Programming Exercise 2 (In-class)

- Write a MATLAB program to find the current distribution and input impedance of a centre-fed dipole
  - Hallen's equation (Exact Kernel)

$$\frac{1}{4\pi}\int_{-h}^{h} I_z K dz' = \xi_0 \cos(kz) - \frac{jV_0}{2\eta}\sin(k|z|)$$

$$K = \int_{-\pi}^{\pi} \frac{e^{-jk\sqrt{4a^2\sin^2(\phi/2)+(z-z')^2}}}{2\pi\sqrt{4a^2\sin^2(\phi/2)+(z-z')^2}}\, d\phi$$

  - Piece-wise constant function
  - Magnetic frill source
  - Point-matching method

Dipole (Hallen's Equation)

- Exact Kernel
- Define z coordinate as symbol
- Radius of cylindrical dipole, $a = 0.00065$ m
- Half-length dipole, $h = 0.005$ m
- Operation frequency, $f = 1$ GHz
- Light velocity, $c = 3\times10^8$ m/s

Dipole (Hallen's Equation)

- Permittivity of free space, $\varepsilon_o = 8.8541878176 \times 10^{-12}$ F/m
- Permeability of free space, $\mu_o = 4\pi \times 10^{-7}$ H/m
- Intrinsic impedance of free space, $\eta = 377\ \Omega$
- Relative permittivity, $\varepsilon_r = 1$ (air)
- Angular frequency, $\omega = 2\pi f$
- Propagation constant, $k = 2\pi f (\varepsilon_r)^{0.5} / c$

Dipole (Hallen's Equation)

- Number of segment (pulses), $N = 9$ (odd number)
- Segment length, $\Delta z = 2h/N$
- Match point, z_m at segment m
- $z_m = (m - 0.5)\Delta z - h \qquad m = 1, 2, 3, \ldots, N$
- Discretization of A_{mn}

$$K = \int_{-\pi}^{\pi} \frac{e^{-jk\sqrt{4a^2\sin^2(\phi/2)+(z-z')^2}}}{2\pi\sqrt{4a^2\sin^2(\phi/2)+(z-z')^2}}\, d\phi$$

$$\boxed{\frac{1}{4\pi}\int_{-h}^{h} I_z K dz'} = \xi_0 \cos(kz) - \frac{jV_0}{2\eta}\sin(k|z|)$$

$$A_{mn} = \frac{1}{4\pi}\times \int_{z_n-\Delta z/2}^{z_n+\Delta z/2}\int_{-\pi}^{\pi} \frac{e^{-jk\sqrt{4a^2\sin^2(\phi/2)+(z_m-z')^2}}}{2\pi\sqrt{4a^2\sin^2(\phi/2)+(z_m-z')^2}}\, d\phi\, dz'$$

Dipole (Hallen's Equation)

- The double integral A_{mn} can be solved by the numerical integration (*integral2* function in MATLAB toolbox) for point z_1 at $m = 1$ segment.
- $[A_{11}\ ,\ A_{12}\ , A_{13}\ ,\ A_{14}\ , A_{15}\ ,\ A_{16}\ , A_{17}\ ,\ A_{18}\ , A_{19}\ ]$

Dipole (Hallen's Equation)

- To create a matrix:

$$\begin{bmatrix} A_{11}/2 & A_{12} & A_{13} & A_{14} & \cdots & A_{19} \\ 0 & A_{11}/2 & A_{12} & A_{13} & \cdots & A_{18} \\ 0 & 0 & A_{11}/2 & A_{12} & \cdots & A_{17} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & A_{11}/2 \end{bmatrix}$$

Dipole (Hallen's Equation)

- To create a complete matrix, $[A_{mn}]$:

$$\begin{bmatrix} A_{11} & A_{12} & A_{13} & A_{14} & \cdots & A_{19} \\ A_{12} & A_{11} & A_{12} & A_{13} & \cdots & A_{18} \\ A_{13} & A_{12} & A_{11} & A_{12} & \cdots & A_{17} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ A_{19} & A_{18} & A_{17} & A_{16} & \cdots & A_{11} \end{bmatrix}$$

Dipole (Hallen's Equation)

- Discretization B_m and C_m.

$$\frac{1}{4\pi}\int_{-h}^{h} I_z K dz' = \xi_0 \cos(kz) - \frac{jV_0}{2\eta}\sin(k|z|)$$

- Create transpose matrix, $[B_m]$ and matrix, $[C_m]$:

$$B_m = \cos(kz_m) = \begin{bmatrix} \cos(kz_1) \\ \cos(kz_2) \\ \vdots \\ \cos(kz_9) \end{bmatrix}$$

$$C_m = -\frac{j}{2\eta}\sin(k|z_m|) = -\frac{j}{2\eta}\begin{bmatrix} \sin(k|z_1|) \\ \sin(k|z_2|) \\ \vdots \\ \sin(k|z_9|) \end{bmatrix}$$

Dipole (Hallen's Equation)

- Thus, the discretization of can be written in matrix form as: $[I_n][A_{mn}] = \xi_o [B_m] + [C_m]$
- where $A_{11} = A_{22} = A_{33} = \ldots = A_{99}$.
- The current, I_n along the dipole can be determined by inverting the matrix, $[A_{mn}]$:
- $$[I_n] = \xi_o [A_{mn}]^{-1} [B_m] + [A_{mn}]^{-1} [C_m] \\ = \xi_o [D_n] + [S_n]$$

Dipole (Hallen's Equation)

- where $$[D_n] = [A_{mn}]^{-1}[B_m]$$

$$[S_n] = [A_{mn}]^{-1}[C_m]$$

- As it is known that the current, $I_9 = 0$ at both ends ($z = \pm h$) of the dipole vanishes, thus

$$\xi_o [D_9] + [S_9] = 0$$

Dipole (Hallen's Equation)

- The value of ξ_o is determined by enforcing the boundary conditions at the ends of the dipole, yields

$$\xi_o = -\frac{[S_9]}{[D_9]}$$

Programming Exercise 4 (Homework)

- Write a MATLAB program to find the current distribution and input impedance of a centre-fed dipole
  - Hallen's equation (Reduced Kernel)

$$\frac{1}{4\pi}\int_{-h}^{h} I_z K dz' = \xi_0 \cos(kz) - \frac{jV_0}{2\eta}\sin(k|z|)$$

$$K = \int_{-\pi}^{\pi} \frac{e^{-jk\sqrt{a^2+(z-z')^2}}}{2\pi\sqrt{a^2+(z-z')^2}} d\phi$$

  - Piece-wise constant function
  - Magnetic frill source
  - Point-matching method

Dipole (Hallen's Equation)

- Define z coordinate as symbol
- Radius cylindrical dipole, $a = 0.00065$ m
- Half-length dipole, $h = 0.005$ m
- Operation frequency, $f = 1$ GHz
- Light velocity, $c = 3\times10^8$ m/s
- Permittivity of free space, $\varepsilon_o = 8.8541878176\times10^{-12}$ F/m
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- Intrinsic impedance of free space, $\eta = 377\ \Omega$

Dipole (Hallen's Equation)

- Relative permittivity, $\varepsilon_r = 1$ (air)
- Angular frequency, $\omega = 2\pi f$
- Propagation constant, $k = 2\pi f (\varepsilon_r)^{0.5} / c$
- Number segment (pulses), $N = 9$ (odd number)
- Segment length, $\Delta z = 2h / N$
- Match point, z_m at segment m
- $z_m = (m - 0.5)\Delta z - h \qquad m = 1, 2, 3, \ldots, N$

Dipole (Hallen's Equation)

- Discretization of A_{mn}

$$A_{mn} = \frac{1}{4\pi}\int_{z_n-\Delta z/2}^{z_n+\Delta z/2} \frac{e^{-jk\sqrt{a^2+(z_m-z')^2}}}{\sqrt{a^2+(z_m-z')^2}}dz'$$

- The double integral A_{mn} can be solved by numerical integration (*integral* function in MATLAB toolbox) for point z_1 at $m = 1$ segment.
- $[A_{11}, A_{12}, A_{13}, A_{14}, A_{15}, A_{16}, A_{17}, A_{18}, A_{19}]$

Dipole (Hallen's Equation)

- To create a matrix: we assume that $z_m = z_n$ and solve the integral from $z_n - (\Delta z/2)$ to $z_n + (\Delta z/2)$ and yield the matrix:

$$\begin{bmatrix} A_{11}/2 & A_{12} & A_{13} & A_{14} & \cdots & A_{19} \\ 0 & A_{11}/2 & A_{12} & A_{13} & \cdots & A_{18} \\ 0 & 0 & A_{11}/2 & A_{12} & \cdots & A_{17} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & A_{11}/2 \end{bmatrix}$$

Dipole (Hallen's Equation)

- To create a complete matrix, $[A_{mn}]$:

$$\begin{bmatrix} A_{11} & A_{12} & A_{13} & A_{14} & \cdots & A_{19} \\ A_{12} & A_{11} & A_{12} & A_{13} & \cdots & A_{18} \\ A_{13} & A_{12} & A_{11} & A_{12} & \cdots & A_{17} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ A_{19} & A_{18} & A_{17} & A_{16} & \cdots & A_{11} \end{bmatrix}$$

Dipole (Hallen's Equation)

- Discretization C_m and B_m. Create transpose matrix, $[C_m]$ and matrix, $[B_m]$:

$$C_m = -\frac{j}{2\eta}\sin\left(k\left|z_m\right|\right) = -\frac{j}{2\eta}\begin{bmatrix} \sin\left(k\left|z_1\right|\right) \\ \sin\left(k\left|z_2\right|\right) \\ \vdots \\ \sin\left(k\left|z_9\right|\right) \end{bmatrix}$$

$$B_m = \cos\left(kz_m\right) = \begin{bmatrix} \cos\left(kz_1\right) \\ \cos\left(kz_2\right) \\ \vdots \\ \cos\left(kz_9\right) \end{bmatrix}$$

Dipole (Hallen's Equation)

- Thus, the discretization can be written in matrix form as:

$$[I_n][A_{mn}] = \xi_o [B_m] + [C_m]$$

- where $A_{11} = A_{22} = A_{33} = \ldots = A_{99}$. The current, I_n along the dipole can be determined by inverting the matrix, $[A_{mn}]$:

$$[I_n] = \xi_o [A_{mn}]^{-1} [B_m] + [A_{mn}]^{-1} [C_m]$$
$$= \xi_o [D_n] + [S_n]$$

Dipole (Hallen's Equation)

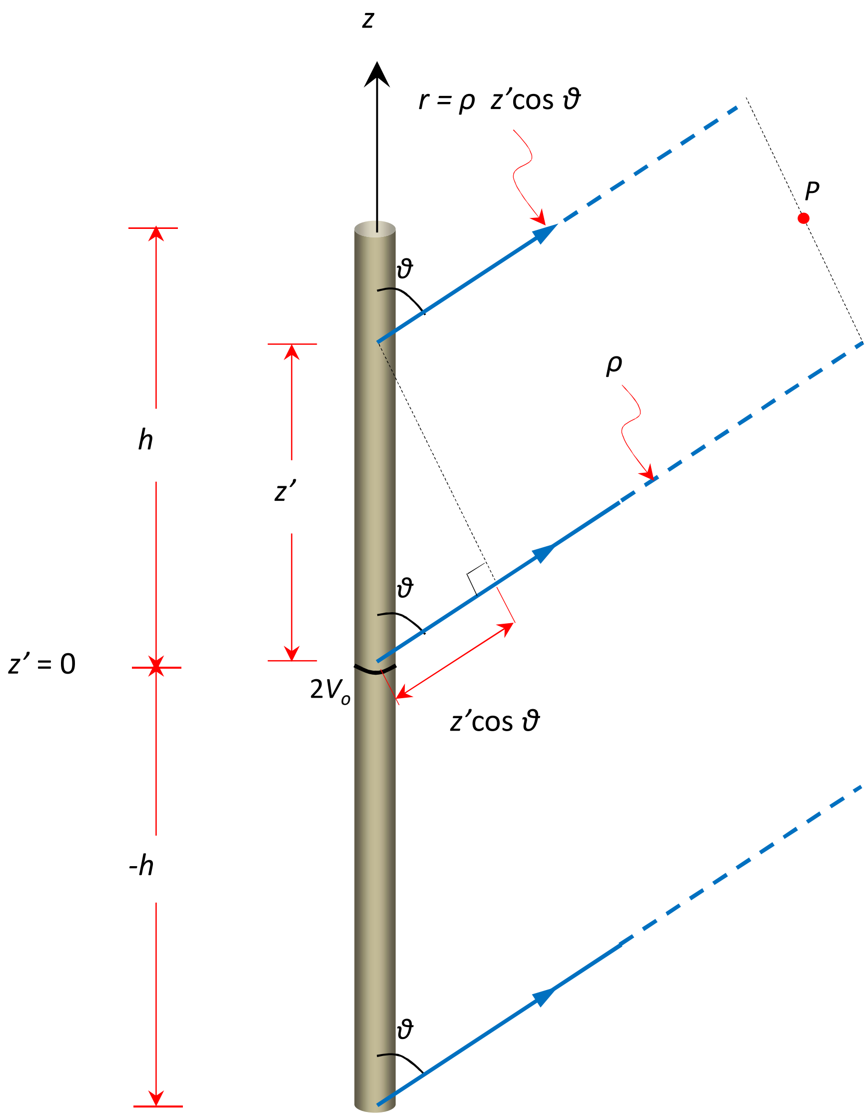
- where

$$[D_n] = [A_{mn}]^{-1}[B_m] \quad [S_n] = [A_{mn}]^{-1}[C_m]$$

- As it is known that the current, $I_9 = 0$ at both ends ($z = \pm h$) of the dipole vanishes, thus $\xi_o[D_9] + [S_9] = 0$

- The value of ξ_o is determined by enforcing the boundary conditions at the ends of the dipole:

$$\xi_o = -\frac{[S_9]}{[D_9]}$$



$$r \approx \rho - z' \cos\theta$$

Fig. *Parallel radiated line approximation for far field analysis*

Far-field radiation of dipole

$$A_z = \frac{1}{4\pi}\int_{-h}^{h} I_z \frac{e^{-jkr}}{r} dz'$$

$$A_z \cong \frac{1}{4\pi}\int_{-h}^{h} I_z \frac{e^{-jk(\rho - z'\cos\theta)}}{\rho} dz'$$

$$= I_z \frac{e^{-jk\rho}}{4\pi\rho}\int_{-h}^{h} e^{jkz'\cos\theta} dz'$$

$$E_\theta = -j\omega\mu A_\theta \hat{\theta}$$

$$= j\omega\mu \sin\theta A_z \hat{\theta}$$

Far-field radiation of dipole

$$E_\theta = j\omega\mu \sin\theta I_z \frac{e^{-jk\rho}}{4\pi\rho}\int_{-h}^{h} e^{jkz'\cos\theta} dz'\hat{\theta}$$

$$= j\omega\mu \sin\theta I_z \frac{e^{-jk\rho}}{4\pi\rho}\left(\frac{e^{jkh\cos\theta} - e^{-jkh\cos\theta}}{jk\cos\theta}\right)\hat{\theta}$$

$$= j\omega\mu \sin\theta I_z h \frac{e^{-jk\rho}}{2\pi\rho}\frac{\sin\left(kh\cos\theta\right)}{kh\cos\theta}\hat{\theta}$$