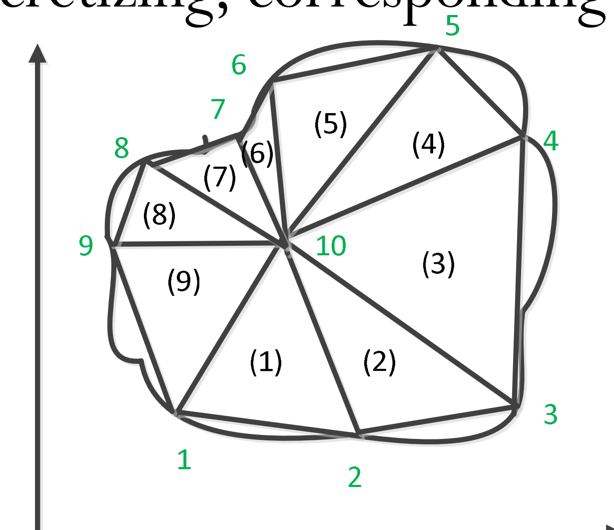
Introduction

- To solve these equations,
  - we need to invert the matrix on the LHS
- Since this matrix is time independent,
  - it needs to be filled and
  - solved only once
- **PML**
- Uniaxial PML gives better performance than split-field or stretched coordinate based PML

Introduction

- Doubts raised:
- Q1. While you are discretizing any finite element, how you decide which section to divide into triangular section or rectangular section or any other kind of section? Like in example (page-3) you have divided the finite element into triangular section. So can I divide it in rectangular sections and if I do this…..will it yield same solution when you divide it into triangular section?
- Ans: Depending on the shape you are discretizing, corresponding shape or interpolation functions must be used



Introduction

- Doubts raised:
- Q2. Except Triangular and Rectangular section, what other geometrical figures we use for Discretization whose voltage equation is known to us?
- Ans:
- We have already discussed abut piecewise linear shape functions. For 3-D shapes, you can even use pyramidal shapes

Introduction

- Doubts raised:
- Q3. In page-58 and 59, two methods are discussed such as BAND MATRIX METHOD. ITERATION METHOD. Which method gives more accurate result because in the example discussed in page-58,59 there is slight variation comes in the solution.
- Ans: No hard and fast rule, but for iteration based results, your initial choice and number of iterations effect the final solution
- For band matrix method, you have to do sorting of free nodes and prescribed nodes

Introduction

- Q4. In page 76, To satisfy EULER'S equation we need to find F(x,phi,phi'). How to find F assuming Phi(x) is given?
- Ans:
- That is technique to find unknown function in FEM
- If unknown function is already known, we do not seek for any further ways of finding it
- If you still want to find it, we do not consider such problems in FEM, that will be purely a mathematical problem (depends of what you are looking for)

Introduction

- Q5. In page-71, Why we have to find extremum value of I(phi)?
- Ans:
- FEM has two approaches
  - Direct approach:
    - Solve for unknown function in PDE by applying Galerkin's Weighted residual method
  - Indirect approach:
    - Find an equivalent problem where minimizing the functional gives the solution of the PDE (Physically it is stable solution for energy or equilibrium points)

Introduction

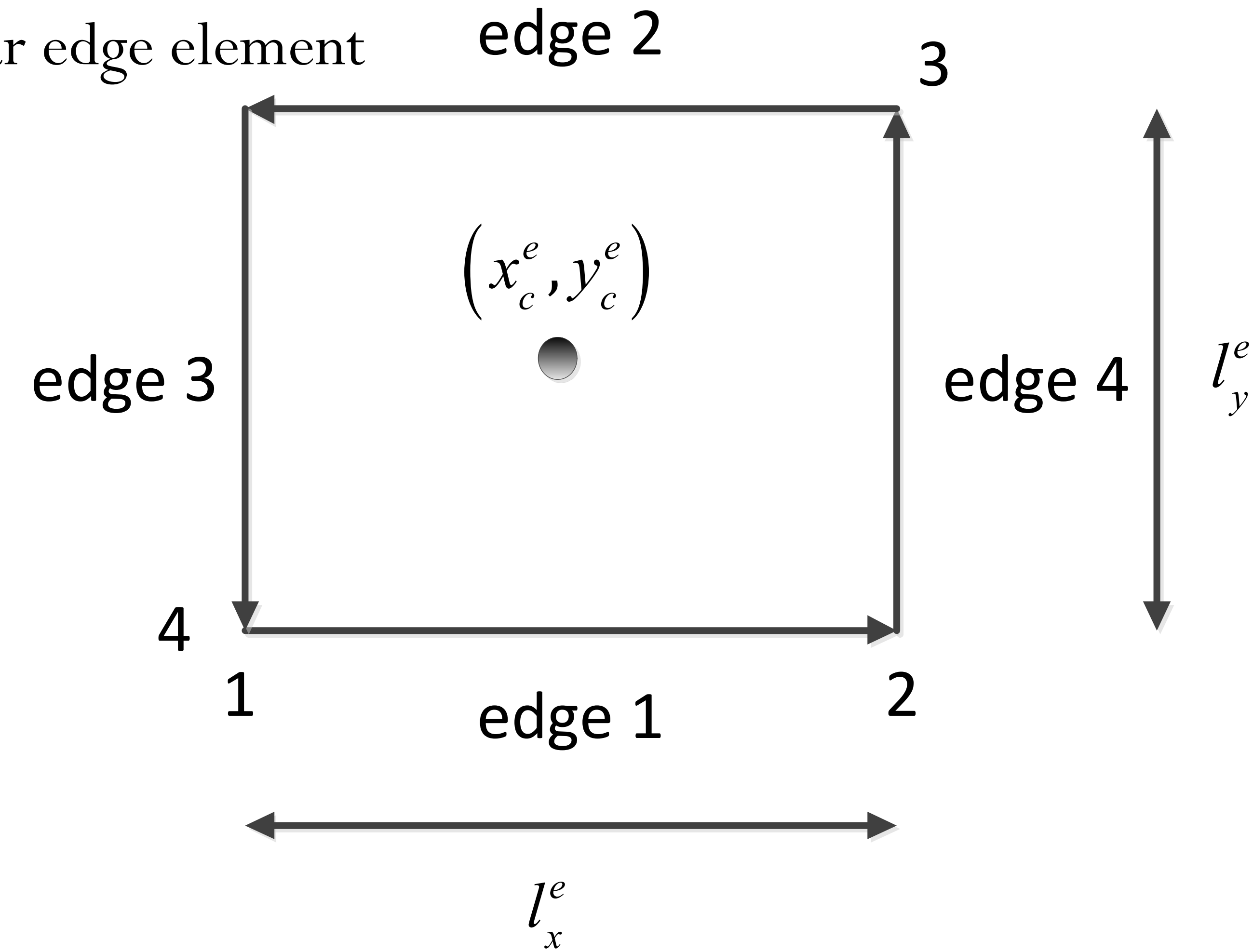
- **Vector edge elements**
- Field can be approximated as

$$\vec{E}_e \approx \sum_{i=1}^{4} \vec{N}_i^e E_i^e$$

- Directed edge elements
  - We have considered finding potentials which is scalar
  - We also find fields,
    - fields are vectors,
      - so logical to use vector shape or interpolation functions

Introduction

- Rectangular edge element



Introduction

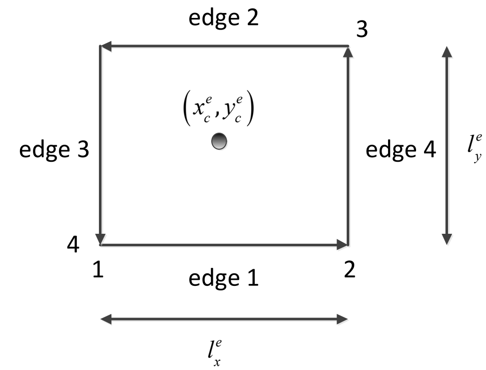
- where $\vec{N}_i^e$ are vector basis function which are defined as

$$\vec{N}_1^e = \frac{1}{l_y^e}\left(y_c^e - y + \frac{l_y^e}{2} \right)\hat{x} \qquad \vec{N}_2^e = \frac{1}{l_y^e}\left(y - y_c^e + \frac{l_y^e}{2} \right)\hat{x}$$

$$\vec{N}_3^e = \frac{1}{l_x^e}\left(x_c^e - x + \frac{l_x^e}{2} \right)\hat{y} \qquad \vec{N}_4^e = \frac{1}{l_x^e}\left(x - x_c^e + \frac{l_x^e}{2} \right)\hat{y}$$

Introduction

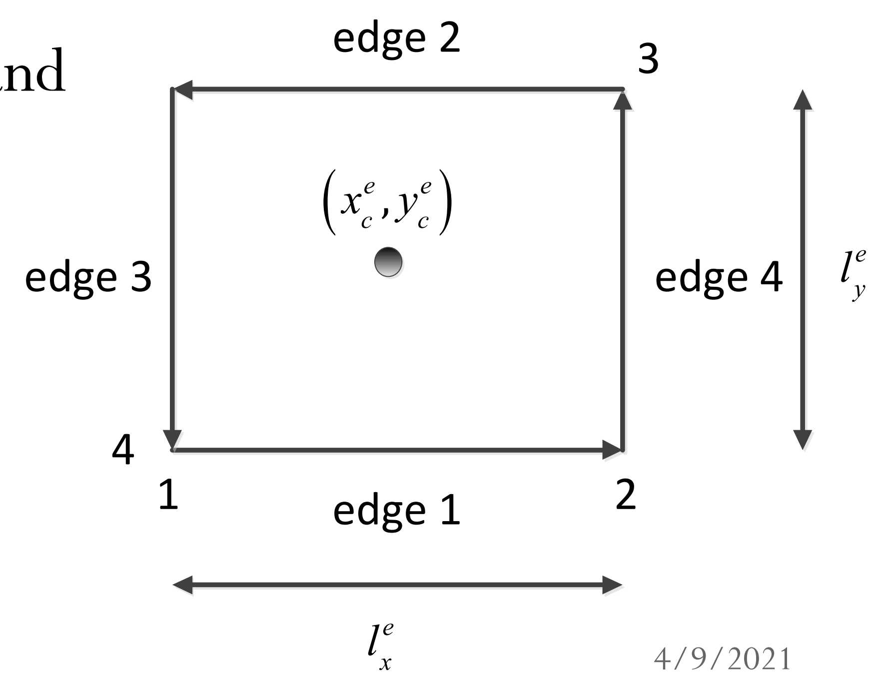
$$\vec{N}_1^e = \frac{1}{l_y^e}\left(y_c^e - y + \frac{l_y^e}{2}\right)\hat{x}$$



- Note that $\vec{N}_1^e$ is zero on edge 2
- On edges 3 and 4 it increases linearly from the top to the bottom
- It is purely normal along these edges
- These basis functions provide a mixed-order approximation of the field
- On the edges the approximation is constant tangentially (along edge 1) and linear normally (edge 3 and edge 4)
- In short from these elements are called CT/LN elements

Introduction

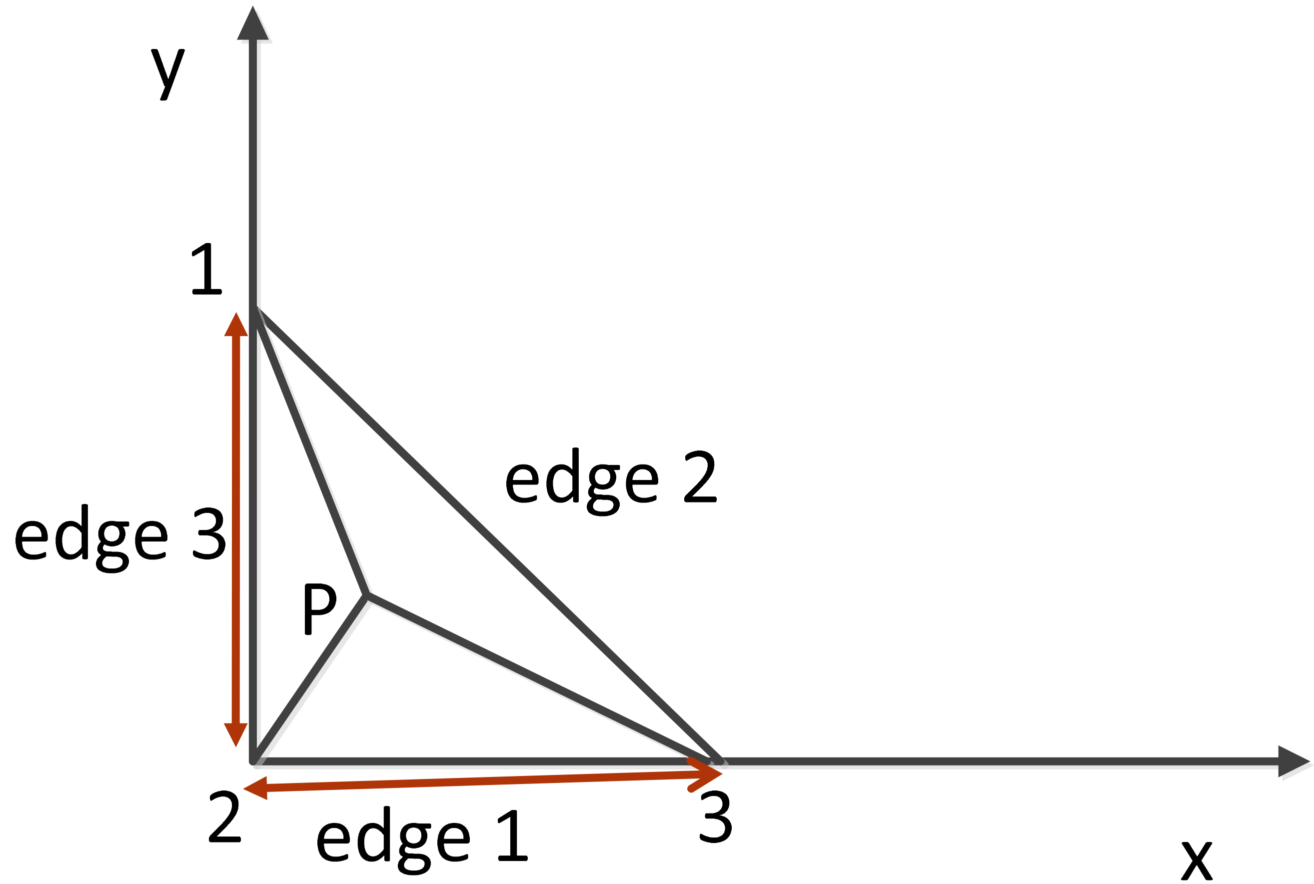
- Field in a rectangular element can be interpolated as
- $\vec{E}^e(x,y) = E_1^e\vec{N}_1^e + E_2^e\vec{N}_2^e + E_3^e\vec{N}_3^e + E_4^e\vec{N}_4^e$
- where E_1^e is the tangential
- component of the field along
- the edge 1 of the element e and
- $\vec{N}_1^e$ is the interpolation
- or basis function



Introduction

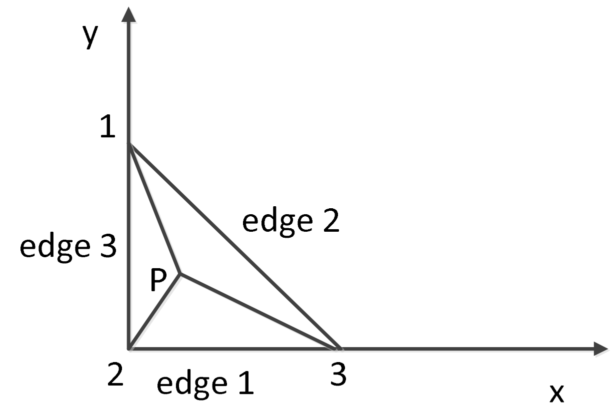
- These properties of CT/LN permit enforcing tangential continuity (such as in electric field and it is called 1-form) without affecting the normal components (in 2-form for magnetic field and current density it is CN/LT)
- It is precisely the boundary conditions required by electric and magnetic fields
- **Vector elements on triangles- the Whitney element**
- Consider a right-angled triangle of unit length along x-axis and y-axis
- Even though we derive Whitney function for right-angled triangle it is true for any triangle (D.B. Davidson's book)

Introduction



- Fig. Right-angled triangle [Whitney element {P(x,y)}]

Introduction

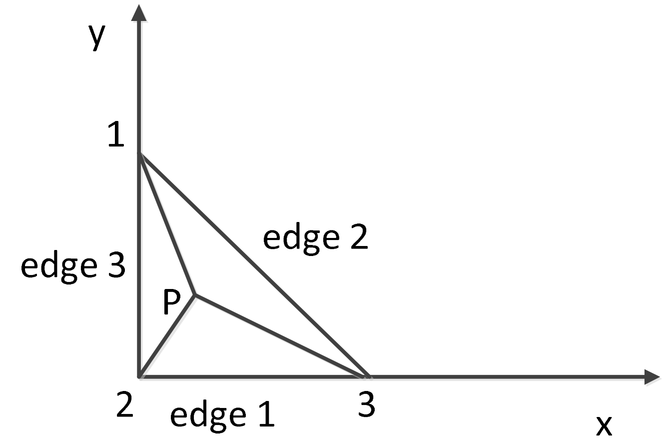


- The simplex coordinates are

$$\lambda_1 = \frac{area_{\Delta P23}}{area_{\Delta 123}} = \frac{\frac{1}{2} base \times height}{\frac{1}{2}} = y$$

- Note that area of triangle 123 is ½, and the base of the triangle P23 is unity and height is y

Introduction



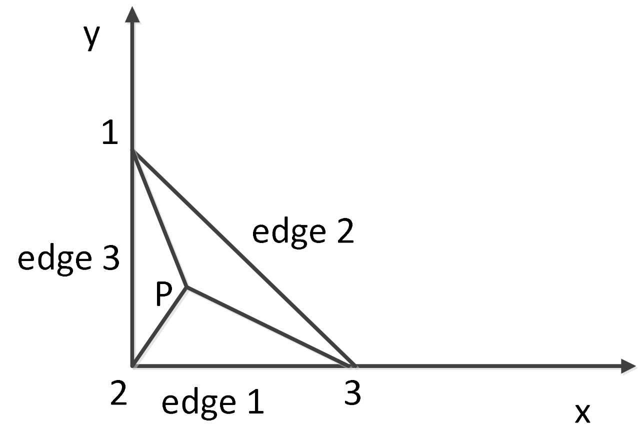
- Similarly

$$\lambda_3 = \frac{area_{\Delta P12}}{area_{\Delta 123}} = \frac{\frac{1}{2} base \times height}{\frac{1}{2}} = x$$

- and

$$\because \sum_{i=1}^{3} \lambda_i = 1; \lambda_2 = 1 - \lambda_3 - \lambda_1 = 1 - (x + y)$$

Introduction



- Their gradients

$$\nabla\lambda_1 = \hat{y}; \nabla\lambda_2 = -\hat{x} - \hat{y}; \nabla\lambda_3 = \hat{x}$$

- Note that $\nabla\lambda_1$ is normal to edge 1 (which was along x-axis) and similarly $\nabla\lambda_2$ and $\nabla\lambda_3$ are normal to edges 2 and 3 respectively
- Now the Whitney elements function for first order (higher order terms may also exist) can be expressed as

$$\vec{w}_{ij} = \lambda_i \nabla\lambda_j - \lambda_j \nabla\lambda_i$$

Introduction

- For instance,

$$\vec{N}_1 = \vec{w}_{23} = \lambda_2 \nabla \lambda_3 - \lambda_3 \nabla \lambda_2$$
$$= (1 - x - y)\hat{x} - x(-\hat{x} - \hat{y})$$
$$= (1 - y)\hat{x} + x\hat{y}$$

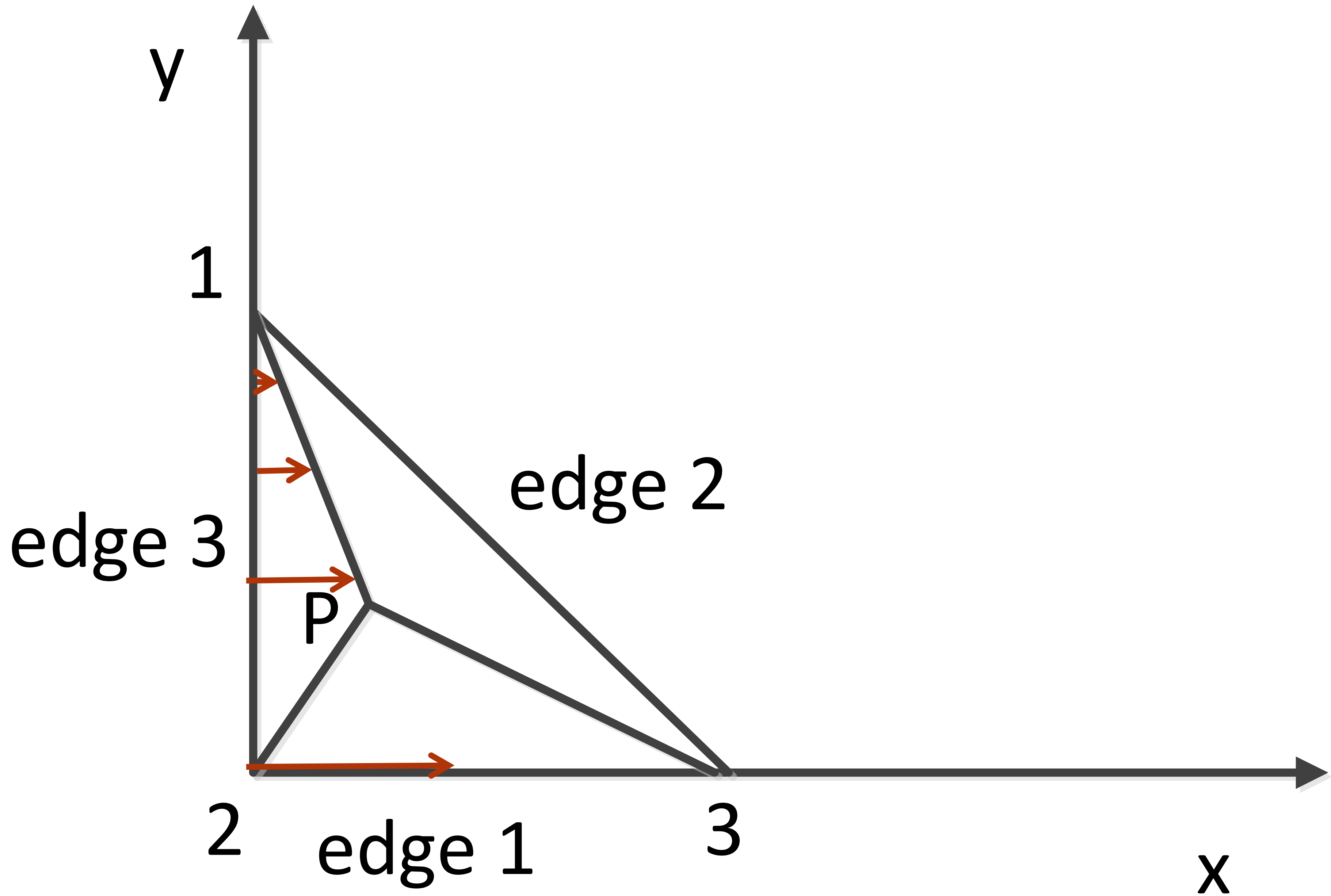
- Similarly,

$$\vec{N}_2 = \vec{w}_{31} = \lambda_3 \nabla \lambda_1 - \lambda_1 \nabla \lambda_3$$
$$= x\hat{y} - y\hat{x}$$

$$\vec{N}_3 = \vec{w}_{12} = \lambda_1 \nabla \lambda_2 - \lambda_2 \nabla \lambda_1$$
$$= -y\hat{x} + (-1 + x)\hat{y}$$

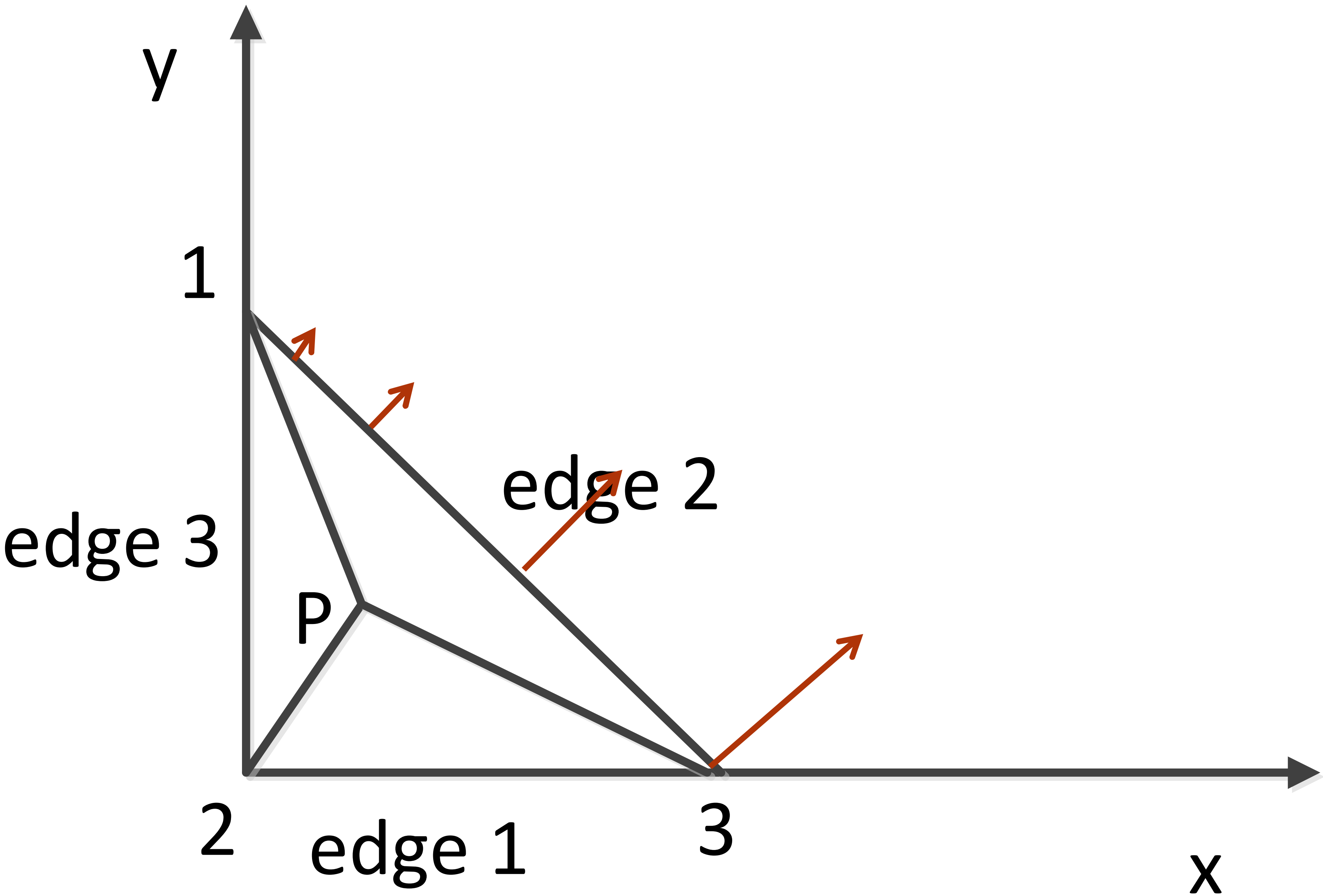
Introduction

- Interpretations of $\vec{N}_1 = \vec{w}_{23} = (1-y)\hat{x} + x\hat{y}$
- Along edge 3 (along y-axis, x=0) the function is purely normal
- It increases linearly from node 1 (x=0) to node 2 (x=0) along edge 3


y
1
edge 3
edge 2
P
2
edge 1
3
x

Introduction

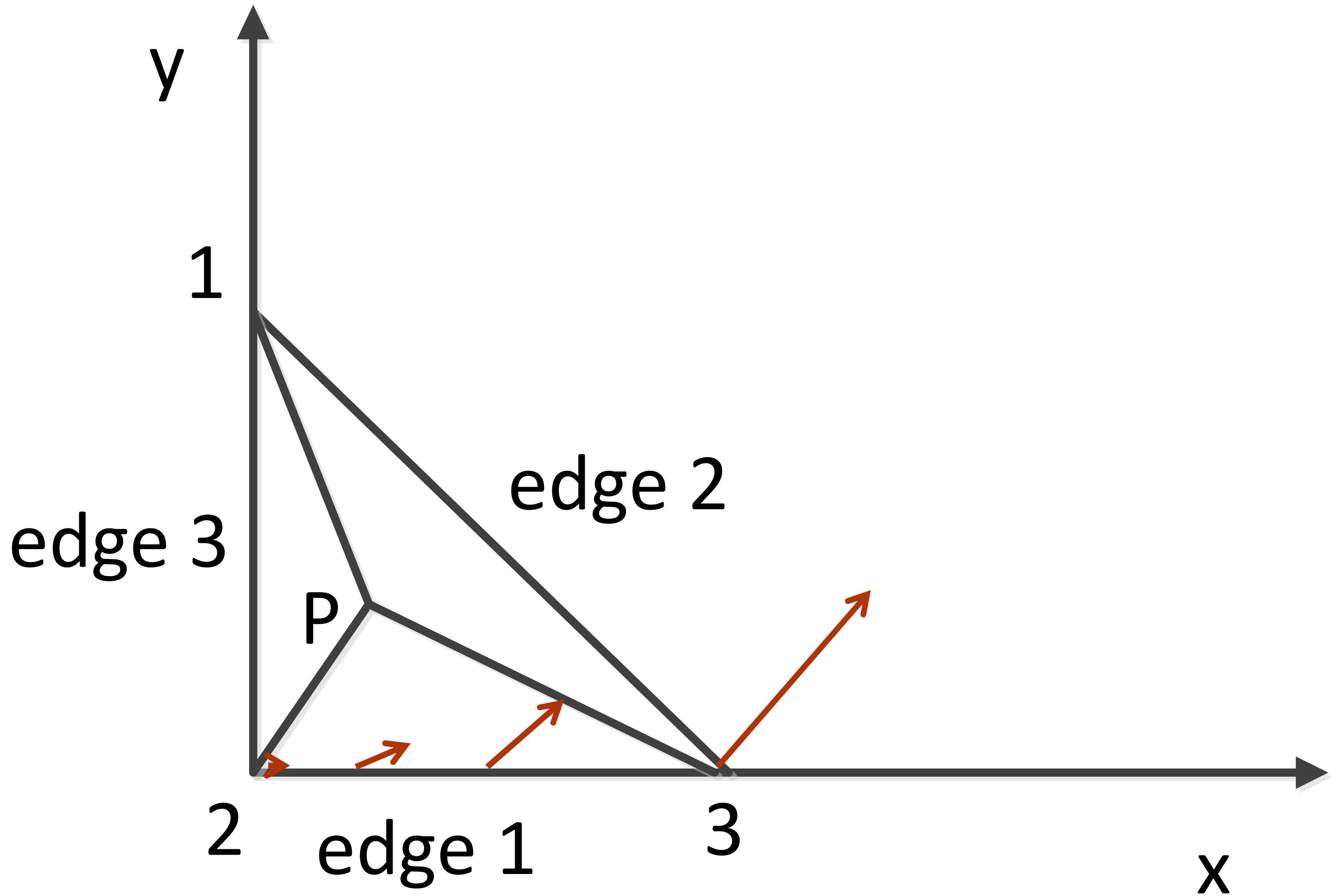
- Interpretations of $\vec{N}_1 = \vec{w}_{23} = \left(1 - y\right)\hat{x} + x\hat{y}$
- Along edge 2 (hypotenuse of the right angled triangle) the function is purely normal
- It also increases linearly from node 1 (x=0) to node 3 (y=0) along edge 2



y
1
edge 2
edge 3
P
2
edge 1
3
x

Introduction

- Interpretations of $\vec{N}_1 = \vec{w}_{23} = (1-y)\hat{x} + x\hat{y}$
- Along edge 1 (y=0), it has both tangential (*it is constant and equal to 1*)
- and normal (*it is linearly increasing with x*) components
- Divergence of Whitney function is also zero


y
1
edge 3
P
edge 2
2
edge 1
3
x

Introduction

- Thus on this edge tangential x-component is constant
- But the normal y-component keeps on increasing linearly
- Summary
- $\vec{N}_1 = \vec{w}_{23}$ is a basis function with a constant tangential component and
  - linearly increasing normal component
- These Whitney basis functions has the same mixed order CT/LN behaviour
  - as the rectangular element studied earlier

Introduction

- Whitney elements are used in HFSS
- Field in a triangular element can be interpolated as
- $\vec{E}^e(x,y) = E_1^e \vec{N}_1^e + E_2^e \vec{N}_2^e + E_3^e \vec{N}_3^e$
- where E_1^e is the tangential
- component of the field along
- the edge 1 of the element e and
- $\vec{N}_1^e$ is the interpolation
- or basis function