EE540 Advance Electromagnetic Theory & Antennas

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Electromagnetic Theorems and Concepts

- **Uniqueness theorem:**
- For a given set of sources and boundary conditions in a lossy medium,
  - the solution to the Maxwell's equation is unique
- **Field equivalence principle (FEP):**
- FEP 1:
- Consider a set of current sources in a homogeneous medium
  - producing electromagnetic fields $\vec{E}$ and $\vec{H}$ everywhere
- Enclose all sources by a surface S,
  - separating the entire space into two parts: volume 1 and volume 2

Electromagnetic Theorems and Concepts

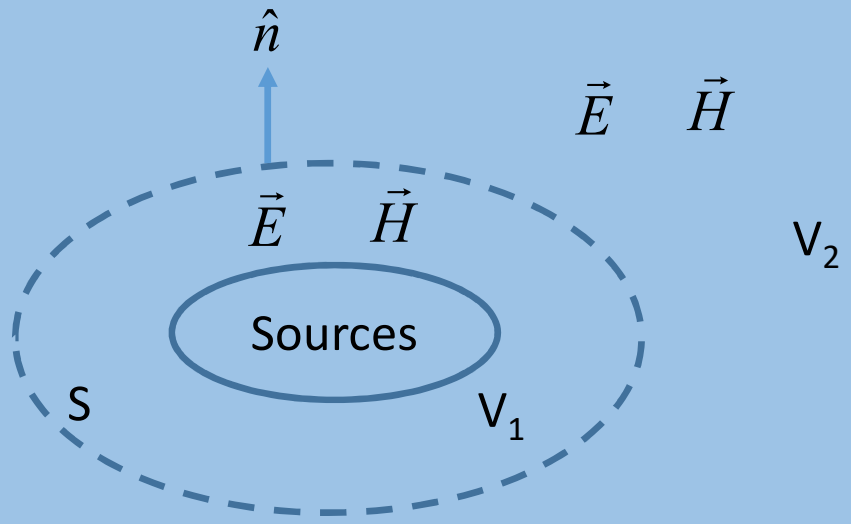

Fig. Fields and sources

Electromagnetic Theorems and Concepts

- Volume 1 contains sources and volume 2 is source free
- Assume surface S is also source free
  - and $\hat{n}$ is unit normal to surface S from V_1 to V_2
- According to FEP 1,
  - the fields in V_2 can be also generated by an equivalent set of virtual sources on surface S, given by $\vec{J}_S = \hat{n} \times \vec{H} \quad \vec{M}_S = \vec{E} \times \hat{n} = -\hat{n} \times \vec{E}$
- where $\vec{E}$ and $\vec{H}$ are the fields on the surface S produced
  - by the original set of sources in volume V_1
- Further the set of virtual sources produce null fields everywhere in V_1

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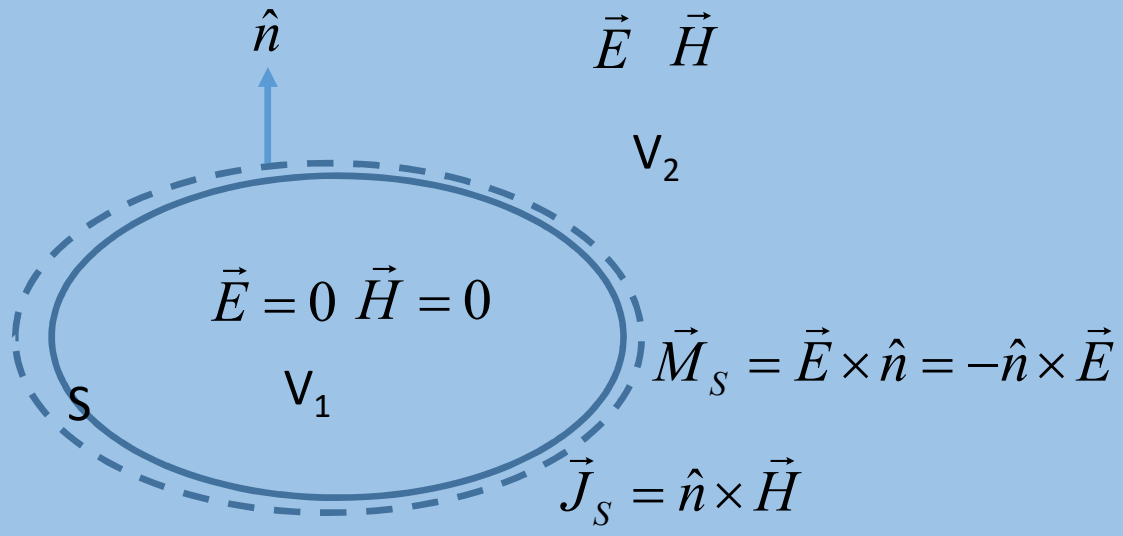

Fig. FEP 1: introduction of surface current densities (electric and magnetic) on the surface S

Electromagnetic Theorems and Concepts

- *Justification using uniqueness theorem:*
- Consider a situation where the fields in volume V_2 are the same as before
- Then we delete all the sources in V_1
  - and assume the fields are identically zero everywhere in V_1
- At the boundary surface S the fields are discontinuous
- Hence they cannot be supported
  - unless we introduce sources on the discontinuity surface

Electromagnetic Theorems and Concepts

- Specifically we introduce surface current sheets on S such that

$$\vec{J}_S = \hat{n} \times \vec{H} \qquad \vec{M}_S = \vec{E} \times \hat{n} = -\hat{n} \times \vec{E}$$

  - so that boundary conditions are satisfied
- Since the tangential $\vec{E}$ and $\vec{H}$ satisfy the boundary conditions,
  - it is a solution to Maxwell's equations,
  - and from Uniqueness theorem,
  - it is the only solution

Electromagnetic Theorems and Concepts

- Thus the original sources in V_1
  - and the new set of surface current sources produce the same fields in the volume V_2
- These are *equivalent problems*
  - as far as the fields in volume V_2 are concerned
- It is the first FEP
  - and the most general form
- We can use this FEP
  - provided we can find tangential electric and magnetic fields on the surface S