Physics II: Electromagnetism

PH 102

Lecture 12

Bibhas Ranjan Majhi
Indian Institute of Technology Guwahati

bibhas.majhi@iitg.ac.in

January-May 2019

Electric fields in matter

Plan is to discuss about: Polarisation, Dielectrics,

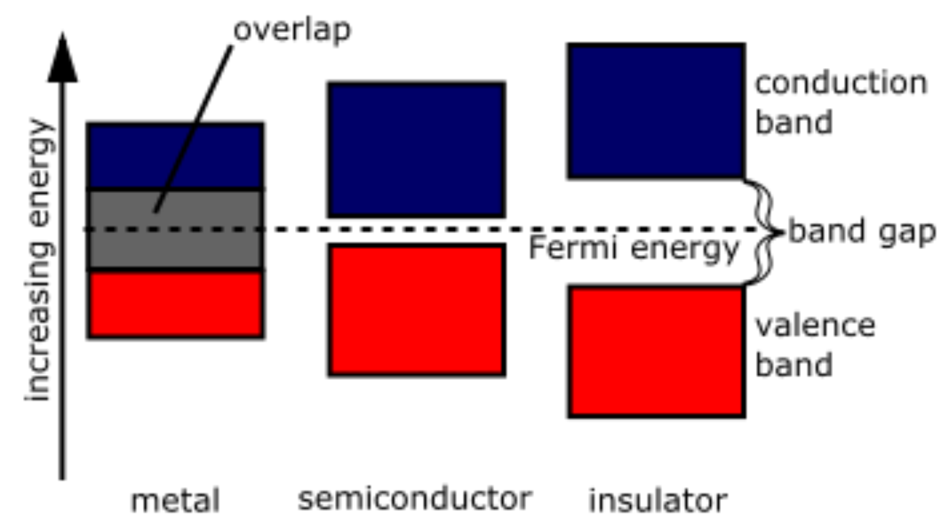
Broad classification of materials

Insulators (Dielectrics)

Semiconductors

Conductors

Superconductors



Recall: Conductors have "free" electrons, which are detached from the atoms.

Insulators have bound electrons, attached to its atoms.

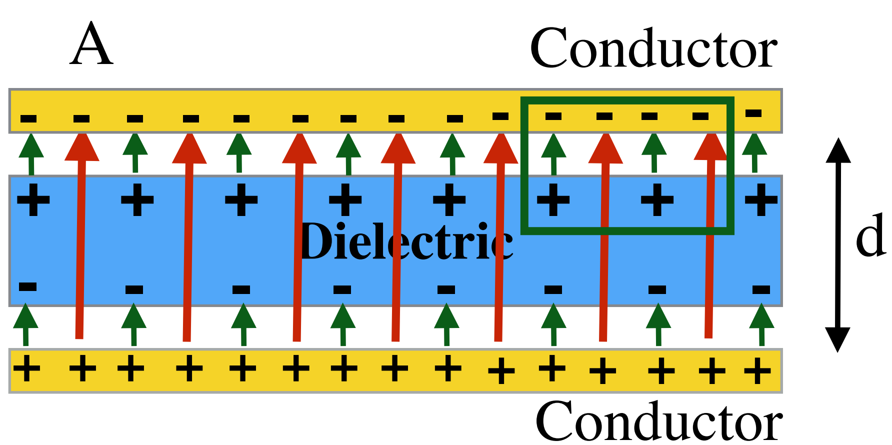
We have already discussed about conductors (metals)

We will discuss about insulators or dielectrics!

Why should we be bothered about insulators?

After all insulators/dielectrics do not conduct electricity. At first you might think that there should not be any effect of the electric field on the insulators!

Faraday showed that capacitance of a capacitor increases if we place a dielectric between its plates. If the dielectric completely fills up the space between the plates, then the capacitance increases by an amount k which depends only on the nature of the material. k is the property of the dielectric and is called dielectric constant.



We know that the capacitance of a parallel plate capacitor is

$$C = \frac{\epsilon_0 A}{d} \qquad \text{where,} \qquad Q = CV$$

Putting an insulating material between the plates increases C

That means voltage is lower for the same charge!

But, voltage diff. is the integral of electric field across the capacitor!

Hence electric field is reduced, even though charges remain same!

HOW??

Gauss law: Flux $\propto$ charge enclosed! The only way electric field can reduce if the net charge inside the Gaussian surface is lower than it would be without the material.

Hence there must be positive charges on the surface of the dielectric. Since the field is reduced, but not zero, we would expect this positive charge to be smaller than the negative charge on the conductor!

When a dielectric is placed in an electric field, positive charges get induced on one surface and negative on the other

Induced dipole

What happens to a neutral atom when it is placed in an electric filed ?

- Due to the presence of a positively charged core in an atom with electrons surrounding it, the nucleus is pushed towards the electric filed.

- The two opposing forces : electric field pulling the electron and nucleus apart and their mutual attractions drawing them together reach a balance : Atom is polarised

With plus and minus charges shifted slightly results in a dipole moment $\vec{p}$ pointing in the same direction as of the electric field.

- Typically this induced dipole moment is approximately proportional to the field:

$$\vec{p} = \alpha \vec{E}$$

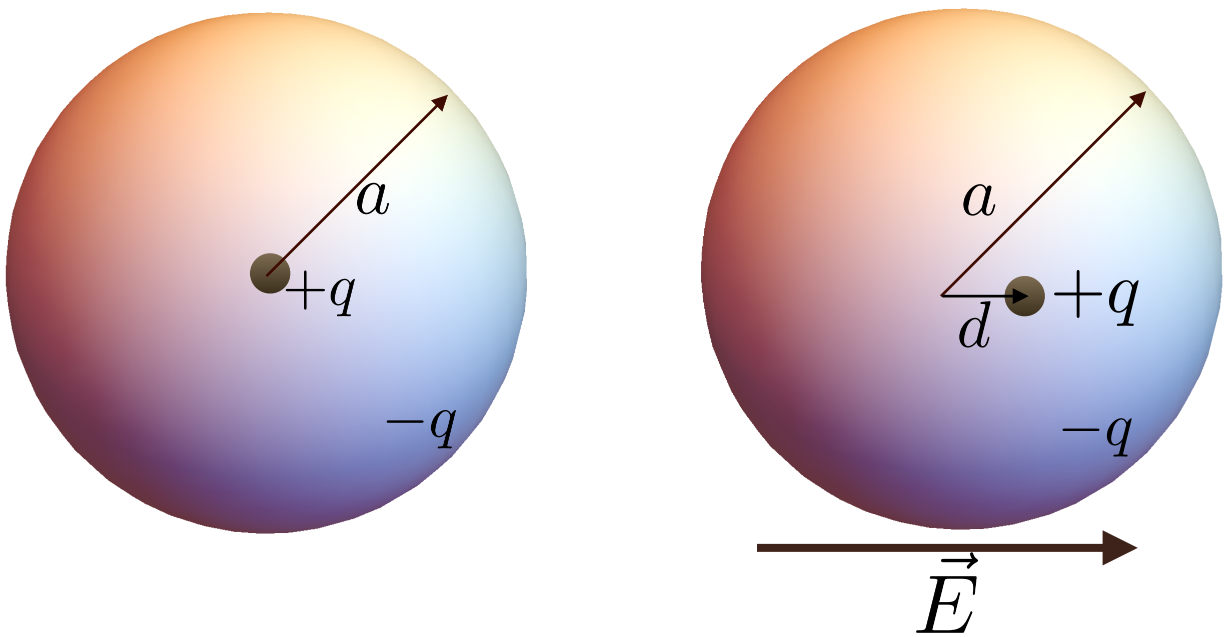
Atomic polarisability

H	He	Li	Be	C	Ne	Na	Ar	K	Cs
0.667	0.205	24.3	5.60	1.76	0.396	24.1	1.64	43.4	59.6

Atomic polarizabilities ($\alpha/4\pi\epsilon_0$, in units of $10^{-30}\ m^3$)

A quick calculation on atomic polarisability

A primitive atomic model: A point nucleus of charge +q surrounded by a uniformly charged sphere of charge -q: Crude approximation



- We assume that the electron cloud remains same in external field
- Suppose equilibrium occurs when the nucleus is 'd' distance apart

At this point, the external field pushing the nucleus will balance the internal field pulling it to left $\vec{E} = \vec{E}_e$

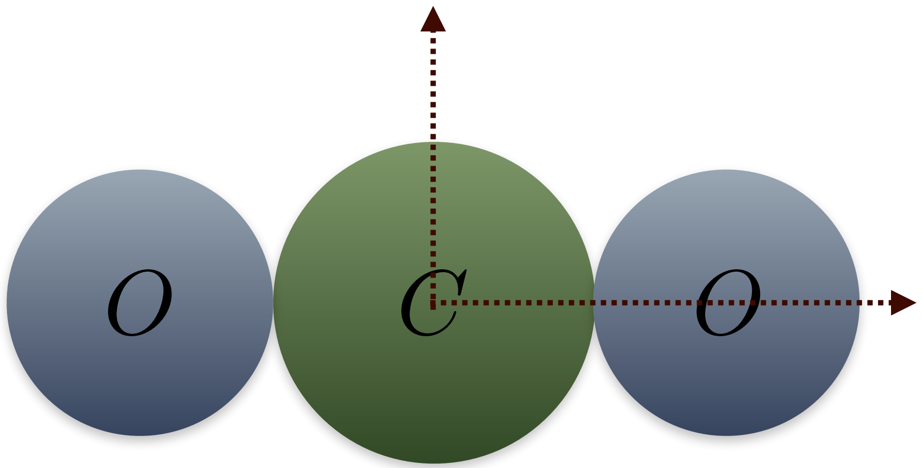
The field at a distance 'd' inside a uniformly charged sphere:

$$\vec{E}_e = \frac{\rho d}{3\epsilon_0}\hat{r} = \frac{1}{4\pi\epsilon_0}\frac{qd}{a^3}\hat{r} = \vec{E} \longrightarrow E = \frac{1}{4\pi\epsilon_0}\frac{qd}{a^3}$$

Dipole moment: $p = qd = (4\pi\epsilon_0 a^3)E = \alpha E$

Atomic polarisability : $\boxed{\alpha = 4\pi\epsilon_0 a^3 = 3\epsilon_0 V}$ V is volume of the atom

The case of molecular polarizability



- For molecules situations are more complex.

Polarization often depend on the direction of electric field

CO_2 has polarisability $4.5 \times 10^{-40} \;\; C^2.m/N$ when the field is along the axis of the molecule; $2 \times 10^{-40} \;\; C^2.m/N$ when the field is perpendicular. $\longrightarrow \{\alpha_\perp, \alpha_{||}\}$

- When the field is at some angle, one must resolve it in parallel and perpendicular components $\longrightarrow$

$$\vec{p} = \alpha_\perp \vec{E}_\perp + \alpha_{||} \vec{E}_{||}$$

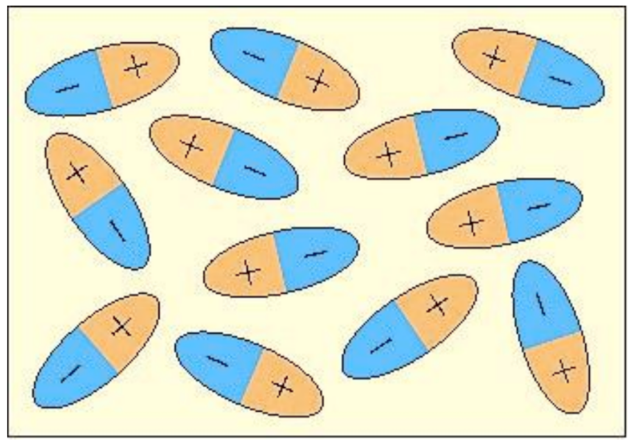
For a completely asymmetric molecule:

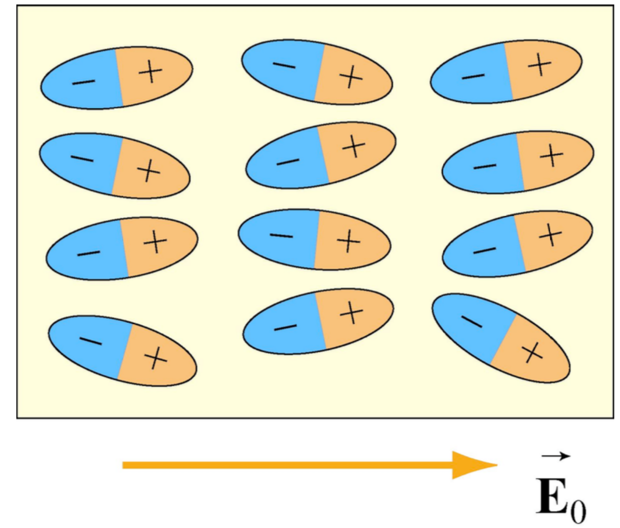
$$p_x = \alpha_{xx} E_x + \alpha_{xy} E_y + \alpha_{xz} E_z$$
$$p_y = \alpha_{yx} E_x + \alpha_{yy} E_y + \alpha_{yz} E_z$$
$$p_z = \alpha_{zx} E_x + \alpha_{zy} E_y + \alpha_{zz} E_z$$

α_{ij} forms the components of polarisability tensor.

Two types of dielectrics

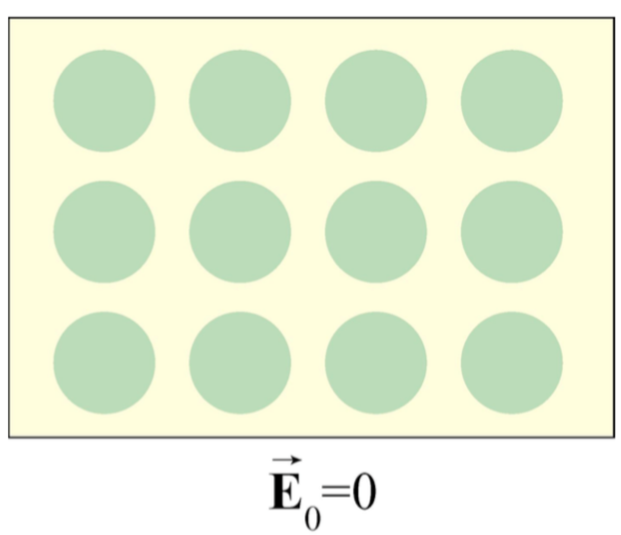
Polar dielectrics - having permanent electric dipole moments. (Example: water)

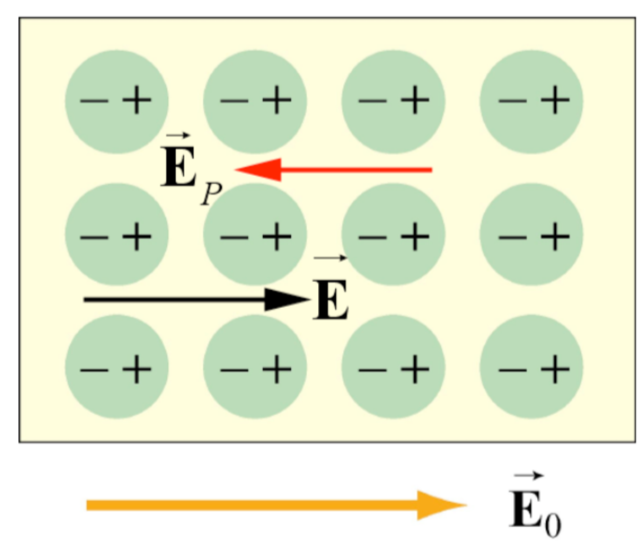




The orientation of **Polar** molecules is random in the absence of an external electric field. When in electric field molecules align with the electric field. However, the alignment is not complete due to random thermal motion. The aligned molecules generate an electric field that is opposite to the applied field but smaller in magnitude.

Non-Polar dielectrics - No permanent electric dipole moments.



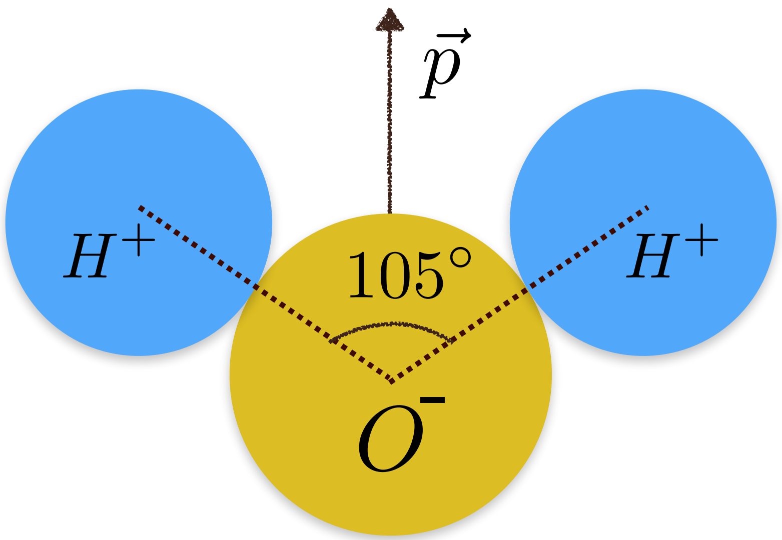


$$\vec{E} = \vec{E}_0 + \vec{E}_p$$

$$|\vec{E}| < |\vec{E}_0|$$

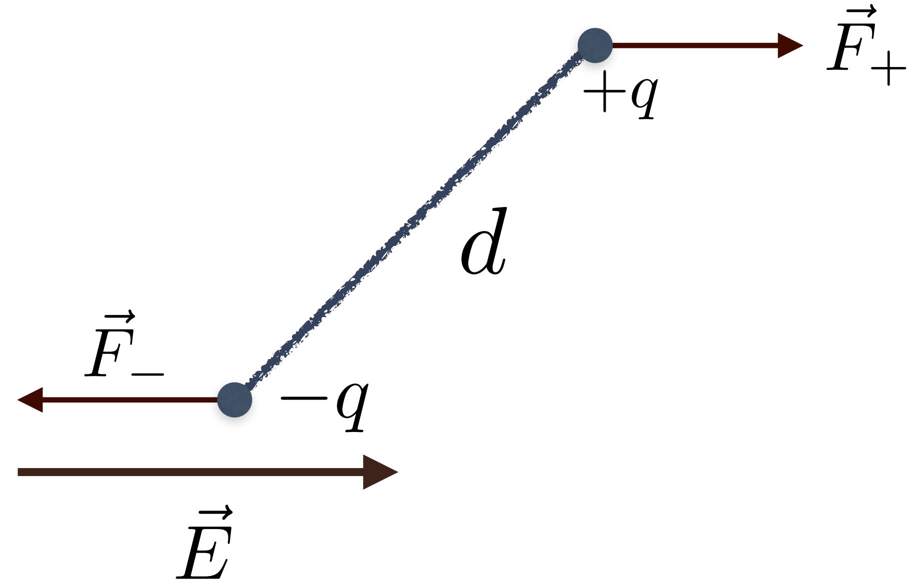
Polar molecules in electric field

Consider molecules which has built in dipole moment. Ex: Water molecule



$$p = 6.1 \times 10^{-30}\ C.m$$

What happens when we bring such polar molecules in electric field ?



In case the field is uniform the force cancel at both end, with a residual torque on the dipole

$$\vec{N} = [(\vec{d}/2) \times (q\vec{E})] + [(-\vec{d}/2) \times (-q\vec{E})]$$

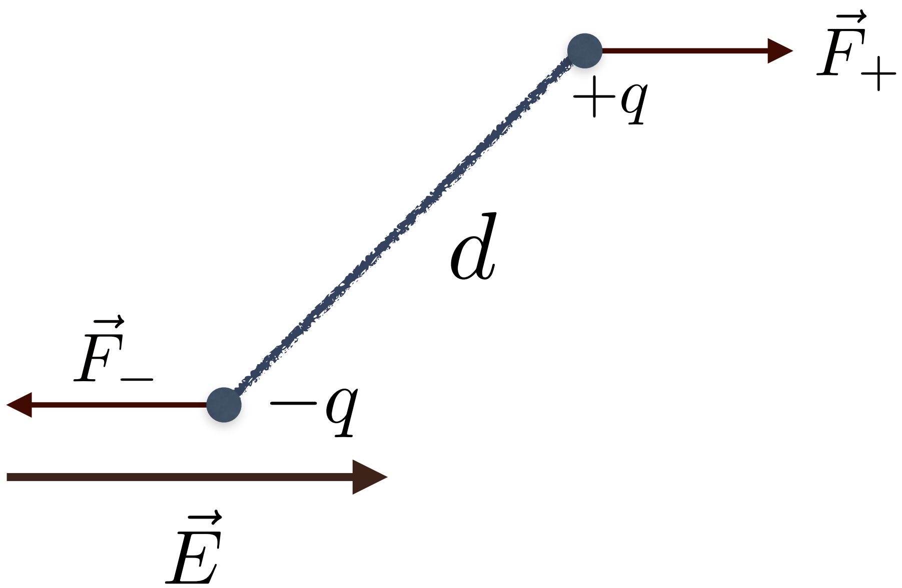
$$= q\vec{d} \times \vec{E} = \vec{p} \times \vec{E}$$

$$\vec{p} = q\vec{d}$$

Dipole moment

A polar molecule that is free to rotate will swing around till it points in direction of the applied field.

Polar molecule in non-uniform field



There will be a net force on the dipole

$$\vec{F} = \vec{F}_+ + \vec{F}_- = q(\vec{E}_+ - \vec{E}_-) = q(\Delta\vec{E})$$

$\Delta\vec{E}$ is the difference in electric field at both ends.

Now the change in x-component of the field

$$\Delta E_x = \frac{\partial E_x}{\partial x}\Delta x + \frac{\partial E_x}{\partial y}\Delta y + \frac{\partial E_x}{\partial z}\Delta z = (\vec{d}.\vec{\nabla})E_x$$

- Above formula works for a very small dipole
- Similarly one may write the change in the electric field in y and z directions

Hence, one may write $\Delta\vec{E} = (\vec{d}.\vec{\nabla})\vec{E}$ ➡ $\vec{F} = q(\vec{d}.\vec{\nabla})\vec{E} = (\vec{p}.\vec{\nabla})\vec{E}$

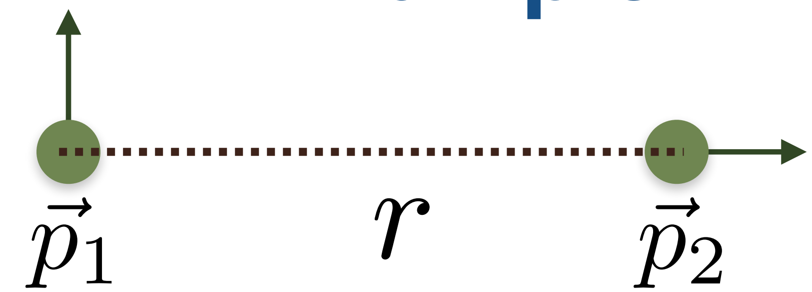
For a perfect dipole in non-uniform field, torque about the centre of dipole remains same

$$\vec{N} = \vec{p} \times \vec{E}$$

- However, the torque about an arbitrary point becomes

$$\vec{N} = (\vec{p} \times \vec{E}) + (\vec{r} \times \vec{F})$$

Example...



Although it might seem to be the same, it is actually not !

What is the torque on $\vec{p}_1$ due to $\vec{p}_2$ and on $\vec{p}_2$ due to $\vec{p}_1$?

Electric field due to $\vec{p}_1$ at the position of $\vec{p}_2$: $\vec{E}_1 = \frac{p_1}{4\pi\epsilon_0 r^3}\hat{\theta}$ ← points down

Recall, $\vec{E} = \frac{p}{4\pi\epsilon_0 r^3}(2\cos\theta\hat{r} + \sin\theta\hat{\theta})$ and use $\theta = 90^\circ$

Hence, torque on $\vec{p}_2$ due to $\vec{p}_1$:

$$\vec{N} = \vec{p}_2 \times \vec{E}_1 = p_2 E_1 \sin 90^\circ(-\hat{z}) = \frac{p_1 p_2}{4\pi\epsilon_0 r^3}(-\hat{z})$$

points into the page

Electric field due to $\vec{p}_2$ at the position of $\vec{p}_1$: $\vec{E}_2 = \frac{p_2}{4\pi\epsilon_0 r^3}(-2\hat{r})$

Again, use: $\vec{E} = \frac{p}{4\pi\epsilon_0 r^3}(2\cos\theta\hat{r} + \sin\theta\hat{\theta})$ with $\theta = \pi$

Hence, torque on $\vec{p}_1$ due to $\vec{p}_2$:

$$\vec{N} = \vec{p}_1 \times \vec{E}_2 = p_1 E_2 \sin 90^\circ(-\hat{z}) = \frac{2p_1 p_2}{4\pi\epsilon_0 r^3}(-\hat{z})$$

Twice than the other one

Polarization

- What happens to a piece of dielectric material in an external electric field ?

What we said so far:

- Presence of a atom or molecule in external field will induce a tiny dipole moment aligned in the direction of the field
- If the material is a polar object it will feel a torque to align the dipole along the external field

Hence, we can summarise, that a material placed in external field will produce a lot of tiny little dipoles along the direction of the field: Material is polarised

We define hence, a parameter called Polarisation as

$$\vec{P} = \text{dipole moment per unit volume}$$

We will first study the field a polarised material itself produces and then study the effect of such material in external electric field

The Field of a polarized object

Suppose we have a piece of polarized object: an object containing a lot of microscopic dipoles lined up. The dipole moment per unit volume is given as $\vec{P}$

Q. What is the field produced by this object?

(Not the field that causes the polarization, but the field that the polarization itself has caused.)

Strategy: We know the field of an individual dipole, so we can chop the material up into infinitesimal dipoles and integrate to get the total field.

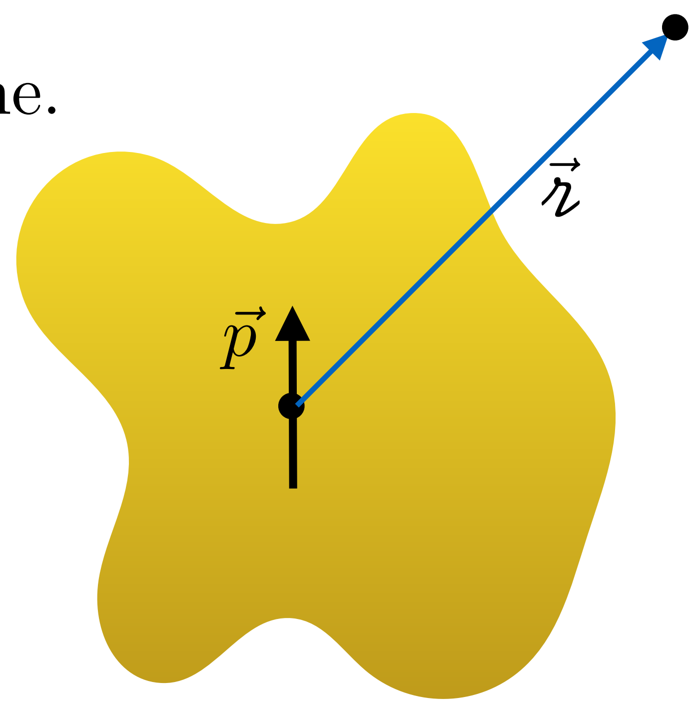
Since it is easier to work with potentials, the potential for a single dipole:

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0}\frac{\vec{p}.\hat{\mathscr{r}}}{\mathscr{r}^2}$$

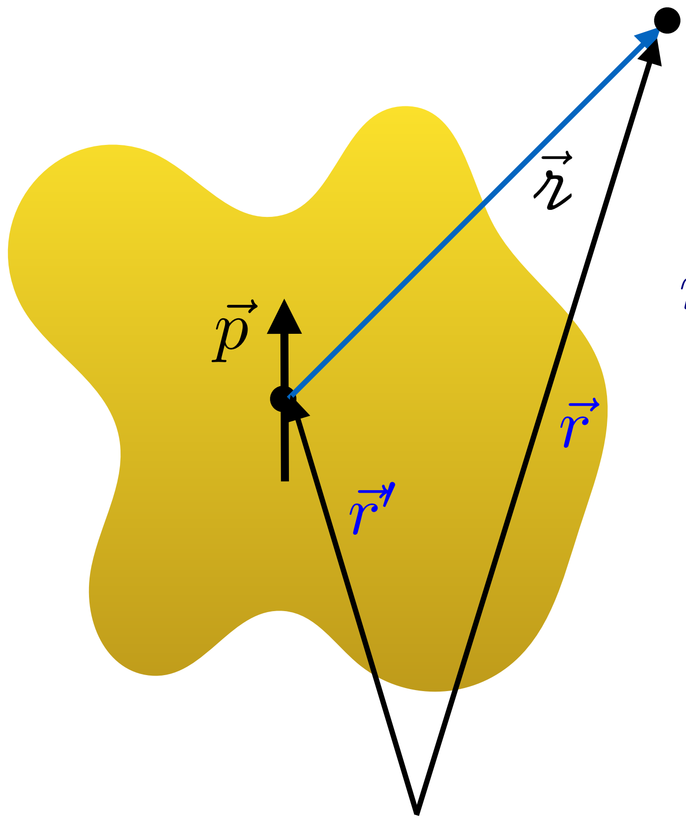
The polarisation is given by $\vec{P}(\vec{r}\,')d\tau'$ in an elemental volume.

The total potential at $\vec{\mathscr{r}}$ is then given by

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0}\int_{\mathcal{V}}\frac{\vec{P}(\vec{r}\,').\hat{\mathscr{r}}}{\mathscr{r}^2}d\tau'$$



The Field of a polarized object



Potential at $\vec{r}$ is given by

$$\vec{r} - \vec{r}\,' = \vec{\mathscr{r}}$$

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int_{\mathcal{V}} \frac{\vec{P}(\vec{r}\,').\hat{\mathscr{r}}}{\mathscr{r}^2} d\tau'$$

But, remember that $\vec{\nabla}' \left(\frac{1}{\mathscr{r}} \right) = \frac{\hat{\mathscr{r}}}{\mathscr{r}^2}$

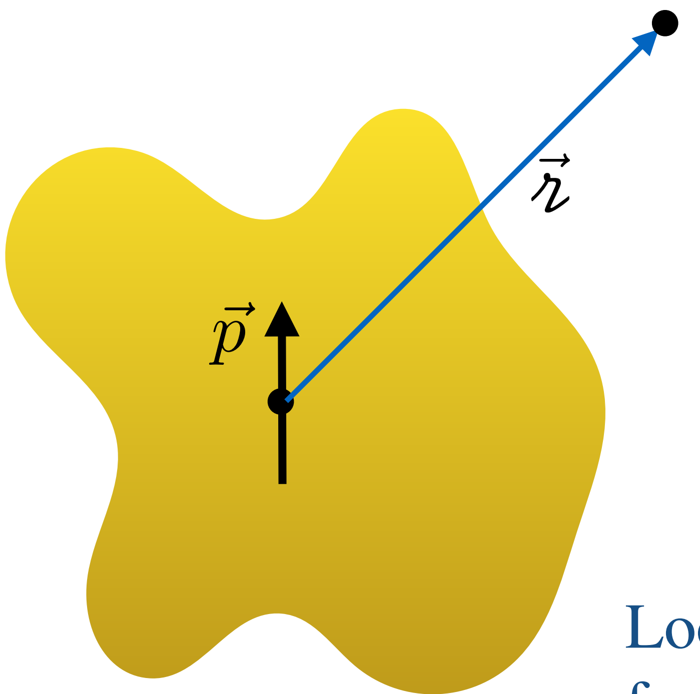
i.e. $\vec{\nabla}' \left(\frac{1}{|\vec{r} - \vec{r}\,'|} \right) = \frac{\vec{r} - \vec{r}\,'}{|\vec{r} - \vec{r}\,'|^3}$

Therefore: $$\frac{\vec{P}(\vec{r}\,').\hat{\mathscr{r}}}{\mathscr{r}^2} = \vec{P}(\vec{r}\,').\vec{\nabla}' \left(\frac{1}{\mathscr{r}} \right) = \vec{\nabla}'. \left(\frac{\vec{P}(\vec{r}\,')}{\mathscr{r}} \right) - \frac{1}{\mathscr{r}} \left(\vec{\nabla}'.\vec{P}(\vec{r}\,') \right)$$

Here, we have used the vector identity

$$\vec{\nabla}.(f\vec{A}) = f\vec{\nabla}.\vec{A} + (\vec{\nabla} f).\vec{A}$$

The Field of a polarized object



Hence the potential is

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0}\int_{\mathcal{V}} \frac{\vec{P}(\vec{r}\,').\hat{\mathscr{r}}}{\mathscr{r}^2}d\tau'$$

$$\Longrightarrow V(\vec{r}) = \frac{1}{4\pi\epsilon_0}\left[\int_{\mathcal{V}} \vec{\nabla}'.\left(\frac{\vec{P}(\vec{r}\,')}{\mathscr{r}}\right)d\tau' - \int_{\mathcal{V}} \frac{1}{\mathscr{r}}(\vec{\nabla}'.\vec{P}(\vec{r}\,'))d\tau'\right]$$

Using divergence theorem

Looks like potential for a volume charge

$$= \frac{1}{4\pi\epsilon_0}\oint_{\mathcal{S}} \frac{1}{\mathscr{r}}\vec{P}(\vec{r}\,').\hat{n}'da' - \frac{1}{4\pi\epsilon_0}\int_{\mathcal{V}} \frac{1}{\mathscr{r}}(\vec{\nabla}'.P(\vec{r}\,'))d\tau'$$

Looks like potential for a surface charge

$$= \frac{1}{4\pi\epsilon_0}\oint_{\mathcal{S}} \frac{\sigma_b(\vec{r}\,')}{\mathscr{r}}da' + \frac{1}{4\pi\epsilon_0}\int_{\mathcal{V}} \frac{\rho_b(\vec{r}\,')}{\mathscr{r}}d\tau'$$

Bound surface charge density

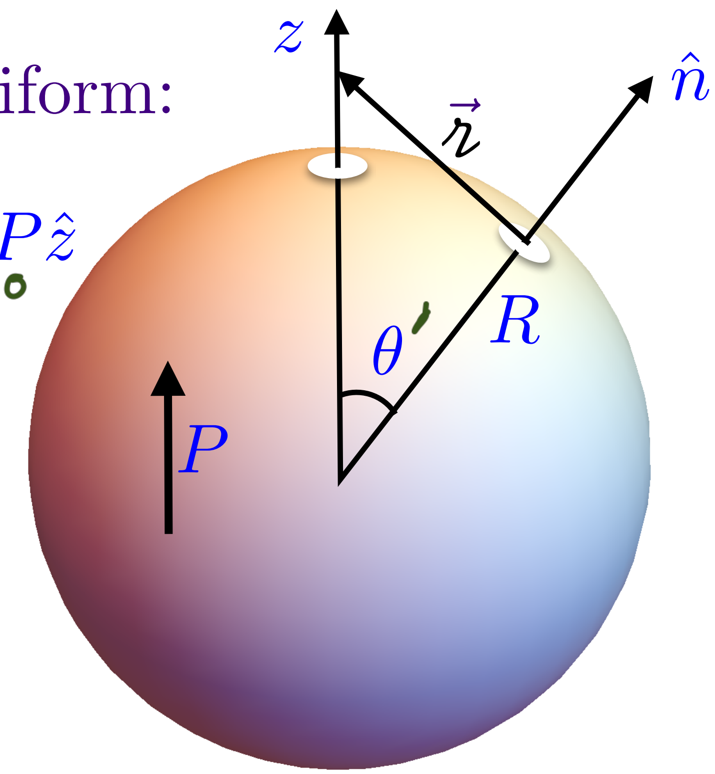
$$\sigma_b = \vec{P}(\vec{r}).\hat{n}$$

Bound volume charge density

$$\rho_b = -\vec{\nabla}.P(\vec{r})$$

This means that the potential (hence the field also) of a polarized object is same as that produced by a volume charge density $\rho_b = -\vec{\nabla}.\vec{P}$ plus a surface charge density $\sigma_b = \vec{P}.\hat{n}$.

Example: To find the electric field produced by a uniformly polarized sphere of radius R



Clearly the volume bound charge density is zero, since $\vec{P}_0$ is uniform:

$$\rho_b = -\vec{\nabla}.\vec{P}_0 = 0 \qquad \text{Let } \vec{P}_0 = P_0\hat{z}$$

Surface bound charge density $\sigma_b = \vec{P}_0.\hat{n} = P_0 \cos\theta'$

$$\therefore V(\vec{r}) = \frac{1}{4\pi\epsilon_0}\oint_{\mathcal{S}} \frac{\sigma_b(\vec{r}\,')}{\mathscr{r}} da' + \frac{1}{4\pi\epsilon_0}\int_{\mathcal{V}} \frac{\rho_b(\vec{r}\,')}{\mathscr{r}} d\tau'$$

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0}\oint_{\mathcal{S}} \frac{\sigma_b(\vec{r}\,')}{\mathscr{r}} da' = \frac{1}{4\pi\epsilon_0}\oint_{\mathcal{S}} \frac{P_0\cos\theta'}{\sqrt{R^2+z^2-2Rz\cos\theta'}} R^2 \sin\theta' d\theta' d\phi'$$

$$= \frac{P_0 R^2}{4\pi\epsilon_0}\int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} \frac{\cos\theta' \sin\theta' d\theta' d\phi'}{\sqrt{R^2+z^2-2Rz\cos\theta'}}$$

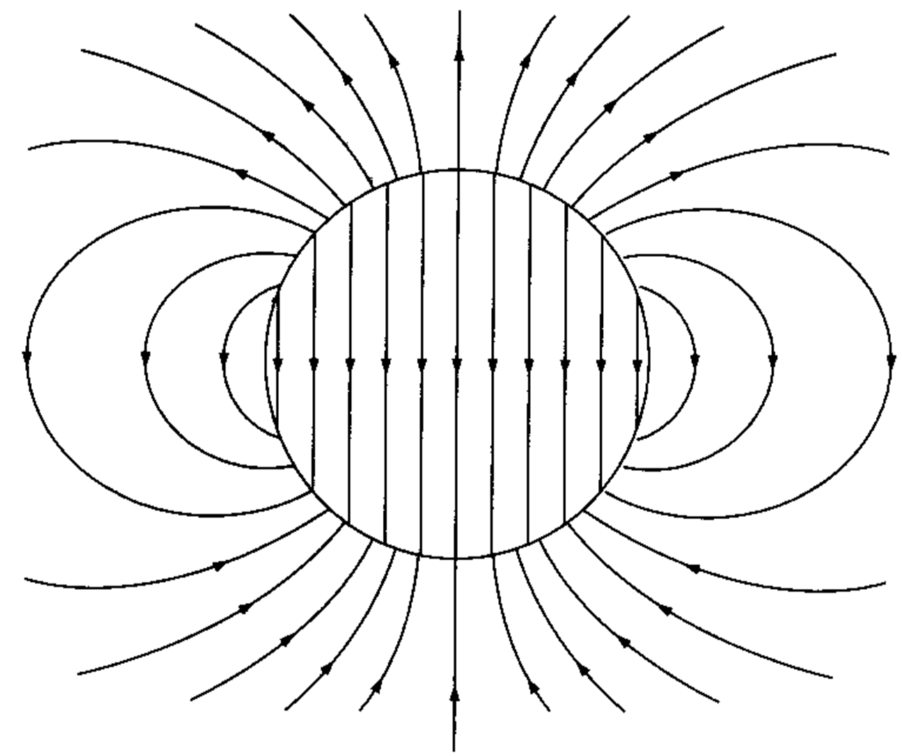
Substitute: $R^2 + z^2 - 2Rz\cos\theta' = t^2$

$$V(\vec{r}) = \frac{P_0 R^2}{2\epsilon_0}\int_{|R-z|}^{R+z} \frac{(R^2+z^2-t^2)}{2(Rz)^2} dt$$

Example: To find the electric field produced by a uniformly polarized sphere of radius R

After the integration: $$V(r,\theta) = \begin{cases} \frac{P}{3\epsilon_0} r\cos\theta & \text{if } r \le R; \\ \frac{P}{3\epsilon_0}\frac{R^3}{r^2}\cos\theta & \text{if } r \ge R. \end{cases}$$ (Remember: $z = r\cos\theta$)

$\frac{P_0}{3\epsilon_0} z$ $\quad$ $\frac{P_0 R^3}{3\epsilon_0 z^2}$



Inside the sphere: $\vec{E} = -\vec{\nabla} V = -\frac{P}{3\epsilon_0}\hat{z} = -\frac{\vec{P}}{3\epsilon_0}$ for $r < R$.

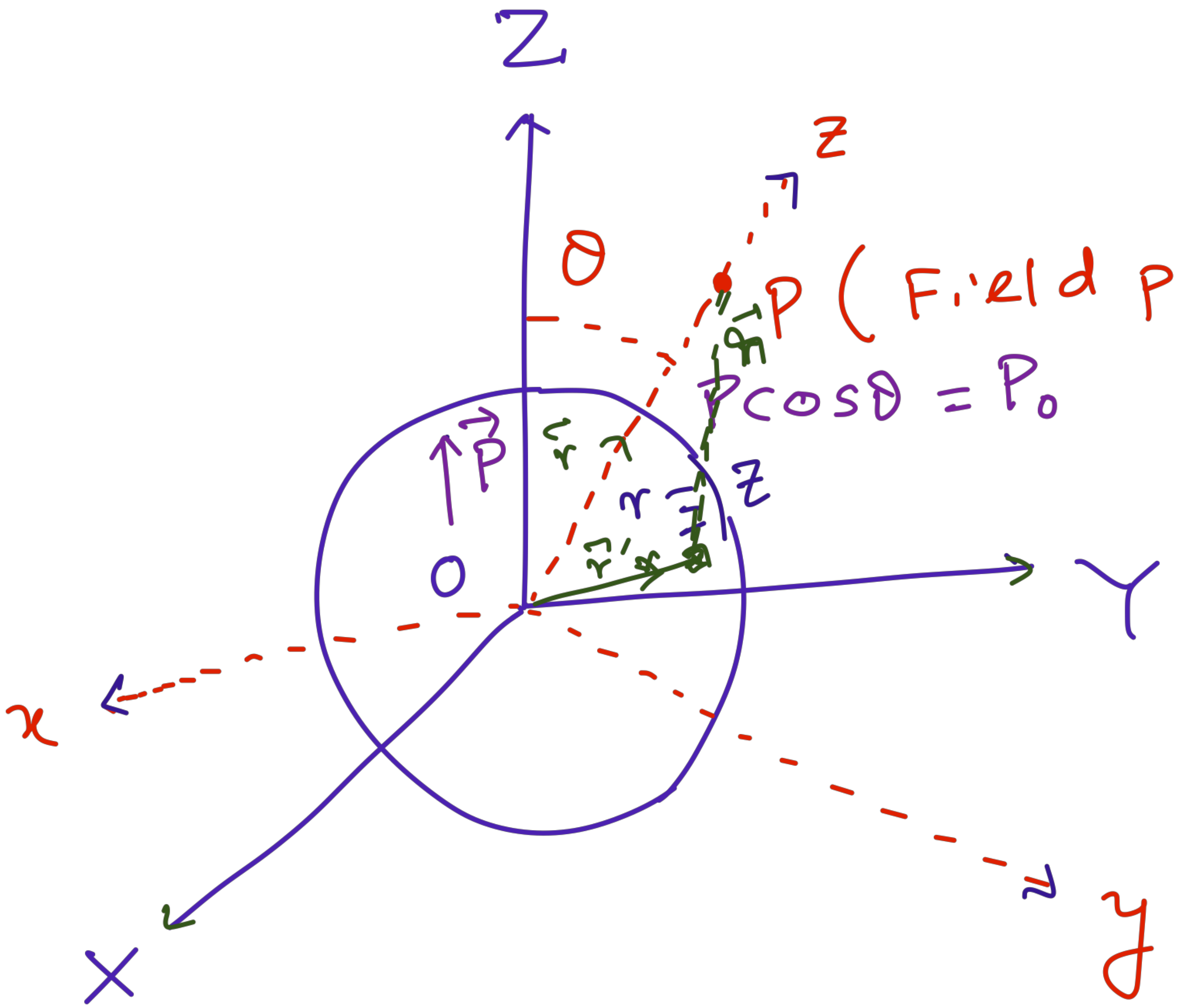
i.e. electric field inside is uniform!

Also note, the potential outside $(r \ge R)$

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0}\frac{\frac{4}{3}\pi R^3 P}{r^2}\cos\theta = \frac{1}{4\pi\epsilon_0}\frac{\vec{p}.\hat{r}}{r^2}.$$

where $p = \int P d\tau = \frac{4}{3}\pi R^3 P$ is the total dipole moment of the sphere

i.e. Outside the sphere the potential is identical to that of a perfect dipole at the origin, whose dipole moment $(\vec{p})$ is the total dipole moment of the sphere!



$(0, 0, z)$ w.r.t. (x, y, z) (Earlier coordinates)

(r, θ) OR (X, Y, Z) w.r.t. XYZ coordinates

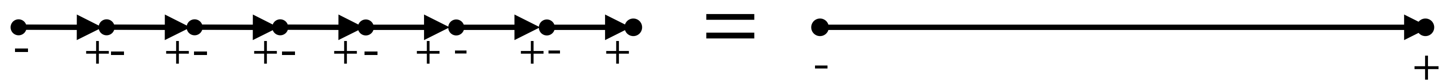
$\Rightarrow$ $Z = r$

Notation: $p \equiv p_0 \equiv p\cos\theta$

$$\therefore \; V(r,\theta) = \begin{cases} \dfrac{p_0 Z}{3\epsilon_0} = \dfrac{p\cos\theta \cdot r}{3\epsilon_0} & (r \le R) \\[2ex] \dfrac{p_0 R^3}{3\epsilon_0 Z^2} = \dfrac{p\cos\theta \cdot R^3}{3\epsilon_0 r^2} & \text{for } (r \geq R). \end{cases}$$

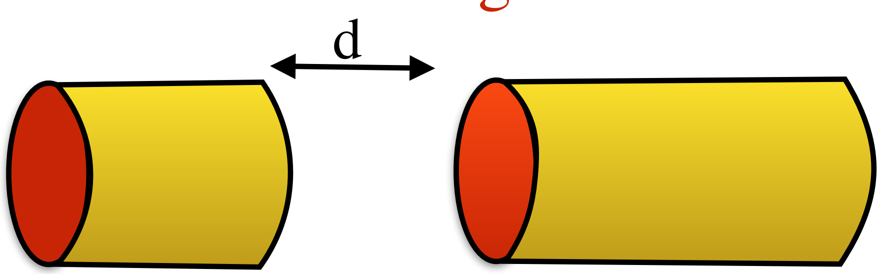
How to see the bound charges in action?

The field of a polarized object is identical to the field that would be produced by a certain distribution of ‘bound charges’, σ_b and ρ_b

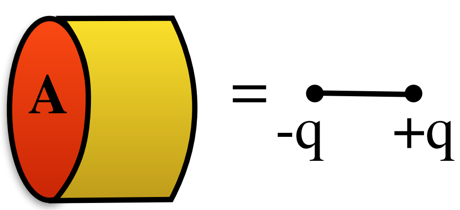


genuine accumulation of charges

Long string of dipoles: head of one effectively cancels the tail of the neighbour. At the end two charges left over → ‘bound charges’ → they can not be remove.



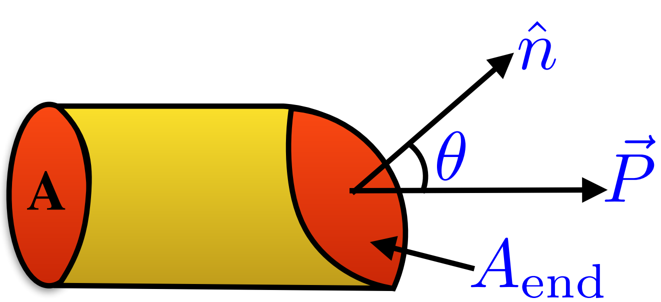
To calculate actual amount of bound charges resulting from a given polarization: Take a tube of dielectric parallel to $\vec{P}$



→ This chunk of dielectric has the dipole moment $P(Ad)$, where A is the area of cross section and d is the length of the chunk.

In terms of the charge (q) at the end, this same dipole moment is : qd. The bound charge that piles up to the right of the tube is : $q = PA$

If the ends are cut perpendicularly, then the surface charge density is: $\sigma_b = \dfrac{q}{A} = P$



For the oblique cut, the charge is still the same but A is $A_{\text{end}} \cos\theta$

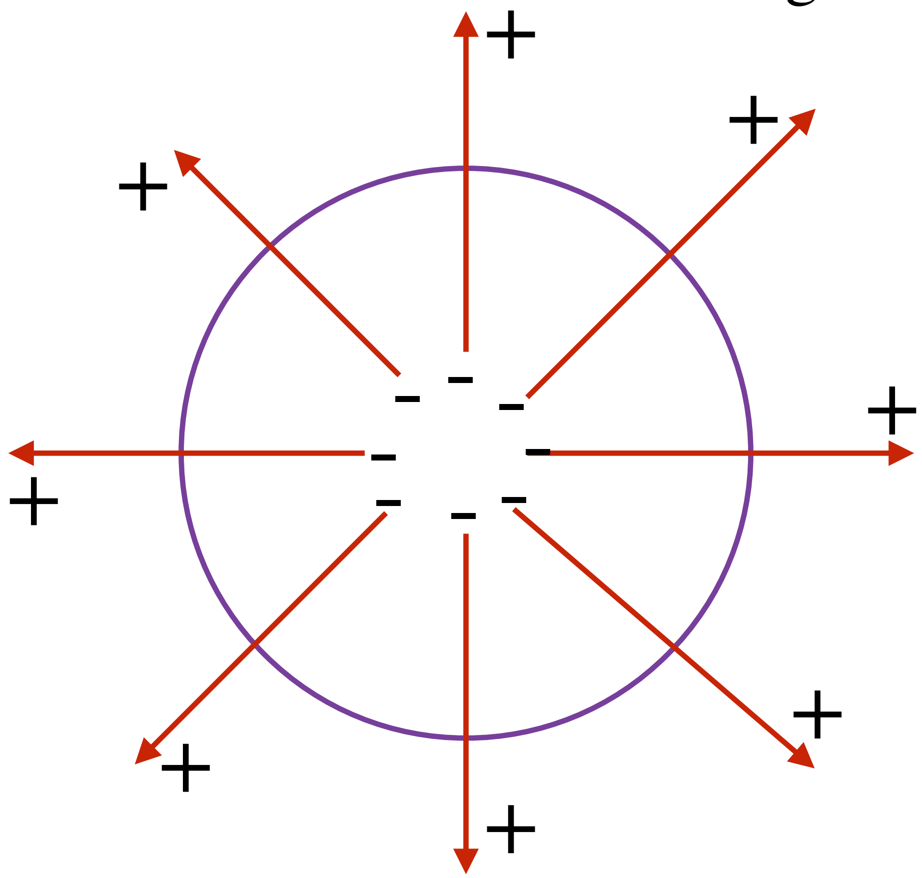
$$\sigma_b = \frac{q}{A_{\text{end}}} = P\cos\theta = \vec{P}.\hat{n}$$

This is the origin of the surface bound charges!

How to see the bound charges in action?

If the polarization is non uniform, bound charge starts accumulating within the material as well as on the surface

Suppose we have a diverging $\vec{P}$, then it would definitely mean that there will be accumulation of negative charges in the bulk. See figure:



The net bound charge $\int \rho_b d\tau$ in a given volume is equal and opposite to the amount that has been pushed out through the surface.

The latter is $\vec{P}.\hat{n}$ per unit area.

$$\int_{\mathcal{V}} \rho_b d\tau = -\oint_{\mathcal{S}} \vec{P}.d\vec{a} = -\int_{\mathcal{V}} (\vec{\nabla}.\vec{P}) d\tau.$$

Hence we have $\rho_b = -\vec{\nabla}.\vec{P}$

Example:

Find bound charges in a spherical dielectric (radius R, centred at origin) and polarization given by

$$\vec{P}(\vec{r}) = k\vec{r}$$

Where k is a constant and r is the vector from the centre. What is the net charge on the sphere?

- Bound surface charge density is given by $\sigma_b = \vec{P}.\hat{n} = kR\hat{r}.\hat{r} = kR$.
- Total bound surface charge is $4\pi kR^3$.
- Bound volume charge density is $\rho_b = -\vec{\nabla}.\vec{P} = -3k$.
- Total bound volume charge is $\frac{4}{3}\pi R^3\rho_b = -4\pi kR^3$.
- Net charge in material is zero.

What will be the electric field both inside and outside this polarized sphere?

- for $r < R$: Enclosed charge is $\frac{4}{3}\pi r^3\rho_b$.
- Applying Gauss's law inside the sphere: $\vec{E} = \frac{\rho_b r}{3\epsilon_0}\hat{r} \implies \vec{E} = -\frac{k}{\epsilon_0}\vec{r}$.
- for $r > R$: Enclosed charge is zero. Therefore field outside will be zero.

Gauss's Law for Dielectrics

• We just saw that effect of polarization is to produce accumulation of bound charge: $\rho_b = -\vec{\nabla}.\vec{P}$ within the dielectric and $\sigma_b = \vec{P}.\hat{n}$ on the surface.

• Field due to polarization of medium is just the field due to this bound charge.

• We also want to accommodate fields due to everything else (excluding the field due to polarization) into the picture.

• Call "this everything else" ρ_f: free charge density.

• Total charge density: $\rho = \rho_b + \rho_f$.

• Therefore the Gauss's law reads: $\epsilon_0 \vec{\nabla}.\vec{E} = \rho = \rho_b + \rho_f \implies -\vec{\nabla}.\vec{P} + \rho_f$

$$\implies \vec{\nabla}.(\epsilon_0 \vec{E} + \vec{P}) = \rho_f \implies \boxed{\vec{\nabla}.\vec{D} = \rho_f}$$

Gauss's law for dielectrics

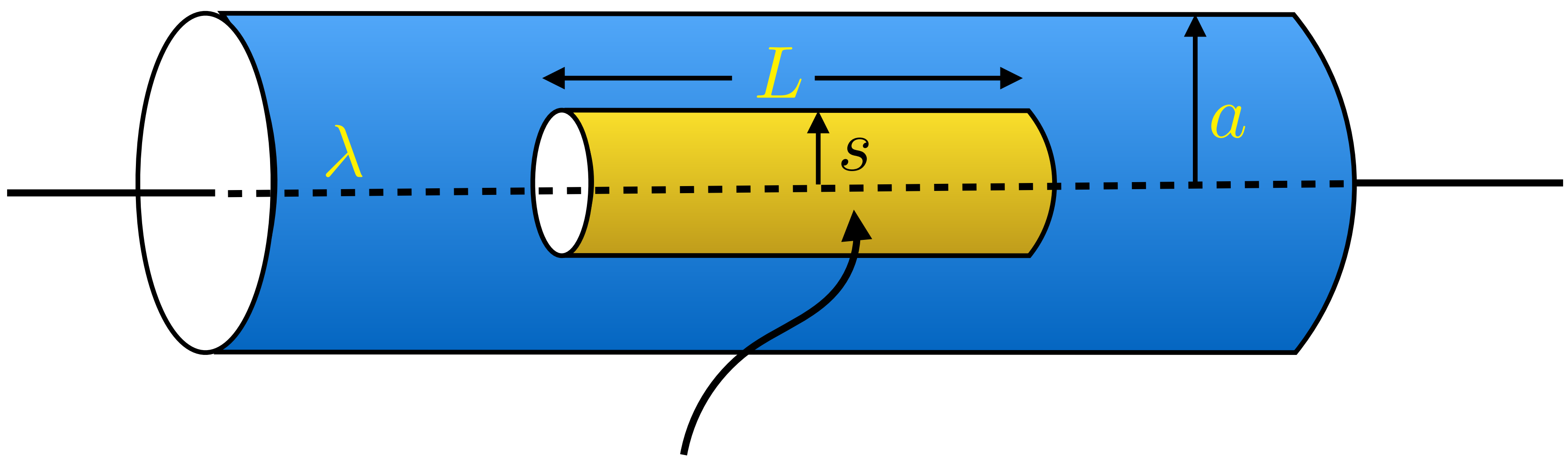
• Here $\vec{D} \equiv \epsilon_0 \vec{E} + \vec{P}$: **Electric displacement**.

• Integral form: $\oint \vec{D}.d\vec{a} = Q_{\mathrm{f_{enc}}}$. → Total free charge enclosed in the volume!

• Note that the Gauss's law in dielectrics makes reference to free charge only. That is good, because, free charge is the stuff we control!

Example:

A long straight wire, carrying uniform line charge density λ, is surrounded by rubber insulation out to a radius a. Find the electric displacement.



Draw a cylindrical Gaussian surface of radius s and length L:

$$D(2\pi s L) = \lambda L$$

$$\vec{D} = \frac{\lambda}{2\pi s}\hat{s}$$

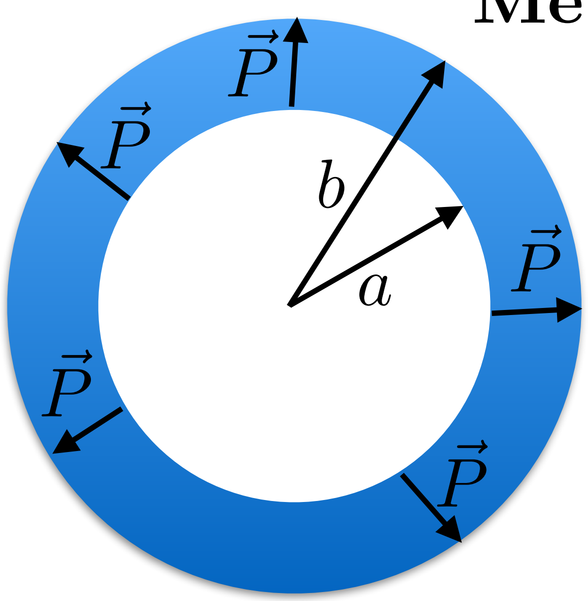
This formula is valid everywhere. Particularly note: outside the rubber, $\vec{P} = 0$:

$$\vec{E} = \frac{\vec{D}}{\epsilon_0} = \frac{\lambda}{2\pi\epsilon_0 s}\hat{s} \quad \text{for } s > a$$

Inside rubber, we do not know $\vec{E}$ since we do not know $\vec{P}$.

Example:

A thick spherical shell of inner radius a and outer radius b is made of dielectric material with a "frozen in" polarization $\vec{P}(\vec{r}) = \frac{k}{r}\hat{r}$, where k is a constant and r is the distance from the centre. There is no free charge in the problem. Find the electric field in all three regions.



Method 1: $\oint \vec{D}.d\vec{a} = Q_{f_{enc}} = 0$, since there is no free charge.

$$\Longrightarrow \vec{D} = 0 \text{ everywhere.}$$

$$\Longrightarrow \vec{D} = \epsilon_0 \vec{E} + \vec{P} = 0 \Longrightarrow \vec{E} = -\frac{1}{\epsilon_0}\vec{P}$$

In $a < r < b$, $\vec{E} = -\frac{k}{\epsilon_0 r}\hat{r}$

In $r < a$ and $r > b$, $\vec{E} = 0$ since $\vec{P} = 0$ there.

Method 2:

$$\rho_b = -\vec{\nabla}.\vec{P} = -\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{k}{r}\right) = -\frac{k}{r^2} \quad \text{and} \quad \sigma_b = \vec{P}.\hat{n} \;=\; +\vec{P}.\hat{r} = \frac{k}{b} \;\; (\text{at } r = b)$$

$$= \; -\vec{P}.\hat{r} = -\frac{k}{a} \;\; (\text{at } r = a).$$

(Normal is outward with respect to the dielectric)

In region $a < r < b$

$Q_{\text{enc}} = (-\frac{k}{a})4\pi a^2 + \int_a^r (-\frac{k}{\bar{r}^2})4\pi\bar{r}^2 d\bar{r} = -4\pi k r$. Therefore: $\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{Q_{\text{enc}}}{r^2}\hat{r} = -(\frac{k}{\epsilon_0 r})\hat{r}$

In $r < a$ and $r > b$, $\vec{E} = 0$