MoM Advances

Spectral domain MoM

Electric type dyadic Green's functions

$$\vec{E}\left(\vec{r}\right)=\int_{V}\overleftrightarrow{G}_{EJ}\left(\vec{r},\vec{r}^{'}\right)\bullet\vec{J}_{e}\left(\vec{r}^{'}\right)d\vec{r}^{'}$$

For 3-D current source, the electric dyadic Green's function will have nine components

$$\overleftrightarrow{G}_{EJ}\left(x,y,x;x^{'},y^{'},x^{'}\right)=G_{EJ}^{xx}\hat{x}\hat{x}+G_{EJ}^{xy}\hat{x}\hat{y}+G_{EJ}^{xz}\hat{x}\hat{z}+G_{EJ}^{yx}\hat{y}\hat{x}+G_{EJ}^{yy}\hat{y}\hat{y}+G_{EJ}^{yz}\hat{y}\hat{z}+G_{EJ}^{zx}\hat{z}\hat{x}+G_{EJ}^{zy}\hat{z}\hat{y}+G_{EJ}^{zz}\hat{z}\hat{z}$$

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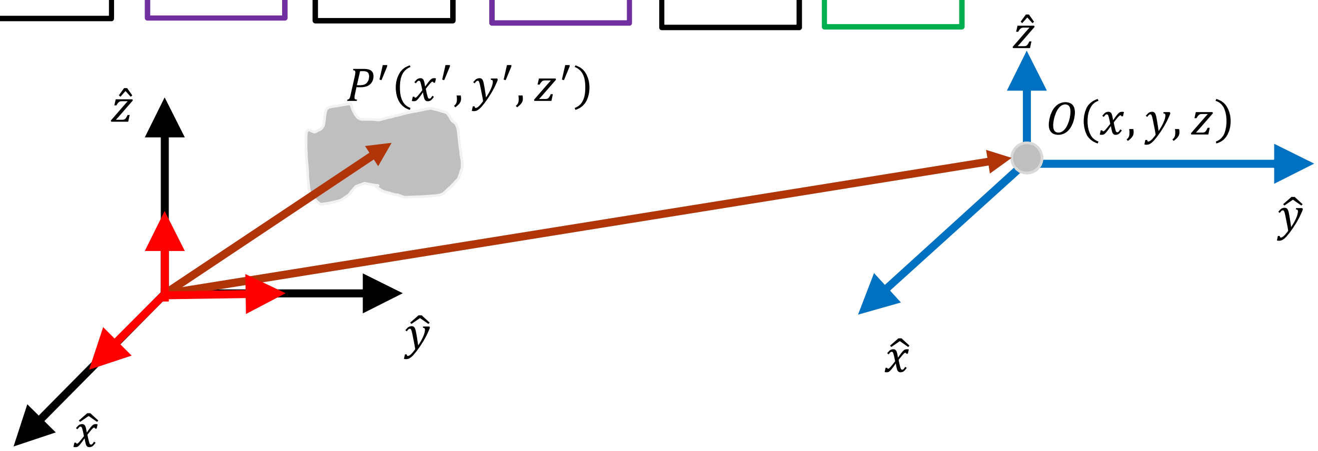
- We are interested in first finding the spectral domain (Fourier transform domain) version
  - of the above spatial domain dyadic Green's functions which will have nine components as follows

$$\tilde{\overleftrightarrow{G}}_{EJ}\left(\vec{k}_t, z, z'\right) = \tilde{G}_{EJ}^{xx}\hat{x}\hat{x} + \tilde{G}_{EJ}^{xy}\hat{x}\hat{y} + \tilde{G}_{EJ}^{yx}\hat{y}\hat{x} + \tilde{G}_{EJ}^{yy}\hat{y}\hat{y} + \tilde{G}_{EJ}^{zx}\hat{z}\hat{x} + \tilde{G}_{EJ}^{xz}\hat{x}\hat{z} + \tilde{G}_{EJ}^{zy}\hat{z}\hat{y} + \tilde{G}_{EJ}^{yz}\hat{y}\hat{z} + \tilde{G}_{EJ}^{zz}\hat{z}\hat{z}$$



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- Usually, one neglects the metal thickness in the analysis which is applicable for perfect electric conductor
- For horizontal current excitation in x-y plane, removing the Green's functions which are dependent on the vertical current excitation component,
- only four Green's function components need to be determined

$$\tilde{\overleftrightarrow{G}}_{EJ}\left(\vec{k}_t, z, z'\right) = \tilde{G}_{EJ}^{xx}\hat{x}\hat{x} + \tilde{G}_{EJ}^{xy}\hat{x}\hat{y} + \tilde{G}_{EJ}^{yx}\hat{y}\hat{x} + \tilde{G}_{EJ}^{yy}\hat{y}\hat{y}$$

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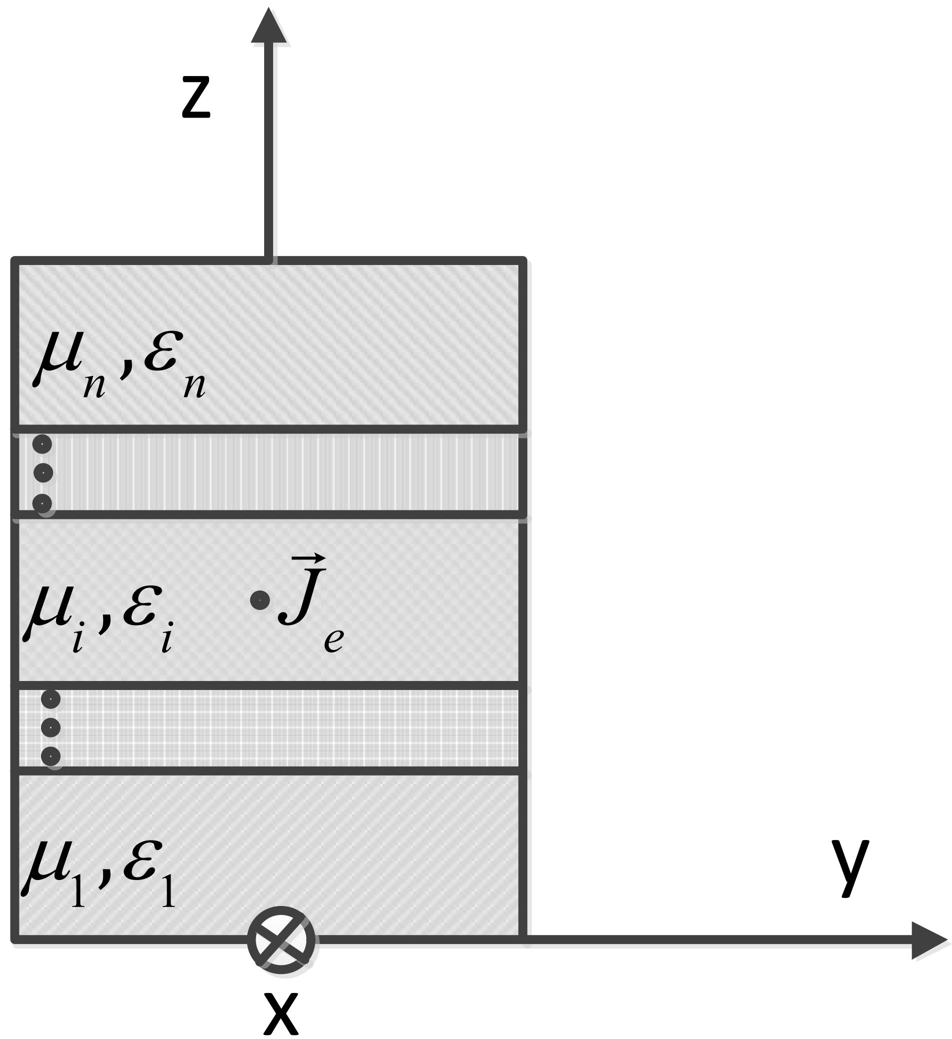
- Fig. One dimensional multilayer structures (n layer of dielectric and magnetic media) excited by three dimensional electric point source



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- The four components of the spectral electric type transverse dyadic Green's function
- can be obtained from the voltages on the transverse electric (TE)/ transverse magnetic (TM) circuit models shown in Fig. as

$$\tilde{\overleftrightarrow{G}}_{EJ}\left(\vec{k}_t, z, z'\right) = -V^{TE}_{horizon}\left(z, z'\right)\left(\hat{k}_t \times \hat{z}\right)\left(\hat{k}_t \times \hat{z}\right) - V^{TM}_{horizon}\left(z, z'\right)\left(\hat{k}_t\right)\left(\hat{k}_t\right)$$

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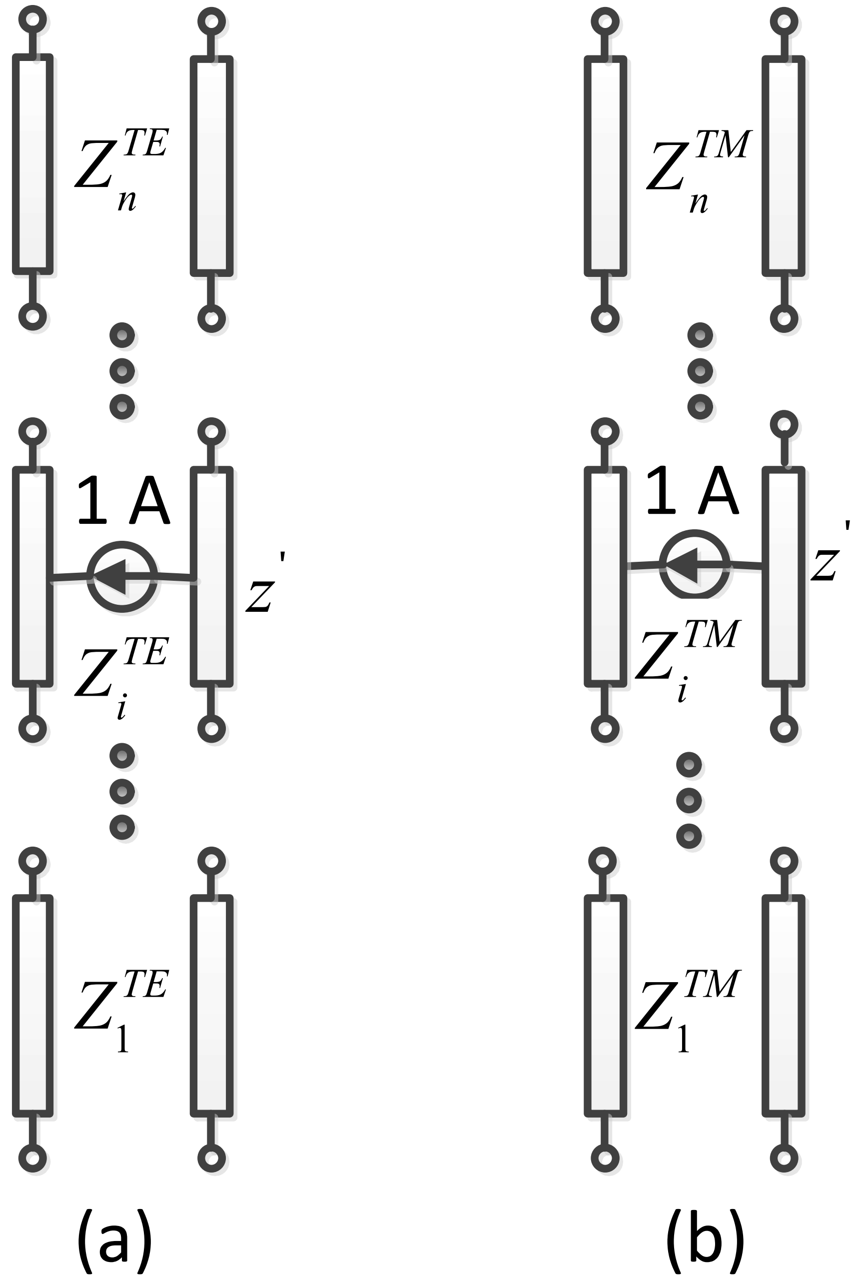
- Fig. Equivalent TE/TM circuits models for one-dimensional multilayer structure (i=1,2,3,…,n) excited by a two-dimensional horizontal (transverse) electric point source (a) TE modes and (b) TM modes



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- using vector and dyadic analysis, we can simplify the above equation as follows

$$\tilde{\overleftrightarrow{G}}_{EJ}\left(\vec{k}_t,z,z'\right)$$

$$=-V_{horizon}^{TE}\left(z,z'\right)\left(\frac{k_x\hat{x}+k_y\hat{y}}{\sqrt{k_x^2+k_y^2}}\times\hat{z}\right)\left(\frac{k_x\hat{x}+k_y\hat{y}}{\sqrt{k_x^2+k_y^2}}\times\hat{z}\right)-V_{horizon}^{TM}\left(z,z'\right)\left(\frac{k_x\hat{x}+k_y\hat{y}}{\sqrt{k_x^2+k_y^2}}\right)\left(\frac{k_x\hat{x}+k_y\hat{y}}{\sqrt{k_x^2+k_y^2}}\right)$$

$$=-V_{horizon}^{TE}\left(z,z'\right)\left(\frac{k_y\hat{x}-k_x\hat{y}}{\sqrt{k_x^2+k_y^2}}\right)\left(\frac{k_y\hat{x}-k_x\hat{y}}{\sqrt{k_x^2+k_y^2}}\right)-V_{horizon}^{TM}\left(z,z'\right)\left(\frac{k_x^2\hat{x}\hat{x}+k_xk_y\hat{x}\hat{y}+k_yk_x\hat{y}\hat{x}+k_y^2\hat{y}\hat{y}}{k_x^2+k_y^2}\right)$$

$$=-V_{horizon}^{TE}\left(z,z'\right)\left(\frac{k_y^2\hat{x}\hat{x}-k_yk_x\left(\hat{x}\hat{y}+\hat{y}\hat{x}\right)+k_x^2\hat{y}\hat{y}}{k_x^2+k_y^2}\right)-V_{horizon}^{TM}\left(z,z'\right)\left(\frac{k_x^2\hat{x}\hat{x}+k_xk_y\left(\hat{x}\hat{y}+\hat{y}\hat{x}\right)+k_y^2\hat{y}\hat{y}}{k_x^2+k_y^2}\right)$$

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- Finally

$$\tilde{\overleftrightarrow{G}}_{EJ}\left(\vec{k}_t,z,z'\right)$$

$$= -\frac{V_{horizon}^{TE}\left(z,z'\right)k_y^2 + V_{horizon}^{TM}\left(z,z'\right)k_x^2}{k_x^2+k_y^2}\hat{x}\hat{x} + \frac{k_x k_y\left(V_{horizon}^{TE}\left(z,z'\right) - V_{horizon}^{TM}\left(z,z'\right)\right)}{k_x^2+k_y^2}\left(\hat{x}\hat{y}+\hat{y}\hat{x}\right)$$

$$-\frac{V_{horizon}^{TE}\left(z,z'\right)k_x^2 + V_{horizon}^{TM}\left(z,z'\right)k_y^2}{k_x^2+k_y^2}\hat{y}\hat{y}$$

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- We have

$$\tilde{G}_{EJ}^{xx}\left(k_x,k_y,z,z^{'}\right) = -\frac{V_{horizon}^{TE}\left(z,z^{'}\right)k_y^2 + V_{horizon}^{TM}\left(z,z^{'}\right)k_x^2}{k_x^2+k_y^2}$$

$$\tilde{G}_{EJ}^{xy}\left(k_x,k_y,z,z^{'}\right) = \tilde{G}_{EJ}^{yx}\left(k_x,k_y,z,z^{'}\right) = \frac{k_x k_y\left(V_{horizon}^{TE}\left(z,z^{'}\right) - V_{horizon}^{TM}\left(z,z^{'}\right)\right)}{k_x^2+k_y^2}$$

$$\tilde{G}_{EJ}^{yy}\left(k_x,k_y,z,z^{'}\right) = -\frac{V_{horizon}^{TE}\left(z,z^{'}\right)k_x^2 + V_{horizon}^{TM}\left(z,z^{'}\right)k_y^2}{k_x^2+k_y^2}$$

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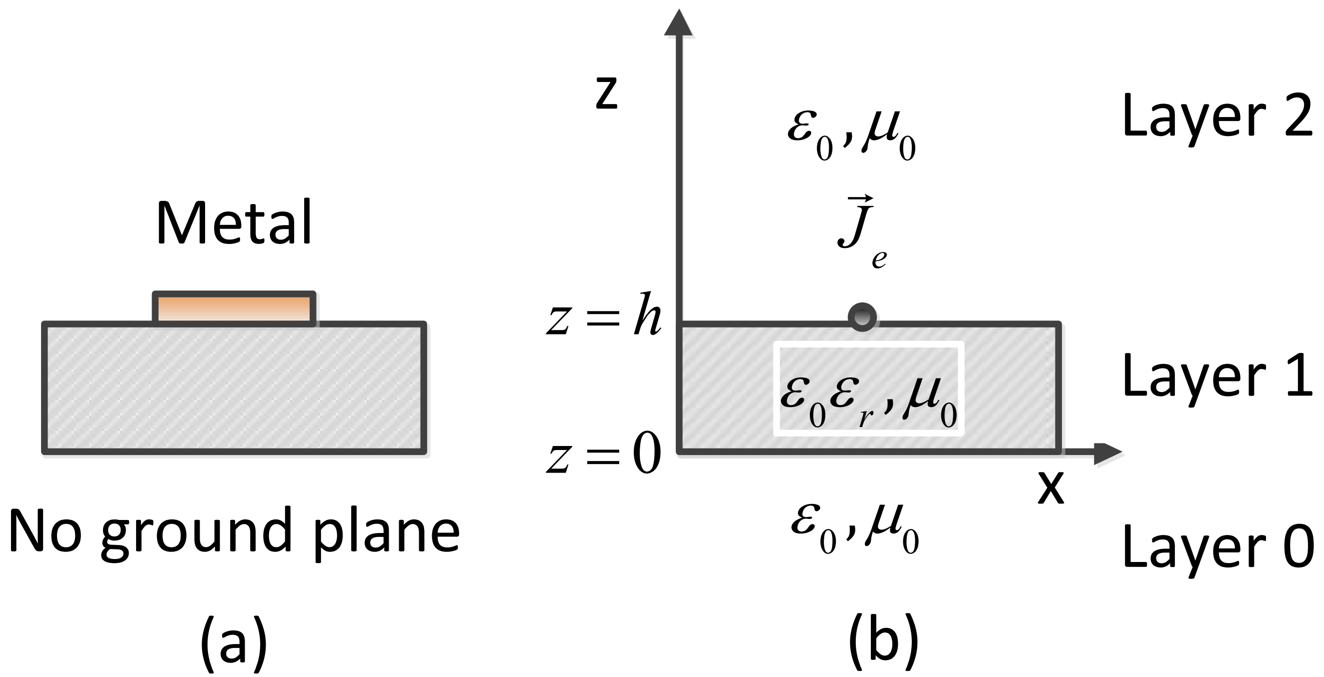
- Fig. (a) Side view of printed monopole antenna (PMA) (b) Transverse (horizontal) electric point source excitation in a three layered structure



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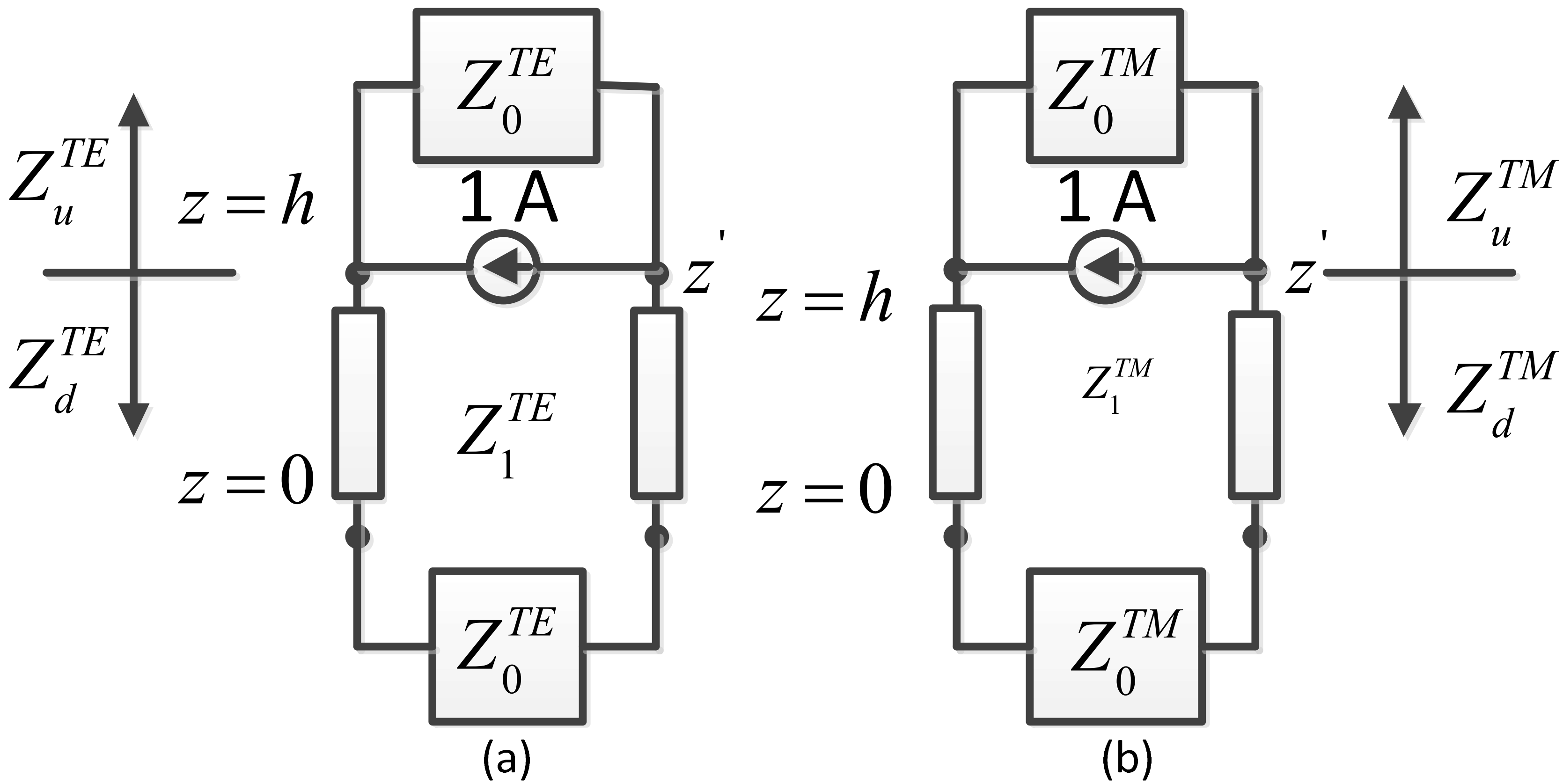

Fig. Equivalent (a) TE and (b) TM circuit models for horizontal (transverse) electric point source excitation

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- From the TE/TM circuit models, the impedance looking up the interface z=h is simply the wave impedance of the free space

$$Z_u^{TE} = Z_0^{TE}; Z_u^{TM} = Z_0^{TM}$$

$$Z_0^{TE} = \frac{k_0 \eta_0}{\beta_0} \qquad Z_0^{TM} = \frac{\beta_0 \eta_0}{k_0}$$

$$\beta_0 = \sqrt{k_0^2 - k_c^2} = \sqrt{k_0^2 - k_x^2 - k_y^2}$$

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- The impedance looking down the interface z=h is simply given by a transmission line of length h terminated with the free space

$$Z_d^{TE} = Z_1^{TE} \frac{Z_0^{TE} + Z_1^{TE} \tanh(\gamma_1 h)}{Z_1^{TE} + Z_0^{TE} \tanh(\gamma_1 h)}; Z_d^{TM} = Z_1^{TM} \frac{Z_0^{TM} + Z_1^{TM} \tanh(\gamma_1 h)}{Z_1^{TM} + Z_0^{TM} \tanh(\gamma_1 h)}$$

$$Z_1^{TE} = \frac{k_1 \eta_1}{\beta_1} = \frac{k_0 \sqrt{\varepsilon_r} \eta_0}{\beta_1 \sqrt{\varepsilon_r}} = \frac{k_0 \eta_0}{\beta_1} \qquad Z_1^{TM} = \frac{\beta_1 \eta_1}{k_1} = \frac{\beta_1 \eta_0}{k_0 \sqrt{\varepsilon_r} \sqrt{\varepsilon_r}} = \frac{\beta_1 \eta_0}{k_0 \varepsilon_r}$$

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- In the above equation the equivalent impedance is the shunt combination of the impedance looking up and down at the interface
- Hence,

$$Z_{eq}^{TE} = \frac{Z_d^{TE} Z_u^{TE}}{Z_d^{TE} + Z_u^{TE}}; Z_{eq}^{TM} = \frac{Z_d^{TM} Z_u^{TM}}{Z_d^{TM} + Z_u^{TM}}$$

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- Finally, we may express the above expression simply

$$Z_{eq}^{TE} = k_0 \eta_0 \frac{1}{D_{TE}} \qquad D_{TE} = \beta_0 + \frac{\left(\beta_0 + j\beta_1 \tan\left(\beta_1 h\right)\right)}{\left(1 + j\dfrac{\beta_0}{\beta_1}\tan\left(\beta_1 h\right)\right)}$$

- Finally,

$$Z_{eq}^{TM} = \frac{\eta_0 \beta_0}{k_0 D_{TM}} \qquad D_{TM} = 1 + \frac{\beta_0 \varepsilon_r \left(\beta_1 + j\varepsilon_r \beta_0 \tan\left(\beta_1 h\right)\right)}{\beta_1 \left(\beta_0 \varepsilon_r + j\beta_1 \tan\left(\beta_1 h\right)\right)}$$

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- Green's function in terms of D_{TE} and D_{TM}

$$\tilde{G}_{EJ}^{xx}\left(\vec{k}_t, z, z' = h\right) = -\frac{1}{\omega\varepsilon_0 k_\rho^2}\left(\frac{k_0^2 k_y^2}{D_{TE}} + \frac{\beta_0 k_x^2}{D_{TM}}\right)$$

$$\tilde{G}_{EJ}^{xy}\left(\vec{k}_t, z, z' = h\right) = \tilde{G}_{EJ}^{yx}\left(\vec{k}_t, z, z' = h\right) = \frac{k_x k_y}{\omega\varepsilon_0 k_\rho^2}\left(\frac{k_0^2}{D_{TE}} - \frac{\beta_0}{D_{TM}}\right)$$

$$\tilde{G}_{EJ}^{yy}\left(\vec{k}_t, z, z' = h\right) = -\frac{1}{\omega\varepsilon_0 k_\rho^2}\left(\frac{k_0^2 k_x^2}{D_{TE}} + \frac{\beta_0 k_y^2}{D_{TM}}\right)$$

$$k_\rho^2 = k_x^2 + k_y^2$$