

An Introduction to Method of Moments

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EE543 Computational Electromagnetics

- Syllabus and reference books available at course webpage
 - https://www.iitg.ac.in/engfac/krs/public_html/lectures/ee543/
- Marks distributions:
 - In-class evaluations: 5 marks
 - Programming exercise: 35 marks
 - Evaluations (online): 20×3

Digression: Introduction to CEM

- Background on commercial softwares
- Who's Who in Computational Electromagnetics (CEM)?

(a) EMSS software

- Originated by *Dr. Ulrich Jacobus*
- *Dr. Francs Meyer* (a student of Prof. D. B. Davidson, a Professor at University of Stellenbosch, South Africa) is one of the main person behind this company

CEM: An Introduction

(b) **IE3D** by Zeland Software Inc.

- Founded by *Dr. Jian-Xiong Zheng*
- Did PhD under Prof. D. C. Chang (President Emeritus of NYU Poly)

(c) **REMCOM XFDTD** is a product of Penn. State University

- Founders are *H. Scott Langdon*,
- *Dr. Raymond Lubbers* and
- *Dr. Christopher Penny*

CEM: An Introduction

(d) SONNET

- *Dr. James C. Rautio* did his PhD from Syracuse university, USA under Prof. R. F. Harrington (considered as originator of MoM in EM area)

(e) GEMS software

- *Dr. Wenhua Yu* who was a member of Prof. Raj Mittra group at Penn. State University, USA founded
- And list goes on
- Can you guess the price of softwares like HFSS?

CEM: An Introduction

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CEM: An Introduction

- What is CEM?
- In CEM
 - Try to solve complex EM problems
 - which do not have tractable analytical solutions
 - Using numerical methods employing computers

CEM: An Introduction

Why CEM?

- Possible to simulate a device/experiment/phenomenon any number of times
- Try to achieve the best or optimal result before actually doing the experiments (preserving resources)
- Some experiments are
 - dangerous to perform like lightning strike on aircraft or
 - not possible to conduct because of limited facilities/resources

CEM: An Introduction

- How to learn CEM?
 - Can be learnt by doing it
 - Once you learn the basic concepts
 - You should write or code simple programs in any language
 - Always start with simple problems to gain confidence
 - and try for more difficult problems later

CEM: An Introduction

- Many CEM methods are available
 - Method of Moments (MoM)
 - Finite Element Method (FEM)
 - Finite Difference Time Domain (FDTD)

10.1 Introduction

- learn how to use method of moments (MoM) to solve
 - electrostatic problems
 - advanced & challenging problems in time-varying fields
- brief discussion on the basic steps of MoM
- solve a simple differential equation using MoM
 - in order to elucidate the steps involved
- MoM for 1-D and 2-D electrostatic problems
- MoM for electrodynamic problems

10.2 Basic Steps in Method of Moments

- Method of Moments (MoM) transforms
 - integro-differential equations into matrix systems of linear equations
 - which can be solved using computers
- Consider the following inhomogeneous equation

$$L(u) = k$$
$$\Rightarrow L(u) - k = 0$$

- where L is a linear integro-differential operator,
- u is an unknown function (to be solved) and
- k is a known function (excitation)

10.2 Basic Steps in Method of Moments

- For example,
- (a) consider the integral equation for a line charge density

$$V_0 = \int \frac{\lambda(x') dx'}{4\pi\epsilon_0 r(x, x')}$$

- Then

$$u = \lambda(x')$$

$$k = V_0$$

$$L = \int \frac{dx'}{4\pi\epsilon_0 r(x, x')}$$

10.2 Basic Steps in Method of Moments

- (b) consider the differential equation of the form

$$\frac{d^2 f(x)}{dx^2} = 3 + 2x^2$$

- Then

$$u = f(x)$$

$$k = 3 + 2x^2$$

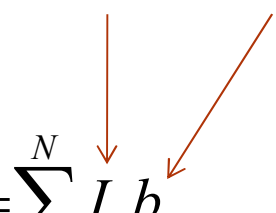
$$L = -\frac{d^2}{dx^2}$$

10.2 Basic Steps in Method of Moments

- To solve u , approximate it by sum of weighted known
 - basis functions or
 - expansion functions

$$f(x) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty} a_n \cos(nx) + \sum_{n=1}^{\infty} b_n \sin(nx)$$


- as given below

$$u \cong \sum_{n=1}^N u_n = \sum_{n=1}^N I_n b_n, \quad n = 1, 2, \dots, N$$



- where b_n is the expansion function,
- I_n is its unknown complex coefficients to be determined,
- N is the total number of expansion functions

10.2 Basic Steps in Method of Moments

- Since L is linear, substitution of the above equation in the
 - integro-differential equation,
- we get,

$$L\left(\sum_{n=1}^N I_n b_n\right) \approx k$$


- where the error or residual is given by

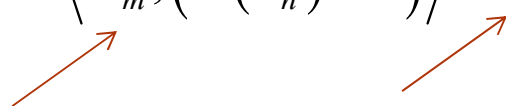
$$R = k - L\left(\sum_{n=1}^N I_n b_n\right)$$


10.2 Basic Steps in Method of Moments

- Mathematicians name this method as Method of Weighted Residuals
- Why?
- Next step in MoM
- Enforcing the boundary condition
- Make inner product of the above equation with each of the
 - testing or
 - weighting functions
- should make residual or error zero

10.2 Basic Steps in Method of Moments

- By replacing u by u_n
 - where $n=1,2,\dots,N$
- taking inner product with a set of w_m
 - weighting or
 - testing functions
- in the range of L , we have,

$$\langle w_m, (L(u_n) - k) \rangle = 0, \quad m = 1, 2, \dots, M$$


10.2 Basic Steps in Method of Moments

- Since I_n is a constant
- we can take it outside the inner product and
- write

$$\sum_{n=1}^N I_n \langle w_m, L(b_n) \rangle = \langle w_m, k \rangle, \quad m = 1, 2, \dots, M$$

- M and N should be infinite theoretically
 - but practically it should be a finite number

10.2 Basic Steps in Method of Moments

- Note that a scalar product $\langle w, g \rangle$ is defined to be a scalar satisfying

$$\langle w, g \rangle = \langle g, w \rangle = \int g(x)w(x)dx$$

$$\langle bf + cg, w \rangle = b\langle f, w \rangle + c\langle g, w \rangle$$

$$\langle g^*, g \rangle > 0 \quad \text{if } g \neq 0 \quad \langle g^*, g \rangle = 0 \quad \text{if } g = 0$$

- b and c are scalars and $*$ indicates complex conjugation

10.2 Basic Steps in Method of Moments

- In matrix form

$$[Z][I] = [V]$$

- with each matrix and vector defined by

$$[I] = [I_1 \quad I_2 \quad \dots \quad I_N]^T \quad [V] = [\langle k, w_1 \rangle \quad \langle k, w_2 \rangle \quad \dots \quad \langle k, w_M \rangle]^T$$

$$[Z] = \begin{bmatrix} \langle w_1, L(b_1) \rangle & \langle w_1, L(b_2) \rangle & \dots & \langle w_1, L(b_N) \rangle \\ \langle w_2, L(b_1) \rangle & \langle w_2, L(b_2) \rangle & \dots & \langle w_2, L(b_N) \rangle \\ \langle w_3, L(b_1) \rangle & \langle w_3, L(b_2) \rangle & \dots & \langle w_3, L(b_N) \rangle \\ \vdots & \vdots & \ddots & \vdots \\ \langle w_M, L(b_1) \rangle & \langle w_M, L(b_2) \rangle & \dots & \langle w_M, L(b_N) \rangle \end{bmatrix}$$