An Introduction to Method of Moments

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EE543 Computational Electromagnetics

- Syllabus and reference books available at course webpage
  - https://www.iitg.ac.in/engfac/krs/public_html/lectures/ee543/
- Marks distributions:
  - In-class evaluations: 5 marks
  - Programming exercise: 35 marks
  - Evaluations (online): 20×3

Digression: Introduction to CEM

- Background on commercial softwares
- Who's Who in Computational Electromagnetics (CEM)?

(a) EMSS software

- Originated by ***Dr. Ulrich Jacobus***
- ***Dr. Francs Meyer*** (a student of Prof. D. B. Davidson, a Professor at University of Stellenbosch, South Africa) is one of the main person behind this company

CEM: An Introduction

(b) **IE3D** by Zeland Software Inc.

- Founded by ***Dr. Jian-Xiong Zheng***
- Did PhD under Prof. D. C. Chang (President Emeritus of NYU Poly)

(c) **REMCOM XFDTD** is a product of Penn. State University

- Founders are ***H. Scott Langdon***,
- ***Dr. Raymond Lubbers*** and
- ***Dr. Christopher Penny***

CEM: An Introduction

(d) **SONNET**

- ***Dr. James C. Rautio*** did his PhD from Syracuse university, USA under Prof. R. F. Harrington (considered as originator of MoM in EM area)

(e) **GEMS** software

- ***Dr. Wenhua Yu*** who was a member of Prof. Raj Mittra group at Penn. State University, USA founded

- And list goes on
- Can you guess the price of softwares like HFSS?

CEM: An Introduction

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CEM: An Introduction

- What is CEM?
- In CEM
  - Try to solve complex EM problems
  - which do not have tractable analytical solutions
  - Using numerical methods employing computers

CEM: An Introduction

Why CEM?

- Possible to simulate a device/experiment/phenomenon any number of times
- Try to achieve the best or optimal result before actually doing the experiments (preserving resources)
- Some experiments are
  - dangerous to perform like lightning strike on aircraft or
  - not possible to conduct because of limited facilities/resources

CEM: An Introduction

- How to learn CEM?
  - Can be learnt by doing it
  - Once you learn the basic concepts
  - You should write or code simple programs in any language
  - Always start with simple problems to gain confidence
  - and try for more difficult problems later

CEM: An Introduction

- Many CEM methods are available
  - Method of Moments (MoM)
  - Finite Element Method (FEM)
  - Finite Difference Time Domain (FDTD)

10.1 Introduction

- learn how to use method of moments (MoM) to solve
  - electrostatic problems
  - advanced & challenging problems in time-varying fields
- brief discussion on the basic steps of MoM
- solve a simple differential equation using MoM
  - in order to elucidate the steps involved
- MoM for 1-D and 2-D electrostatic problems
- MoM for electrodynamic problems

10.2 Basic Steps in Method of Moments

- Method of Moments (MoM) transforms
  - integro-differential equations into matrix systems of linear equations
  - which can be solved using computers
- Consider the following inhomogeneous equation

$$L(u) = k$$

$$\Rightarrow L(u) - k = 0$$

- where L is a linear integro-differential operator,
- u is an unknown function (to be solved) and
- k is a known function (excitation)

10.2 Basic Steps in Method of Moments

- For example,
- (a) consider the integral equation for a line charge density

$$V_0 = \int \frac{\lambda(x')dx'}{4\pi\varepsilon_0 r(x,x')}$$

- Then

$$u = \lambda\left(x'\right)$$

$$k = V_0$$

$$L = \int \frac{dx'}{4\pi\varepsilon_0 r(x,x')}$$

10.2 Basic Steps in Method of Moments

- (b) consider the differential equation of the form

$$-\frac{d^2 f(x)}{dx^2} = 3 + 2x^2$$

- Then

$$u = f\left(x\right)$$

$$k = 3 + 2x^2$$

$$L = -\frac{d^2}{dx^2}$$

10.2 Basic Steps in Method of Moments

- To solve u, approximate it by sum of weighted known
  - basis functions or
  - expansion functions

$$f(x) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty} a_n \cos(nx) + \sum_{n=1}^{\infty} b_n \sin(nx)$$

- as given below

$$u \cong \sum_{n=1}^{N} u_n = \sum_{n=1}^{N} I_n b_n, \qquad n = 1, 2, \ldots, N$$

- where b_n is the expansion function,
- I_n is its unknown complex coefficients to be determined,
- N is the total number of expansion functions

10.2 Basic Steps in Method of Moments

- Since L is linear, substitution of the above equation in the
  - integro-differential equation,
- we get,

$$L\left(\sum_{n=1}^{N} I_n b_n\right) \approx k$$

- where the error or residual is given by

$$R = k - L\left(\sum_{n=1}^{N} I_n b_n\right)$$

10.2 Basic Steps in Method of Moments

- Mathematicians name this method as Method of Weighted Residuals
- Why?
- Next step in MoM
- Enforcing the boundary condition
- Make inner product of the above equation with each of the
  - testing or
  - weighting functions
- should make residual or error zero

10.2 Basic Steps in Method of Moments

- By replacing u by u_n
  - where n=1,2,…,N
- taking inner product with a set of w_m
  - weighting or
  - testing functions
- in the range of L, we have,

$$\left\langle w_m, \left(L\left(u_n\right) - k\right)\right\rangle = 0, \qquad m = 1, 2, \ldots, M$$

10.2 Basic Steps in Method of Moments

- Since I_n is a constant
- we can take it outside the inner product and
- write

$$\sum_{n=1}^{N} I_n \left\langle w_m, L(b_n) \right\rangle = \left\langle w_m, k \right\rangle, \qquad m = 1, 2, \ldots, M$$

- M and N should be infinite theoretically
  - but practically it should be a finite number

10.2 Basic Steps in Method of Moments

- Note that a scalar product $\langle w, g \rangle$ is defined to be a scalar satisfying

$$\langle w, g \rangle = \langle g, w \rangle = \int g(x) w(x) dx$$

$$\langle bf + cg, w \rangle = b\langle f, w \rangle + c\langle g, w \rangle$$

$$\left\langle g^*, g \right\rangle > 0 \quad if \quad g \neq 0 \qquad \left\langle g^*, g \right\rangle = 0 \quad if \quad g = 0$$

- *b* and c are scalars and * indicates complex conjugation

10.2 Basic Steps in Method of Moments

- In matrix form

$$[Z][I]=[V]$$

- with each matrix and vector defined by

$$[I]=[I_1 \quad I_2 \quad \dots \quad I_N]^T \quad [V]=[\langle k,w_1\rangle \quad \langle k,w_2\rangle \quad \dots \quad \langle k,w_M\rangle]^T$$

$$[Z]=\begin{bmatrix} \langle w_1,L(b_1)\rangle & \langle w_1,L(b_2)\rangle & \dots & \langle w_1,L(b_N)\rangle \\ \langle w_2,L(b_1)\rangle & \langle w_2,L(b_2)\rangle & \dots & \langle w_2,L(b_N)\rangle \\ \langle w_3,L(b_1)\rangle & \langle w_3,L(b_2)\rangle & \dots & \langle w_3,L(b_N)\rangle \\ \vdots & \vdots & \ddots & \vdots \\ \langle w_M,L(b_1)\rangle & \langle w_M,L(b_2)\rangle & \dots & \langle w_M,L(b_N)\rangle \end{bmatrix}$$