EE540 Advance Electromagnetic Theory & Antennas

Prof. Rakhesh S. Kshetrimayum

Dept. of EEE, IIT Guwahati, India

Cylindrical, Spherical and Plane Waves (Intuitive)

- *Instantaneous value of a time-harmonic voltage:*
- at an instant t: $v(t) = V_0 sin\omega t$
  - V_0 is the amplitude (peak voltage)
  - Frequency (repetition rate) is the f
  - ω is the angular frequency and equals to $\omega = 2\pi f$
  - Instantaneous phase is $\varphi(t) = \omega t$
- Time period (T) for change
  - $\omega T = 2\pi \Rightarrow T = \frac{2\pi}{\omega} = \frac{1}{f}$
- After each time T
  - the time-harmonic voltage repeats itself over time

Cylindrical, Spherical and Plane Waves (Intuitive)

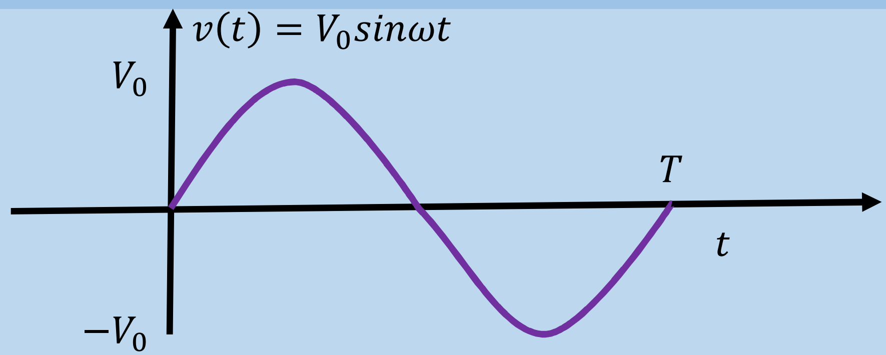


Fig. Time-harmonic voltage

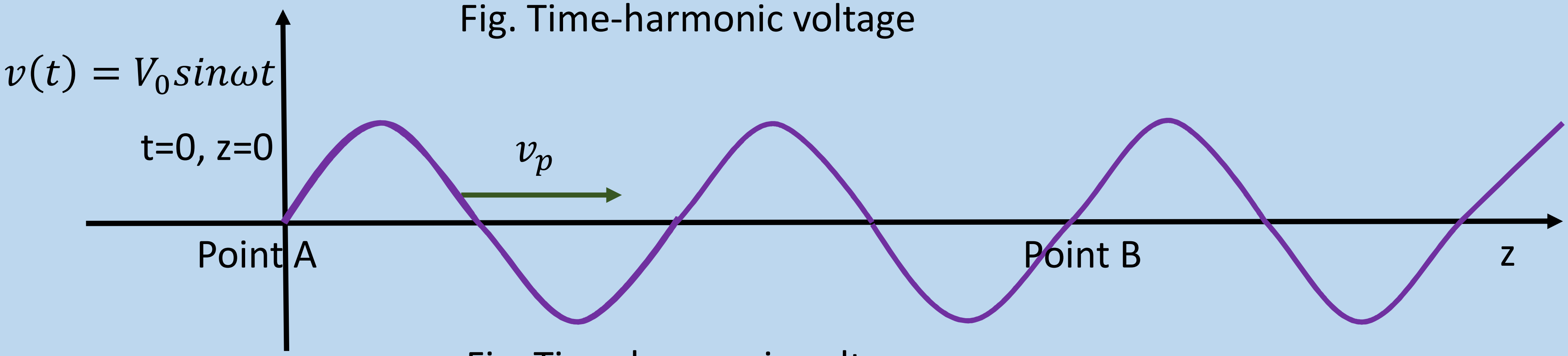


Fig. Time-harmonic voltage wave

Cylindrical, Spherical and Plane Waves (Intuitive)

- At any arbitrary distance z (Point B) from the origin,
  - the wave will have a phase delay of $\varphi_{A-B} = \omega \times \frac{z}{v_p} = \beta z$
  - from the wave at the origin (Point A)
- The time-harmonic voltage wave at point B can be written as
  - $v(t) = V_0 sin(\omega t - \varphi_{A-B}) = V_0 sin(\omega t - \beta z)$
- If the initial wave at time t=0, z=0, was having a initial phase of φ_0
  - $v(t) = V_0 sin(\omega t - \beta z + \varphi_0)$
- If $\varphi_0 = \frac{\pi}{2}$, then
  - $v(t) = V_0 sin\left(\omega t - \beta z + \frac{\pi}{2}\right) = V_0 cos(\omega t - \beta z)$
- It can be also represented as
  - $v(t) = Re\left\{V_0 e^{j(\omega t-\beta\ )}\right\}$=$V_0 Re\{cos(\omega t - \beta z) + j sin(\omega t - \beta z)\}$

Cylindrical, Spherical and Plane Waves (Intuitive)

- **Classification of Waves:**
- Waves can be basically
- classified into three based on their
  - Wavefront shapes (spherical/cylindrical/plane)
    - locus of points/surface of wave of constant phase
    - perpendicular to the wave propagation vector
  - and its sources
    - Spherical waves: Point source
    - Cylindrical waves: Line source
    - Plane waves: Source at infinity/planar source

Cylindrical, Spherical and Plane Waves (Intuitive)

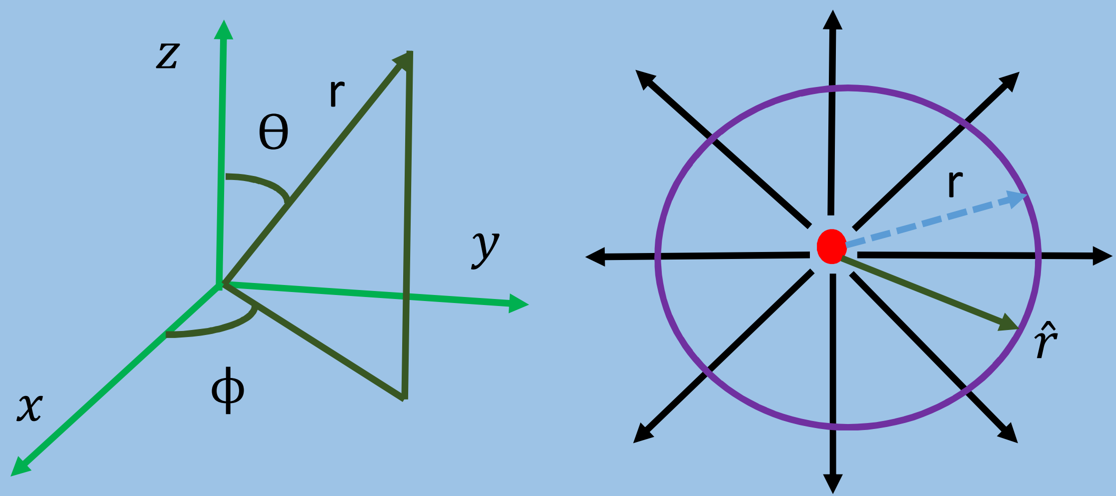
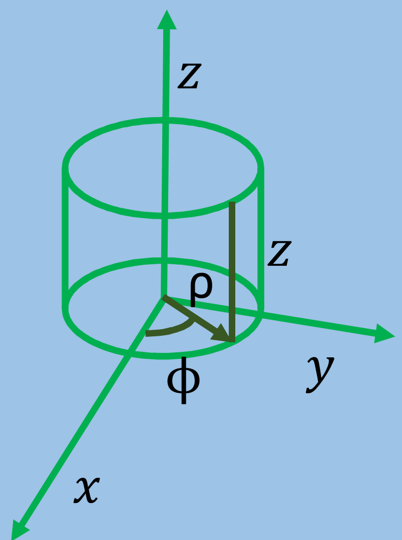


Fig. Spherical waves

Cylindrical, Spherical and Plane Waves (Intuitive)

- *Spherical waves:*
- The wave from point source (0-D) expands
- to a fill a sphere of radius r
  - wavefront cross a spherical surface
  - whose area grows as $4\pi r^2$
- From conservation of energy
  - Power per unit area must be decrease as $\frac{1}{r^2}$ as r increases
  - Then $Power\ \propto \frac{1}{r^2} \times 4\pi r^2 = 4\pi$ which will be a constant & independent of r
  - Since, $Power\ per\ unit\ area\ \propto Amplitude^2$, therefore $Amplitude \propto \frac{1}{r}$
- Hence simple expression for spherical wave function
  - $v(t,r) = \frac{V_0}{r} cos(\omega t - \beta r)$

Cylindrical, Spherical and Plane Waves (Intuitive)



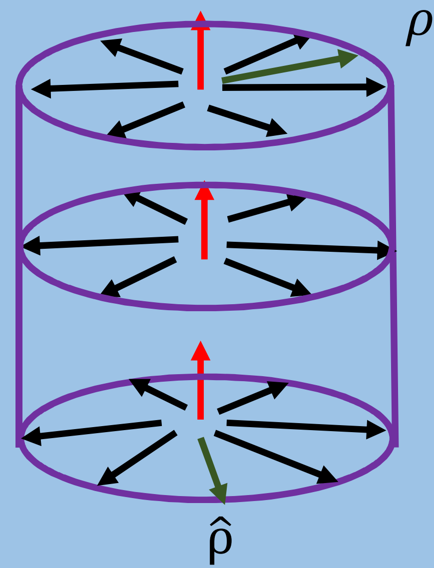


Fig. Cylindrical waves

Cylindrical, Spherical and Plane Waves (Intuitive)

- *Cylindrical Waves:*
- The wave from line source (1-D) expands
- to a fill a cylinder of radius ρ
  - wavefront cross a cylindrical surface
  - whose area grows as $2\pi\rho h$,
  - where h is the height of the cylinder
- From conservation of energy
  - Power per unit area must be decrease as $\frac{1}{\rho}$ as ρ increases
  - Then $Power\ \propto \frac{1}{\rho} \times 2\pi\rho h = 2\pi h$ which will be a constant & independent of ρ
  - Since, $Power\ per\ unit\ area\ \propto Amplitude^2$, therefore $Amplitude \propto \frac{1}{\sqrt{\rho}}$
- Hence simple and possible expression of cylindrical wave function
  - $v(t,\rho) = \frac{V_0}{\sqrt{\rho}} cos(\omega t - \beta\rho)$

Cylindrical, Spherical and Plane Waves (Intuitive)

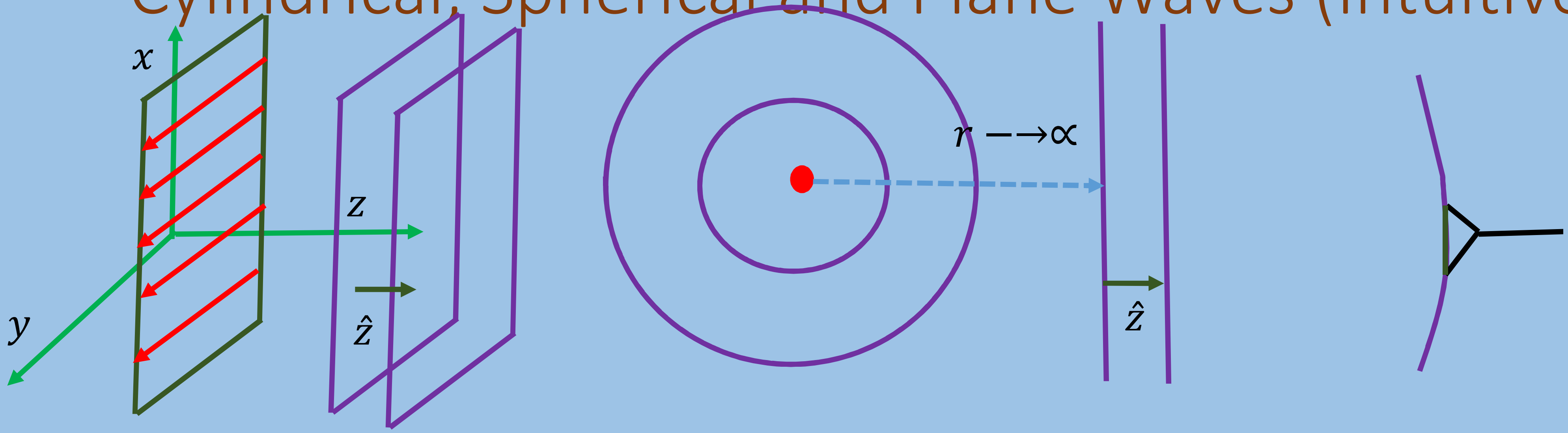


Fig. Plane waves

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Cylindrical, Spherical and Plane Waves (Intuitive)

- *Plane waves:*
- For a planar source (2-D)
    - wavefront form a plane surface
    - parallel to the face of source
    - amplitude is constant
- Hence simple and possible expression
- of plane wave function
    - $v(t,z) = V_0 cos(\omega t - \beta z)$
- For wave moving in x-y-z space
    - $v(t,x,y,z) = V_0 cos\left(\omega t - \beta_x x - \beta_y y - \beta_z z\right)$
- which can be expressed as
    - $v(t,x,y,z) = V_0 Re\left\{e^{j(\omega t - \beta_x x - \beta_y y - \beta_z z)}\right\} = V_0 Re\left\{e^{j\left(\omega t - \vec{\beta}\cdot\vec{r}\right)}\right\}$