Efficient Pebble Placement for Treasure Hunt in Anonymous Graphs

Abstract

We study the problem of treasure hunt in a graph by a mobile agent. The nodes in the graph are anonymous and the edges at any node v of degree $d(v)$ are labeled arbitrarily as $0, 1, \cdots, deg(v) - 1$. An agent, starting from a node, must find a treasure which is situated in an unknown node at a distance D from the initial position from the agent. The agent only finds out the treasure when it reaches the node where the treasure is present. The time of treasure hunt is the number of edges the agent visits before it finds the treasure. The agent does not have any priori knowledge about the graph or the position of the treasure. An oracle, which knows the graph, the initial position of the agent, and the position of treasure, places some pebbles on the nodes of the graph in order to guide the agent towards the treasure. In this paper, we study the tradeoff between number of pebbles and time for treasure hunt. To be specific, we study the following question.

What is the minimum time for treasure hunt if k number of pebbles are used used?

First, we show an algorithm that uses $O(D \log \Delta)$ pebbles to find the treasure in time $O(D \log \Delta)$, where Δ is the maximum degree of a node and D is the distance from the initial position of the agent to the treasure. We propose a lower bound result which shows that regardless of the number of pebbles, the time $O(D \log \Delta)$ can not be improved. Finally, we show that for any $k \leq D - 1$, the time for treasure hunt using k pebbles is $\Theta(k\Delta^{\frac{D}{k+1}})$.