QUADRATIC AND CUBIC NEWTON MAPS OF RATIONAL FUNCTIONS

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Abstract

The Newton map of a rational function R is given by $z - \frac{R(z)}{R'(z)}$. The following are proved on the dynamics of all quadratic and cubic Newton maps of rational functions.

Let R be a rational function such that its Newton map N_R is quadratic. Then N_R is conformally conjugate either to $N_1(z) = z - \frac{1}{\frac{d_1}{z} + \frac{d_2}{z-1}}$ or to $N_2(z) = z + \frac{1}{\frac{e_1}{z} + \frac{e_2}{z-1}}$ for some natural numbers d_1, d_2, e_1, e_2.

1. If N_R is conformally conjugate to N_1 then the Fatou set of N_R is the union of two completely invariant attracting domains and the Julia set is a Jordan curve.
2. If N_R is conformally conjugate to N_2 then the Fatou set of N_R is equal to a completely invariant attracting domain and the Julia set is totally disconnected.

In other words, the Julia set of N_R is either a Jordan curve or totally disconnected.

If a cubic Newton map is conformally conjugate to a polynomial then its Fatou set is $A \bigcup A^*$, where A^* is the completely invariant attracting domain corresponding to a superattracting fixed point and A is one of the following.

1. A is the union of two invariant attracting domains corresponding to two finite attracting fixed points. The Fatou set is the union of infinitely many components and each is simply connected. The Julia set is a self intersecting closed curve.
2. A is the completey invariant attracting domain corresponding to a finite attracting fixed point. In this case, the Julia set is a Jordan curve.

Keyword: Newton method; Rational functions; Fatou and Julia set.
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