PH102: Tutorial Problem set

Tutorial 2

2020-01-21

2.01. If $\vec{F} = 4xz\hat{x} - y^2\hat{y} + yz\hat{z}$, evaluate $\int\int_S \vec{F}\cdot\hat{n}dS$ where S is the surface of the cube bounded by $x = 0$, $x = 1$, $y = 0$, $y = 1$, $z = 0$, $z = 1$.

2.02. Evaluate $\int\int\int_V (2x+y)dV$, where V is the closed region bounded by the cylinder $z = 4-x^2$ and the planes $x = 0, y = 0, y = 2$ and $z = 0$.

2.03. Verify the divergence theorem for $\vec{A} = 4x\hat{x} - 2y^2\hat{y} + z^2\hat{z}$ taken over the region bounded by $x^2 + y^2 = 4$, $z = 0$ and $z = 3$ (see Figure 1).

2.04. Prove $\int\int\int_V \vec{\nabla}\phi\ dV = \int\int_S \phi\ \hat{n}dS$.

2.05. (a) Verify Stoke's theorem by calculating the line integral of $\vec{F} = 2z\hat{x} + x\hat{y} + y\hat{z}$ over a circle of radius R in the xy plane centered at the origin, where the open surface is the hemisphere in $z > 0$ (see Fig. 2).

(b) Calculate the same line integral using Divergence theorem imagining the hemispherical surface as well as the disc on the $x - y$ plane to form a closed surface.

Take home problems

H2.01. Evaluate

$$\int\int_S \vec{A}\cdot\hat{n}dS\ ,$$

where $\vec{A} = 18z\hat{x} - 12\hat{y} + 3y\hat{z}$ and S is that part of the plane $2x + 3y + 6z = 12$ which is located in the first octant (see Figure 3).

H2.02. Find the volume of the region common to the intersecting cylinders $x^2 + y^2 = a^2$ and $x^2 + z^2 = a^2$ (See Figure 4)

H2.03. Prove $\int\int\int_V \left(\phi\nabla^2\psi - \psi\nabla^2\phi\right) = \int\int_S \left(\phi\vec{\nabla}\psi - \psi\vec{\nabla}\phi\right)\cdot d\vec{S}$.

H2.04. Prove the identity $\int\int_S \vec{\nabla}\phi \times d\vec{S} = -\oint_C \phi d\vec{r}$.

H2.05. Prove $\oint d\vec{r} \times \vec{B} = \int\int_S (\hat{n} \times \vec{\nabla}) \times \vec{B}\ dS$.

H2.06. Evaluate $\oint_C \vec{r} \times d\vec{r}$ by using the identity in Problem (2.06) where the loop is on the $x - y$ plane. Check that if the magnitude is twice the area enclosed by the loop C.

H2.07. Prove $\int\int\int_V \vec{\nabla} \times \vec{B}\ dV = \int\int_S \hat{n} \times \vec{B}\ dS$.

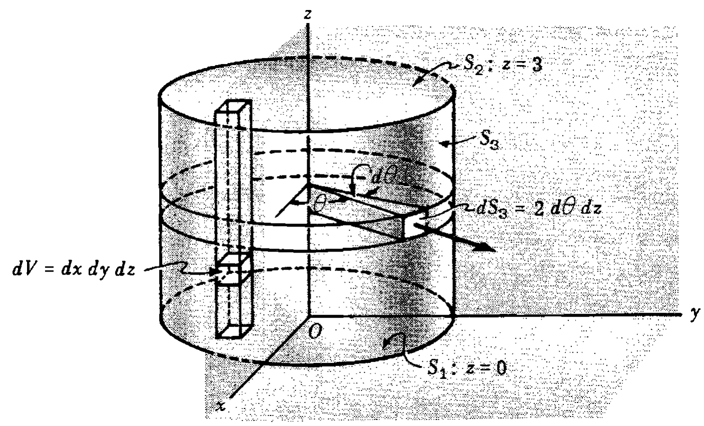

Figure 1: Problem 2.03

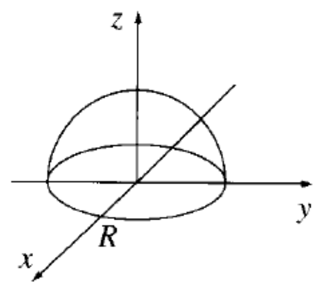

Figure 2: Problem 2.05

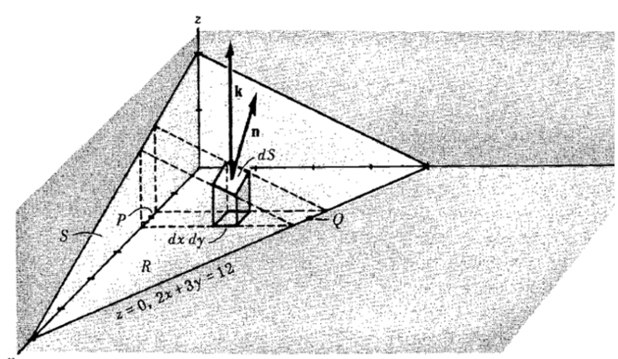

Figure 3: Problem H2.01

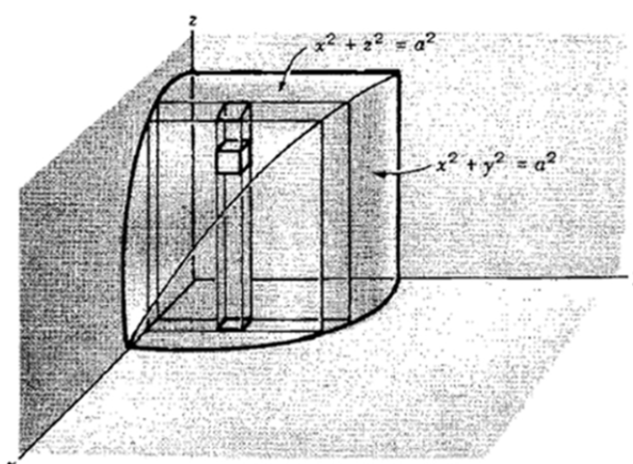

Figure 4: Problem H2.02