'$
- $\Rightarrow (x')^2 + (y')^2 + (z')^2 = 4f^2 + (z')^2 - 4fz'$
- $\Rightarrow (x')^2 + (y')^2 = 4f^2 - 4fz' = 4f(f - z')$
- $\Rightarrow \boxed{(x')^2 + (y')^2 = 4f(f - z')}$
- where $(x')^2 + (y')^2 \leq \frac{d^2}{4}$ and d is the diameter of the reflector

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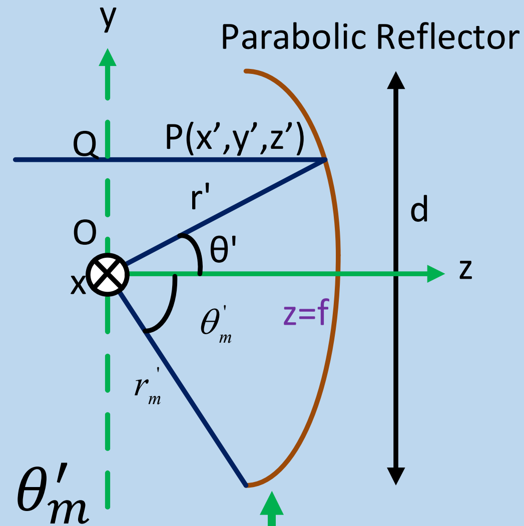
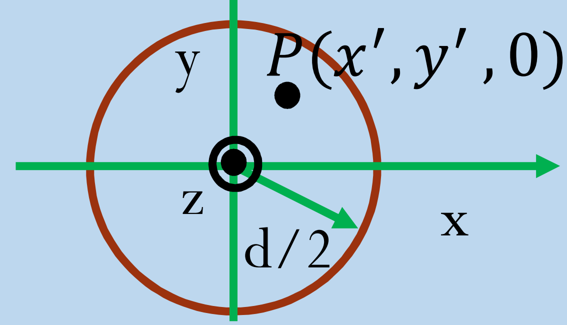
- Note that at the edge minimum value of $z' = z'_m$ and
$$(x')^2 + (y')^2 = \frac{d^2}{4}$$
- $\Rightarrow (x')^2 + (y')^2 = \frac{d^2}{4} = 4f(f - z'_m)$
- $\Rightarrow 4f^2 - \frac{d^2}{4} = 4f z'_m$
- $\Rightarrow z'_m = \frac{4f^2 - \frac{d^2}{4}}{4f} = f - \frac{d^2}{16f}$
- Therefore, at the edge maximum value of $\theta' = \theta'_m$
- $\theta'_m = tan^{-1}\left(\frac{\frac{d}{2}}{z'_m}\right) = tan^{-1}\left(\frac{\frac{d}{2}}{f - \frac{d^2}{16f}}\right)$
- Fig. x-y plane view of reflector





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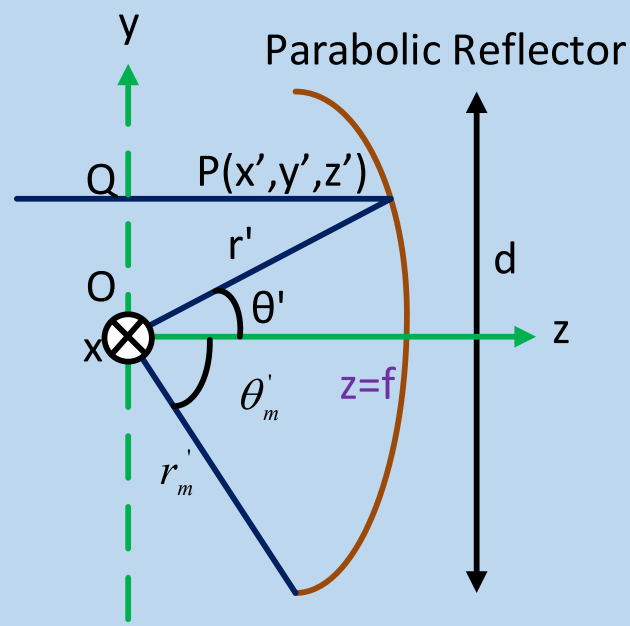
- Divide by $\frac{f}{d^2}$ in both numerator and denominator of the argument
- *(i) θ'_m in terms of the ratio $\frac{f}{d}$*
- $\theta'_m = tan^{-1}\left(\frac{\frac{d}{2}}{z'_m}\right) = tan^{-1}\left(\frac{\frac{1}{2}\frac{f}{d}}{\left(\frac{f}{d}\right)^2 - \frac{1}{16}}\right)$
- which is expressed in terms of ratio $\frac{f}{d}$
- Usually $\frac{f}{d}$ is chosen as 0.5 for prime focus parabolic reflector
- Corresponding value of $\theta'_m = 53^0$
- Similarly, we can also obtain $\frac{f}{d}$ in terms of θ'_m as follows

Aperture antennas

- At the edge, $r' = r'_m$ and $\theta' = \theta'_m$, hence
- $r'_m = \frac{2f}{1+\cos\theta'_m}$
- Also we know that $\sin\theta'_m = \frac{d}{2r'_m}$
- Hence $r'_m = \frac{d}{2\sin\theta'_m}$
- Therefore, $\frac{2f}{1+\cos\theta'_m} = \frac{d}{2\sin\theta'_m}$
- $\Rightarrow \frac{f}{d} = \frac{1+\cos\theta'_m}{4\sin\theta'_m} = \frac{1}{4}\frac{2\cos^2\frac{\theta'_m}{2}}{2\sin\frac{\theta'_m}{2}\cos\frac{\theta'_m}{2}} = \frac{1}{4}\cot\frac{\theta'_m}{2}$
- *(ii) θ'_m in terms of the ratio $\frac{f}{d}$*
- $\frac{f}{d} = \frac{1}{4}\cot\frac{\theta'_m}{2}$

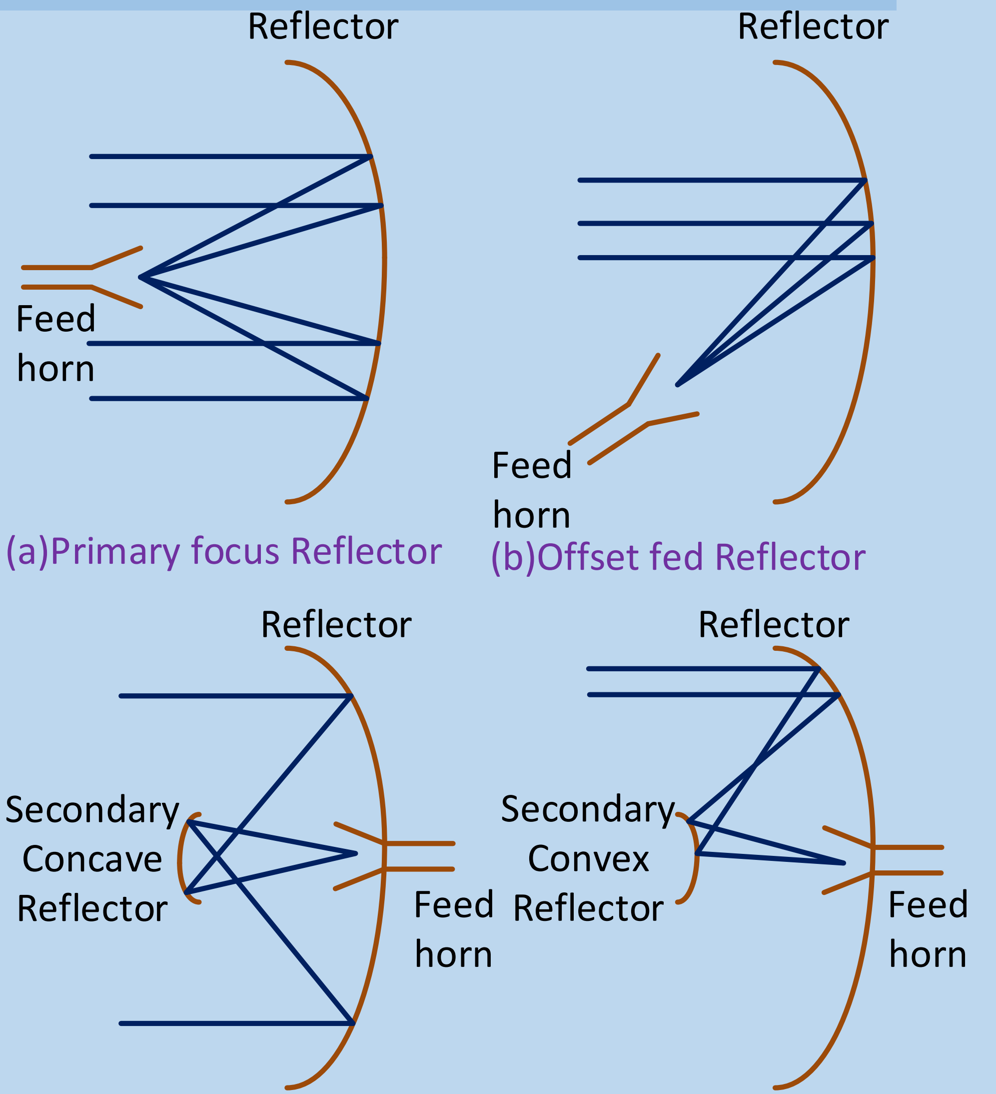


Aperture antennas

- Reflector antenna consists of
  - Feed antenna usually horn antennas (at the reflector's focal point)
  - Reflector generally parabolic in shape (parabolic reflector)
- Four types of Parabolic reflector antennas
  - Prime focus reflector antenna
  - Offset fed reflector
  - Gregorian reflector (Dual-reflector antennas, secondary reflector at the focal point fed by primary feed)
  - Cassegrain reflector (Dual-reflector antennas, , secondary reflector at the focal point fed by primary feed)
- How does these reflectors work?

Aperture antennas

- Fig. Different types
- of Parabolic reflector
- antennas



(a)Primary focus Reflector

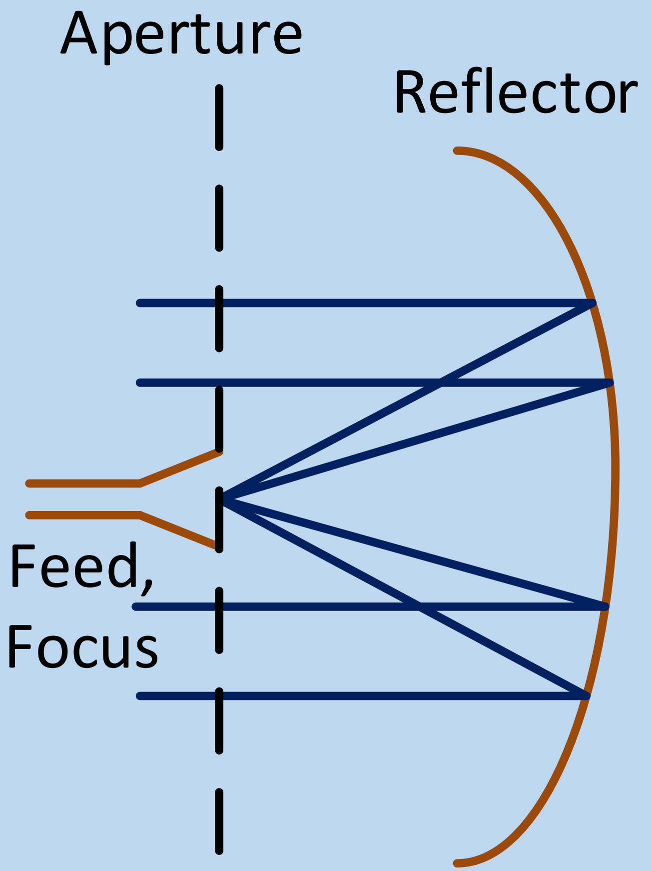
(b)Offset fed Reflector

(c) Gregorian fed Reflector

(d) Cassegrain fed Reflector

Aperture antennas

- How does these reflectors work?
- Reflector antenna concentrates energy received from feed antenna
  - into a narrow beam of radiation
- Paraboloid reflectors converts spherical wavefronts of the source or feed antenna
  - which is usually situated at the geometric focus , O,
- into a wave emanating from the aperture with a plane wave-front
- Fig. Primary focus reflector



Primary focus Reflector

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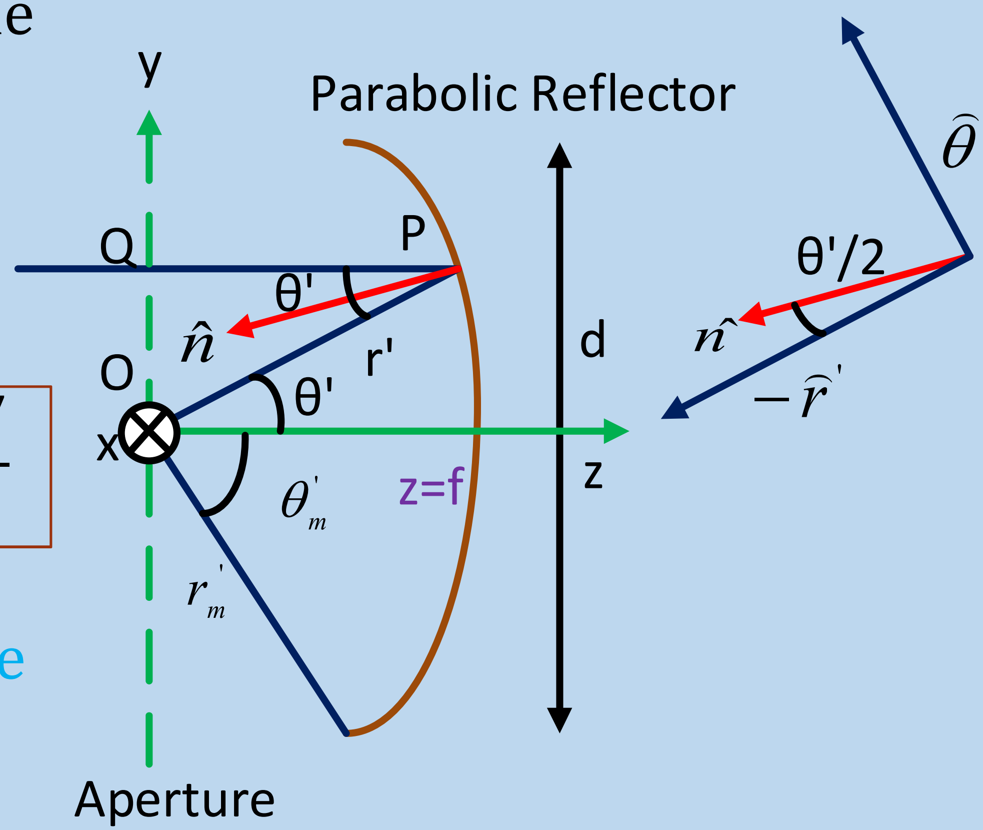
- From the Fig.,
- We can observe that the normal to
- the reflector $\hat{n}$ is bisecting θ'
- Hence,

$$\hat{n} = -\hat{r}'\cos\frac{\theta'}{2} + \hat{\theta}'\sin\frac{\theta'}{2}$$



- Fig. Primary focus reflector (normal to the reflector)

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- Consider a feed antenna located in the focal point and
- reflector is located in the far field region of the feed antenna
- Assume the FF field radiated from the feed antenna as
- $\vec{E}_f(r',\theta',\phi') = E_0\vec{F}(\theta',\phi')\dfrac{e^{-j\beta r'}}{r'}$
- *Note that* r',θ',ϕ' *are coordinates on the reflector surface which may be assumed as a source*
- We may also express the vector function $\vec{F}(\theta',\phi')$ which gives the spatial distribution of the field of the feed antenna as
- $\vec{F}(\theta',\phi') = F_{\theta'}(\theta',\phi')\widehat{\theta'} + F_{\phi'}(\theta',\phi')\widehat{\phi'}$

Aperture antennas

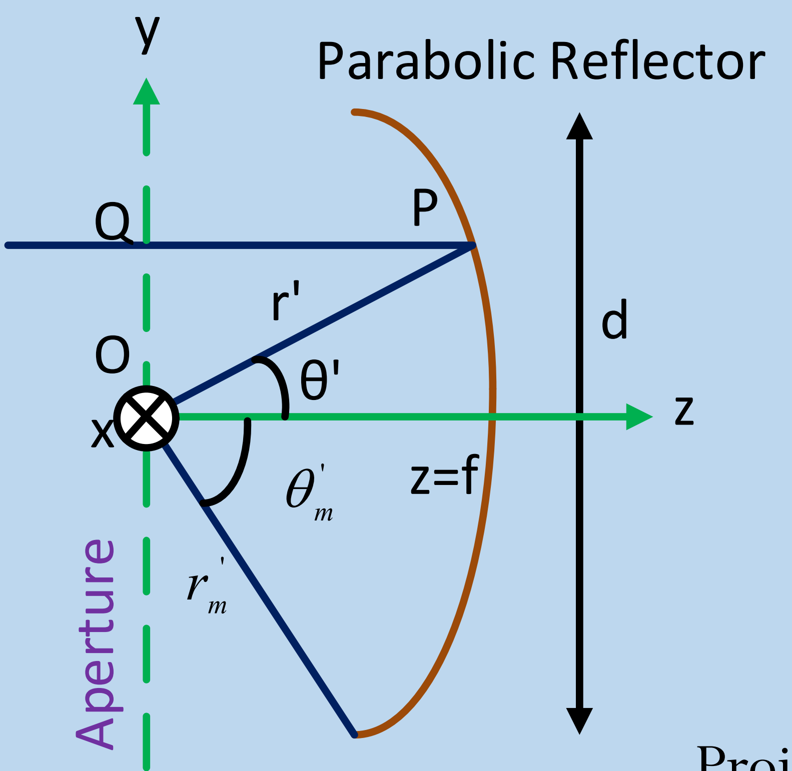
- For very large reflector dimension in which $\frac{d}{\lambda} > 100$,
- geometric optics (GO) can be used to find the approximate fields of the aperture
- *Step 1: Establish the fields in the aperture*
- ***GO for a reflector:***
- On the conducting reflector surface ($\Gamma_{\parallel} = 1$ and $\Gamma_{\perp} = -1$)
- $\left(\vec{E}_i + \vec{E}_r\right) \times \hat{n} = 0$
- and
- $\hat{n} \cdot \left(\vec{E}_i - \vec{E}_r\right) = 0$
- $\Rightarrow \hat{n} \cdot \vec{E}_i = \hat{n} \cdot \vec{E}_r$

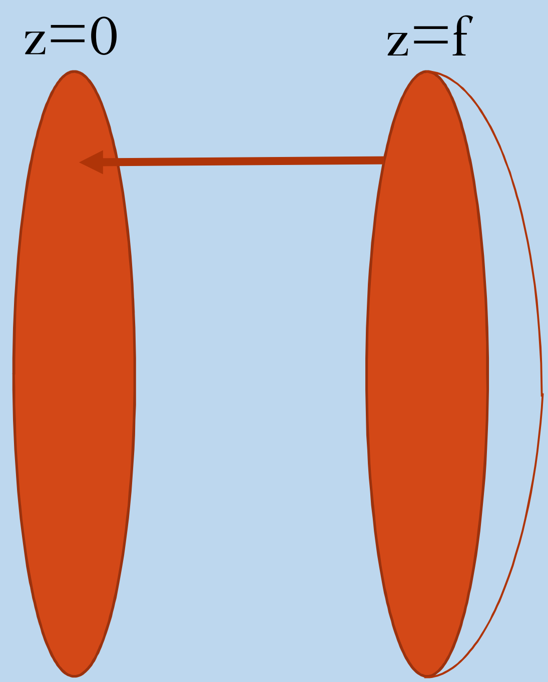
Aperture antennas

- Hence,
- $\hat{n} \times \left(\vec{E}_i + \vec{E}_r\right) \times \hat{n} = 0$
- Applying "bac-cab" rule,
- $\left(\vec{E}_i + \vec{E}_r\right) - \hat{n}\left[\hat{n} \cdot \left(\vec{E}_i + \vec{E}_r\right)\right] = 0$
- $\Rightarrow \left(\vec{E}_i + \vec{E}_r\right) - \hat{n}\left[2\hat{n} \cdot \vec{E}_i\right] = 0$
- $\Rightarrow \boxed{\vec{E}_r = 2\hat{n}\left[\hat{n} \cdot \vec{E}_i\right] - \vec{E}_i}$
- This gives the reflected electric field in terms of the incident electric field

Aperture antennas

- In aperture distribution method, we are interested to find the reflected field at the aperture



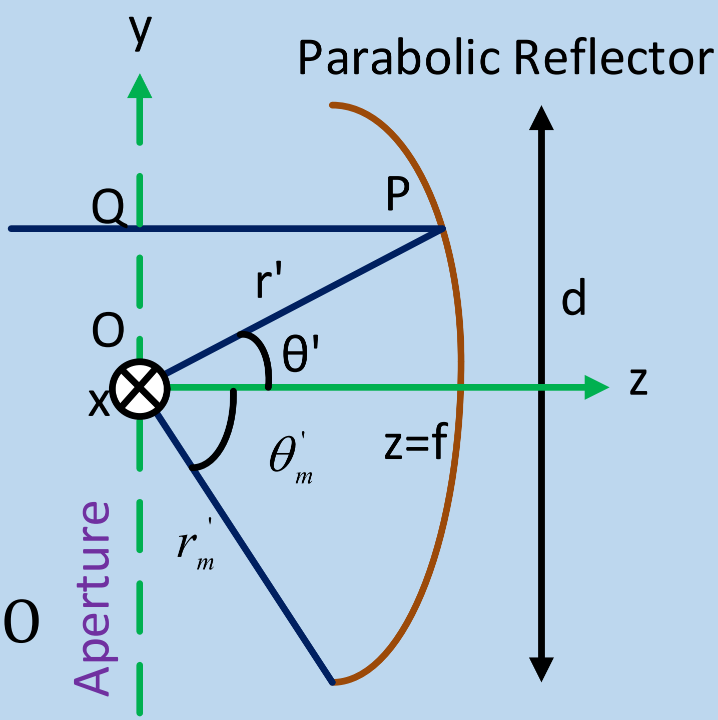


Projected effective aperture at the aperture plane

Parabolic reflector

Aperture antennas

- Therefore, there will be some phase change due to movement of the field from the reflector to the
- aperture (PQ) as shown in Fig.
  - PQ=$r'\cos\theta' = 2f - r'$
  - The field at the aperture is
- $\vec{E}_a = \left[2\hat{n}\left[\hat{n}\cdot\vec{E}_f\right] - \vec{E}_f\right]e^{-j\beta(2f-r')}$
- $\vec{H}_a = \frac{1}{\eta_0}\hat{z}\times\vec{E}_a$
- where $\vec{E}_f$ is the incident field on
- the reflector due to feed antenna at O



Aperture antennas

- Note that the normal unit vector to the reflector is expressed as

$$\hat{n} = -\widehat{r'}\cos\frac{\theta'}{2} + \widehat{\theta'}\sin\frac{\theta'}{2}$$

- and radiated field from the feed antenna is given by
- $\vec{E}_f(r',\theta',\phi') = E_0\left[F_{\theta'}(\theta',\phi')\widehat{\theta'} + F_{\phi'}(\theta',\phi')\widehat{\phi'}\right]\frac{e^{-j\beta r'}}{r'}$
- Therefore,
- $\hat{n}\cdot\vec{E}_f(r',\theta',\phi')$
- $= \left(-\widehat{r'}\cos\frac{\theta'}{2} + \widehat{\theta'}\sin\frac{\theta'}{2}\right)\cdot E_0\left[F_{\theta'}(\theta',\phi')\widehat{\theta'} + F_{\phi'}(\theta',\phi')\widehat{\phi'}\right]\frac{e^{-j\beta r'}}{r'}$
  - $= \left(\sin\frac{\theta'}{2}\right)\cdot E_0[F_{\theta'}(\theta',\phi')]\frac{e^{-j\beta r'}}{r'}$

Aperture antennas

- Therefore
- $2\hat{n}\left[\hat{n}\cdot\vec{E}_f\right]$
- $= 2\left[\left(\sin\frac{\theta'}{2}\right)\cdot E_0\left[F_{\theta'}(\theta',\phi')\right]\frac{e^{-j\beta r'}}{r'}\right]\left(-\widehat{r'}\cos\frac{\theta'}{2}+\widehat{\theta'}\sin\frac{\theta'}{2}\right)$
- Hence
- $2\hat{n}\left[\hat{n}\cdot\vec{E}_f\right]-\vec{E}_f$
- $= E_0\frac{e^{-j\beta r'}}{r'}\left[2\left(\sin\frac{\theta'}{2}\right)\cdot\left[F_{\theta'}(\theta',\phi')\right]\left(-\widehat{r'}\cos\frac{\theta'}{2}+\widehat{\theta'}\sin\frac{\theta'}{2}\right)\right]-E_0\frac{e^{-j\beta r'}}{r'}\left[F_{\theta'}(\theta',\phi')\widehat{\theta'}+F_{\phi'}(\theta',\phi')\widehat{\phi'}\right]$
- $= -E_0\frac{e^{-j\beta r'}}{r'}\left[F_{\theta'}(\theta',\phi')\left(\widehat{r'}\sin\theta'+\widehat{\theta'}\cos\theta'\right)+F_{\phi'}(\theta',\phi')\widehat{\phi'}\right]$

Aperture antennas

- Therefore
- $\vec{E}_a = \left[2\hat{n}\left[\hat{n}\cdot\vec{E}_f\right] - \vec{E}_f\right]e^{-j\beta\left(2f-r'\right)} =$
  $-E_0\frac{e^{-j\beta r'}}{r'}\left[F_{\theta'}(\theta',\phi')\left(\widehat{r'}\sin\theta' + \widehat{\theta'}\cos\theta'\right) + F_{\phi'}(\theta',\phi')\widehat{\phi'}\right]e^{-j\beta\left(2f-r'\right)}$
- Expressing this in Cartesian coordinates,
- $$\begin{pmatrix} E_{ax'} \\ E_{ay'} \\ E_{az'} \end{pmatrix} = \begin{pmatrix} sin\theta' cos\phi' & cos\theta' cos\phi' & -sin\phi' \\ sin\theta' sin\phi' & cos\theta' sin\phi' & cos\phi' \\ cos\theta' & -sin\theta' & 0 \end{pmatrix} \begin{pmatrix} E_{ar'} \\ E_{a\theta'} \\ E_{a\phi'} \end{pmatrix}$$
- $E_{ax'} = -E_0\frac{e^{-j\beta 2}}{r'}\left(F_{\theta'}(\theta',\phi')sin^2\theta' cos\phi' + F_{\theta'}(\theta',\phi')cos^2\theta' cos\phi' - F_{\phi'}(\theta',\phi')sin\phi'\right)$

Aperture antennas

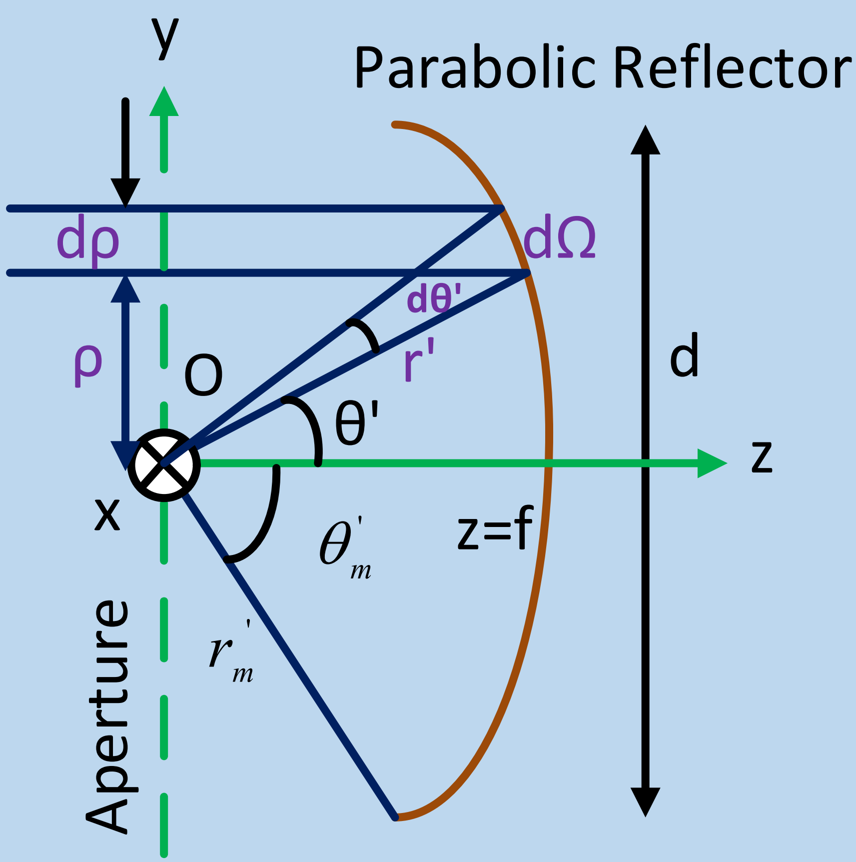
- $\Rightarrow E_{ax'} = -E_0 \frac{e^{-j\beta 2f}}{r'}\left(F_{\theta'}(\theta',\phi')cos\phi' - F_{\phi'}(\theta',\phi')sin\phi'\right)$
- $E_{ay'} = -E_0 \frac{e^{-j\beta 2f}}{r'}\left(F_{\theta'}(\theta',\phi')sin^2\theta' sin\phi' + F_{\theta'}(\theta',\phi')cos^2\theta' sin\phi' - F_{\phi'}(\theta',\phi')cos\phi'\right)$
- $\Rightarrow E_{ay'} = -E_0 \frac{e^{-j\beta 2f}}{r'}\left(F_{\theta'}(\theta',\phi')sin\phi' + F_{\phi'}(\theta',\phi')cos\phi'\right)$
- $\because r' = \frac{2f}{1+\cos\theta'}$, we have,
- $E_{ax'} = -E_0 \frac{e^{-j\beta 2f}}{2f}(1+\cos\theta')\left(F_{\theta'}(\theta',\phi')cos\phi' - F_{\phi'}(\theta',\phi')sin\phi'\right)$
- $E_{ay'} = -E_0 \frac{e^{-j\beta 2f}}{2f}(1+\cos\theta')\left(F_{\theta'}(\theta',\phi')sin\phi' + F_{\phi'}(\theta',\phi')cos\phi'\right)$

Aperture antennas

- *Step 2: Find the 2-D FT the aperture fields*
- Let us find the FT of $\mathrm{E}_{\mathrm{ax}'}$ and $\mathrm{E}_{\mathrm{ay}'}$ over the aperture
- $\tilde{E}_{ax} = \int_0^{d/2} \int_0^{2\pi} \mathrm{E}_{\mathrm{ax}'}\, e^{j\vec{\beta}\cdot\vec{r}'}\, \rho \mathrm{d}\rho \mathrm{d}\phi'$
- where $\vec{r}'$ lies in the aperture plane, then $\theta' = \frac{\pi}{2}$, then
- $\vec{r}' = \rho(\hat{x} cos\phi' + \hat{y} sin\phi')$
- $\vec{\beta} \cdot \vec{r}' = \beta\rho(sin\theta cos\phi cos\phi' + sin\theta sin\phi sin\phi') = \beta\rho\left(sin\theta cos(\phi - \phi')\right)$
- Hence
- $\tilde{E}_{ax} = \int_0^{d/2} \int_0^{2\pi} \mathrm{E}_{\mathrm{ax}'}\, e^{j\beta\rho\left(sin\theta cos(\phi-\phi')\right)}\, \rho \mathrm{d}\rho \mathrm{d}\phi'$

Aperture antennas

- We may convert the above
- integral over the feed angles
- θ' and ϕ' as follows
- $d\rho = r' d\theta'$ and
- $\because r' = f\sec^2\frac{\theta'}{2}$ and
- $\rho = r'\sin\theta' = 2r'\sin\frac{\theta'}{2} cos\frac{\theta'}{2}$
- $\Rightarrow \rho = 2f\sec^2\frac{\theta'}{2}\sin\frac{\theta'}{2} cos\frac{\theta'}{2}$
- $\rho = 2f tan\frac{\theta'}{2}$



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- Therefore, $\rho \mathrm{d}\rho = 2f tan\frac{\theta'}{2} r' d\theta'$
- $\tilde{E}_{ax} = 2f \int_0^{\theta'_m} \int_0^{2\pi} \mathrm{E}_{\mathrm{ax}'}\, e^{j\beta 2f tan\frac{\theta'}{2}\left(sin\theta cos(\phi-\phi')\right)} tan\frac{\theta'}{2} r' d\theta' \mathrm{d}\phi'$
- Similarly,
- $\tilde{E}_{ay} = 2f \int_0^{\theta'_m} \int_0^{2\pi} \mathrm{E}_{\mathrm{ay}'}\, e^{j\beta 2f tan\frac{\theta'}{2}\left(sin\theta cos(\phi-\phi')\right)} tan\frac{\theta'}{2} r' d\theta' \mathrm{d}\phi'$
- The above integrations can be done numerically if analytical integration is not possible

Aperture antennas

- Step 3: *Use equation (24.1) and (24.2) to find FF fields*
- We may use equation (24.1) and (24.2) to find FF fields
- $E_{ff\theta} \cong \frac{j\beta e^{(-j\beta r)}}{4\pi r}\left[\eta\left(\widetilde{H}_{ay}cos\theta cos\phi - \widetilde{H}_{ax}cos\theta sin\phi\right) + \left(\tilde{E}_{ay}sin\phi + \tilde{E}_{ax}cos\phi\right)\right]$ --- (24.1)
- Similarly, $E_{ff\phi} \cong \frac{j\beta e^{(-j\beta r)}}{4\pi r}\left[-\eta\left(\widetilde{H}_{ay}sin\phi + \widetilde{H}_{ax}cos\phi\right) + \left(\tilde{E}_{ay}cos\theta cos\phi - \tilde{E}_{ax}cos\theta sin\phi\right)\right]$ --- (24.2)

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- $\because \vec{H}_a = \frac{1}{\eta_0}\hat{z} \times \vec{E}_a = \frac{1}{\eta_0}\hat{z} \times \left(E_{ax}\hat{x} + E_{ay}\hat{y}\right) = \frac{1}{\eta_0}\left(E_{ax}\hat{y} - E_{ay}x\right)$
- $\therefore H_{ax} = \frac{1}{\eta_0}E_{ax}, H_{ay} = -\frac{1}{\eta_0}E_{ay}$
- Replacing H_{ax} and H_{ay} in the equation $(24.1)$and (24.2), the FF electric field is given by

- $E_{ff\theta} \cong \frac{j\beta e^{(-j\beta r)}}{4\pi r}(1 + cos\theta)\left[\left(\tilde{E}_{ax}cos\phi + \tilde{E}_{ay}sin\phi\right)\right]$
- $E_{ff\phi} \cong \frac{j\beta e^{(-j\beta r)}}{4\pi r}(1 + cos\theta)\left[\left(-\tilde{E}_{ax}sin\phi + \tilde{E}_{ay}cos\phi\right)\right]$

Aperture antennas

- ***Gain of reflector antennas***
- $G = \epsilon_{ap}\left(\frac{4\pi}{\lambda^2}A_p\right) = \epsilon_r\epsilon_{ill}\epsilon_a\left(\frac{4\pi}{\lambda^2}A_p\right) = \epsilon_r\epsilon_t\epsilon_s\epsilon_a\left(\frac{4\pi}{\lambda^2}A_p\right)$
- where ϵ_r is the radiation efficiency
- ϵ_{ill} is the illumination efficiency and is equal to $\epsilon_{ill} = \epsilon_t\epsilon_s$
- ϵ_t is the aperture taper efficiency
- ϵ_s is the spillover efficiency
- ϵ_a is the achievement efficiency and $\epsilon_a = \epsilon_{rs}\epsilon_{cr}\epsilon_{blk}\epsilon_{\phi r}\epsilon_{\phi f}$
- ϵ_{rs} is the random surface error efficiency (due to deviation from the ideal shape of the reflector)
- ϵ_{cr} is the cross-polarization efficiency

Aperture antennas

- ***Gain of reflector antennas***
- $G = \epsilon_{ap}\left(\frac{4\pi}{\lambda^2}A_p\right) = \epsilon_r\epsilon_{ill}\epsilon_a\left(\frac{4\pi}{\lambda^2}A_p\right) = \epsilon_r\epsilon_t\epsilon_s\epsilon_a\left(\frac{4\pi}{\lambda^2}A_p\right)$
- where ϵ_r is the radiation efficiency
- ϵ_{ill} is the illumination efficiency and is equal to $\epsilon_{ill} = \epsilon_t\epsilon_s$
- ϵ_t is the aperture taper efficiency
- ϵ_s is the spillover efficiency
- ϵ_a is the achievement efficiency and $\epsilon_a = \epsilon_{rs}\epsilon_{cr}\epsilon_{blk}\epsilon_{\phi r}\epsilon_{\phi f}$
  - ϵ_{rs} is the random surface error efficiency (due to deviation from the ideal shape of the reflector)
  - ϵ_{cr} is the cross-polarization efficiency (due to feed antenna has a component orthogonal to the desired polarization)

Aperture antennas

- ϵ_{blk} is the aperture blockage efficiency (due to structures placed in the front of the reflector such as the feed, subreflector, support hardware which block rays exiting the aperture and scatter power into the side-lobe regions)
- $\epsilon_{\phi r}$ is the reflector phase error efficiency (due to reflector design shape variation, reflector is defocused (displacement of the feed antenna off the focal point))
- $\epsilon_{\phi f}$ is the reflector phase error efficiency (due to imperfect feed antenna phase centre)

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- Note that the feed pattern will extend beyond the rim of the reflector,
  - the associated power will not be redirected by the reflector into the main beam and
  - hence the gain is reduced,
  - which is referred to as spillover and
- efficiency associated with this is called spillover efficiency
- Taper and spillover efficiencies can be combined to form illumination efficiency
  - which account for the feed pattern and reflector effects

Aperture antennas

- Formula to calculate ϵ_t, ϵ_s and ϵ_{ill}
- Given the feed antenna radiation intensity $U(\theta', \phi')$
- $\epsilon_t = \frac{1}{\pi} cot^2 \frac{\theta'_m}{2} \frac{\left| \int_0^{2\pi} \int_0^{\theta'_m} \sqrt{U(\theta',\phi')} tan \frac{\theta'}{2} d\theta' d\phi' \right|^2}{\int_0^{2\pi} \int_0^{\theta'_m} U(\theta',\phi') sin\theta' d\theta' d\phi'}$
- $\epsilon_s = \frac{\int_0^{2\pi} \int_0^{\theta'_m} U(\theta',\phi') sin\theta' d\theta' d\phi'}{\int_0^{2\pi} \int_0^{\pi} U(\theta',\phi') sin\theta' d\theta' d\phi'}$
- $\epsilon_{ill} = \epsilon_t \epsilon_s = \frac{1}{\pi} cot^2 \frac{\theta'_m}{2} \frac{\left| \int_0^{2\pi} \int_0^{\theta'_m} \sqrt{U(\theta',\phi')} tan \frac{\theta'}{2} d\theta' d\phi' \right|^2}{\int_0^{2\pi} \int_0^{\pi} U(\theta',\phi') sin\theta' d\theta' d\phi'}$
- You can easily derive this