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- %Increase the number of time steps
- NSTEPS=300;
- %Inside the time index loop after injecting the Gaussian pulse
- %Left ABC
- ex(1,1)=ex_left_m2;
- ex_left_m2=ex_left_m1;
- ex_left_m1=ex(1,2);
- %Right ABC

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- ex(1,KE)=ex_right_m2;
- ex_right_m2=ex_right_m1;
- ex_right_m1=ex(1,KE-1);
- Discussion:
- For n=1
- ex(1,1)=0.0;
- ex_left_m2=0.0;
- ex_left_m1=ex(1,2); for t=1

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- For n=2
- ex(1,1)=0.0;
- ex_left_m2=ex(1,2); for t=1
- ex_left_m1=ex(1,2); for t=2
- For n=3
- ex(1,1)=ex(1,2); for t=1
- ex_left_m2=ex(1,2); for t=2
- ex_left_m1=ex(1,2); for t=3

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- For n=4
- ex(1,1)=ex(1,2); for t=2
- ex_left_m2=ex(1,2); for t=3
- ex_left_m1=ex(1,2); for t=4
- We force ex(1,1)=ex(1,2) for previous-to-previous time index
- We also need to store the ex(1,2) for current and previous time index so that they can be used for applying ABC for the future time index
- Note that for every time index we are updating ex for all values of k=2:KE

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- For ε_1=1.0 and ε_2=4.0, we have $\Gamma = -\frac{1}{3}; \tau = \frac{2}{3}$
- *FDTD simulation of Absorbing Boundary Condition* *(*fdtd_1d_ABC_boundary.m*)*
- *(*fdtd_1d_no_boundary.m*)*
- In calculating the E field,
  - we need to know the surrounding H values;
  - this is the fundamental assumption in FDTD
- At the edge of the problem space,
  - we will not have the value to one side,
  - but we know there are no sources outside the problem space

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- The wave moves $\Delta z/2$ ($=c_0 \cdot \Delta t$) distance in one time step,
  - so it takes two time steps for a wave front to cross one cell
- Suppose we are looking for a boundary condition at the end where k=1
- Now if we write the E field at k=1 as
  - $E_x(1,n) = E_x(2, n-2)$,
  - then the fields at the edge will not reflect
- This condition must be applied at both ends

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2. FDTD Solution to Maxwell's equations in 2-D Space

- In deriving 2-D FDTD formulation, we choose between one of two groups of three vectors each:
  - (a) Transverse magnetic (TM^z) mode,
    - which is composed of Ez, Hx, and Hy or
  - (b) Transverse electric (TE^z) mode,
    - which is composed of Ex, Ey, and Hz

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- Unlike time-harmonic guided waves,
  - none of the fields vary with z so that
  - there is no propagation in the z-direction
- But in general propagation along x- or y- directions or both is possible

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- *FDTD simulation of TM mode wave propagation* (fdtd_2d_TM_1.m)
- Expanding the Maxwell's curl equations with
  - Ex = 0, Ey = 0, Hz = 0 and $\partial/\partial z = 0$,
- we obtain,
- 3 equations from the 6 equations

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$$\frac{\partial E_x}{\partial t}=\frac{1}{\varepsilon_0\varepsilon_r}\left(\frac{\partial H_z}{\partial y}-\frac{\partial H_y}{\partial z}\right)$$

$$\frac{\partial E_y}{\partial t}=\frac{1}{\varepsilon_0\varepsilon_r}\left(\frac{\partial H_x}{\partial z}-\frac{\partial H_z}{\partial x}\right)$$

$$\frac{\partial E_z}{\partial t}=\frac{1}{\varepsilon_0\varepsilon_r}\left(\frac{\partial H_y}{\partial x}-\frac{\partial H_x}{\partial y}\right)$$

$$\frac{\partial H_x}{\partial t}=\frac{1}{\mu_0\mu_r}\left(\frac{\partial E_y}{\partial z}-\frac{\partial E_z}{\partial y}\right)$$

$$\frac{\partial H_y}{\partial t}=\frac{1}{\mu_0\mu_r}\left(\frac{\partial E_z}{\partial x}-\frac{\partial E_x}{\partial z}\right)$$

$$\frac{\partial H_z}{\partial t}=\frac{1}{\mu_0\mu_r}\left(\frac{\partial E_x}{\partial y}-\frac{\partial E_y}{\partial x}\right)$$

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- 3 equations as follows

$$\frac{\partial E_z}{\partial t} = \frac{1}{\varepsilon}\left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right)$$

$$\frac{\partial H_x}{\partial t} = -\frac{1}{\mu}\left(\frac{\partial E_z}{\partial y}\right)$$

$$\frac{\partial H_y}{\partial t} = \frac{1}{\mu}\left(\frac{\partial E_z}{\partial x}\right)$$

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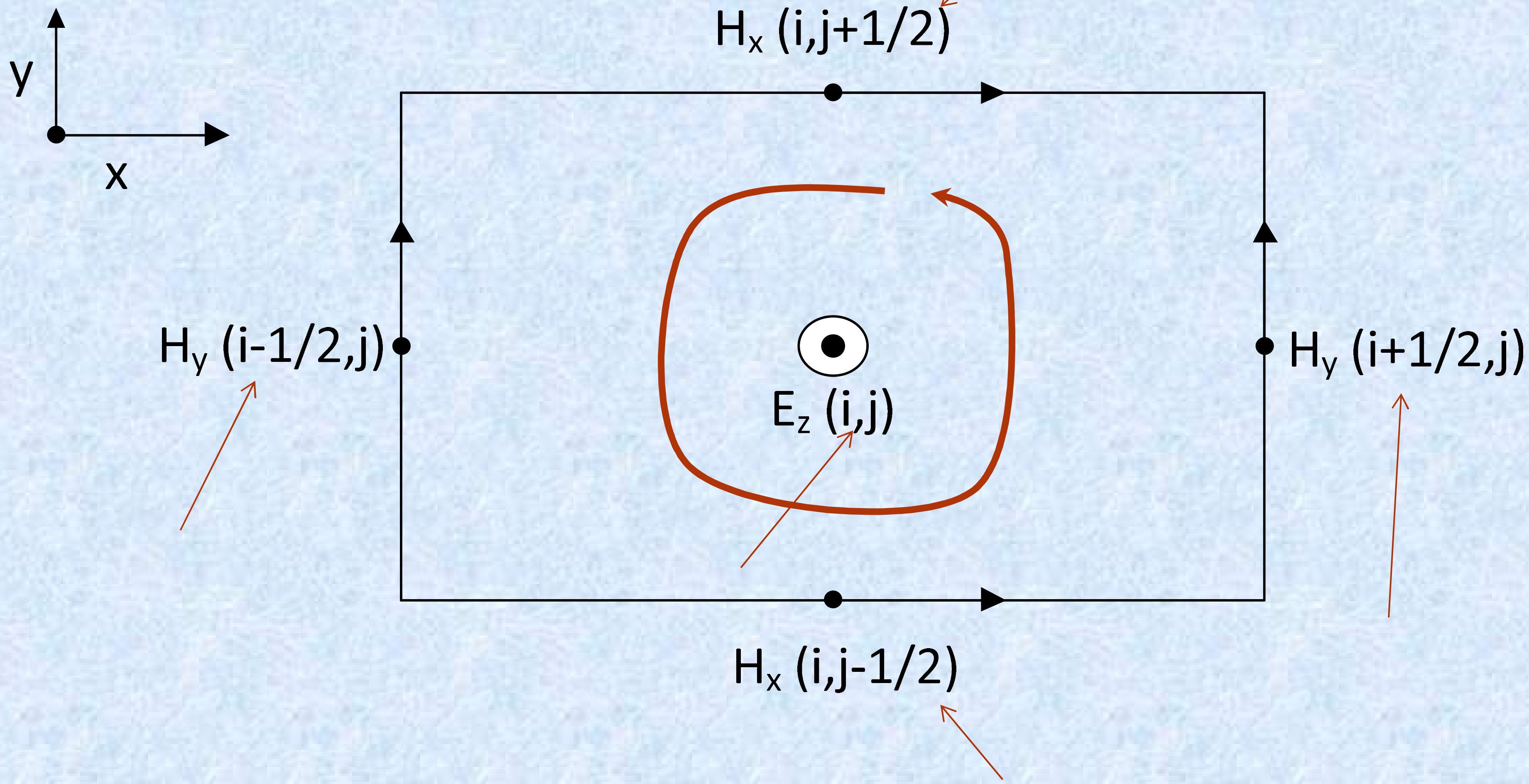

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- Observations:
  - Electric fields are calculated at integer space steps
  - Electric fields are calculated at half integer time steps
  - Magnetic fields are calculated at half integer space step and integer space step
  - Magnetic fields are calculated at integer time steps
- For example,
  - $H_y(i-1/2,j)$, $H_x(i,j-1/2)$, $H_y(i+1/2,j)$, $H_x(i,j+1/2)$

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- Central finite difference approximations of the 1st equation is as follows

$$\frac{\partial E_z}{\partial t} = \frac{1}{\varepsilon}\left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right)$$

$$\frac{E_z\left(i, j, n+\frac{1}{2}\right) - E_z\left(i, j, n-\frac{1}{2}\right)}{\Delta t}$$

$$= \frac{1}{\varepsilon_0 \varepsilon_r}\left[\frac{H_y\left(i+\frac{1}{2}, j, n\right) - H_y\left(i-\frac{1}{2}, j, n\right)}{\Delta x} - \frac{H_x\left(i, j+\frac{1}{2}, n\right) - H_x\left(i, j-\frac{1}{2}, n\right)}{\Delta y}\right]$$

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- Central finite difference approximations of the next equation is as follows

$$\frac{\partial H_x}{\partial t} = -\frac{1}{\mu}\left(\frac{\partial E_z}{\partial y}\right)$$

$$\frac{H_x\left(i, j+\frac{1}{2}, n+1\right) - H_x\left(i, j+\frac{1}{2}, n\right)}{\Delta t} = -\frac{1}{\mu_0 \mu_r} \frac{E_z\left(i, j+1, n+\frac{1}{2}\right) - E_z\left(i, j, n+\frac{1}{2}\right)}{\Delta y}$$

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- Central finite difference approximations of the next equation is as follows

$$\frac{\partial H_y}{\partial t} = \frac{1}{\mu}\left(\frac{\partial E_z}{\partial x}\right)$$

$$\frac{H_y\left(i+\frac{1}{2},j,n+1\right)-H_x\left(i+\frac{1}{2},j,n\right)}{\Delta t}=\frac{1}{\mu_0\mu_r}\frac{E_z\left(i+1,j,n+\frac{1}{2}\right)-E_z\left(i,j,n+\frac{1}{2}\right)}{\Delta x}$$

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- Rearranging the above equations, we can write the final 3 update equations' expressions as
- Electric field update equation

$$E_z\left(i,j,n+\frac{1}{2}\right)$$

$$=E_z\left(i,j,n-\frac{1}{2}\right)+\frac{\Delta t}{\varepsilon_0\varepsilon_r}\left[\frac{H_y\left(i+\frac{1}{2},j,n\right)-H_y\left(i-\frac{1}{2},j,n\right)}{\Delta x}-\frac{H_x\left(i,j+\frac{1}{2},n\right)-H_x\left(i,j-\frac{1}{2},n\right)}{\Delta y}\right]$$

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- Magnetic field update equation

$$H_x\left(i, j+\frac{1}{2}, n+1\right)=H_x\left(i, j+\frac{1}{2}, n\right)-\frac{\Delta t}{\mu_0\mu_r}\frac{E_z\left(i, j+1, n+\frac{1}{2}\right)-E_z\left(i, j, n+\frac{1}{2}\right)}{\Delta y}$$

$$H_y\left(i+\frac{1}{2}, j, n+1\right)=H_x\left(i+\frac{1}{2}, j, n\right)+\frac{\Delta t}{\mu_0\mu_r}\frac{E_z\left(i+1, j, n+\frac{1}{2}\right)-E_z\left(i, j, n+\frac{1}{2}\right)}{\Delta x}$$

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$$E_z\left(i,j,n+\frac{1}{2}\right)$$

$$= E_z\left(i,j,n-\frac{1}{2}\right)+\frac{\Delta t}{\varepsilon_0\varepsilon_r}\left[\frac{H_y\left(i+\frac{1}{2},j,n\right)-H_y\left(i-\frac{1}{2},j,n\right)}{\Delta x}-\frac{H_x\left(i,j+\frac{1}{2},n\right)-H_x\left(i,j-\frac{1}{2},n\right)}{\Delta y}\right]$$

- Electric field computer update equation is (i+1/2 and i-1/2 are replaced by i or i-1)
- ez(i,j)=ez(i,j)+{dt/(eps_0*eps_r)}*[{Hy(i,j)-Hy(i-1,j)}/dx-{Hx(i,j)-Hx(i,j-1)}/dy]

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$$H_x\left(i,j+\frac{1}{2},n+1\right)=H_x\left(i,j+\frac{1}{2},n\right)-\frac{\Delta t}{\mu_0\mu_r}\frac{E_z\left(i,j+1,n+\frac{1}{2}\right)-E_z\left(i,j,n+\frac{1}{2}\right)}{\Delta y}$$

- Magnetic field computer update equation is (j+1/2 is replaced by j, usually meu_r=1)
- Hx(i,j) = Hx(i,j)-{dt/(mu_0*dy)}*{Ez(i,j+1)-Ez(i,j)}

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$$H_y\left(i+\frac{1}{2},j,n+1\right)=H_x\left(i+\frac{1}{2},j,n\right)+\frac{\Delta t}{\mu_0\mu_r}\frac{E_z\left(i+1,j,n+\frac{1}{2}\right)-E_z\left(i,j,n+\frac{1}{2}\right)}{\Delta x}$$

- Magnetic field computer update equation is (i+1/2 is replaced by i, usually meu_r=1)
- Hy(i,j) = Hy(i,j)+{dt/(mu_0*dx)}*{Ez(i+1,j)-Ez(i,j)}

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- Some new MATLAB commands:
- image: IMAGE(C) displays matrix C as an image
- imagesc: data is scaled to use the full color map
- Colorbar: appends a vertical color scale to the current axis
- Colormap: a matrix may have any number of rows but it must have exactly 3 columns
- Each row is interpreted as a color, with the first element specifying the intensity of red, the second green and the third blue
- Color intensity is specified on the interval of 0.0 to 1.0

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- For example, [0 0 0] is black
- [1 1 1] is white
- [1 0 0] is red
- [0.5 0.5 0.5] is gray
- [127/255 1 212/255] is aquamarine

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FDTD simulation of TM wave hitting a dielectric surface (right side), source at the center (2-D) **(**fdtd_2d_TM_dielectric.m)

*(***Programming Exercise 6***)*

- Modify the above program as follows:
- Specify the position of the dielectric surface (id=..,jd=..)
- Choose a value of eps_r=10.0..
- Modify the constant value C1 accordingly
- Run the program
- In free space wave is propagating freely and
- on the other side reflection and transmission of the wave at the interface