

PH102: Tutorial Problem set

Tutorial 4

2020-02-11

4.01. A rod placed along the x axis has charge density $\lambda(x) = \frac{\lambda_0 x}{L}$ in the interval $-L < x < L$. Find the electric field at a point $x = x_0 > L$ along the x axis. Examine this result for $x_0 \rightarrow \infty$ and show that it falls off like a dipole field $\vec{E} = \hat{x} \frac{\lambda_0 L^2}{3\pi\epsilon_0 x_0^3}$ and find the associated dipole moment. Hint: Expand in a Taylor series to an order that yields a nonzero result.

4.02. A long coaxial cable (see Fig. 1) carries a uniform volume charge density ρ on the inner cylinder (radius a), and a uniform surface charge density on the outer cylindrical shell (radius b). This surface charge is negative and of just the right magnitude so that the cable as a whole is electrically neutral. Find the electric field in each of the three regions:

- (a) inside the inner cylinder ($s < a$);
- (b) between the cylinders ($a < s < b$);
- (c) outside the cable ($s > b$).

Plot $|\vec{E}|$ as a function of s for these three regions.

4.03. Two spheres, each of radius R and carrying uniform charge densities $+\rho$ and $-\rho$, respectively, are placed so that they partially overlap (see Fig. 2). Call the vector from the positive center to the negative center \vec{d} . Show that the field in the region of overlap is constant, and find its value.

4.04. A sphere of radius R carries a charge density $\rho(r) = kr$ (where k is a constant). Find the energy of the configuration. Check your answer by calculating it in at least two different ways.

4.05. Two spherical cavities, of radii a and b , are hollowed out from the interior of a neutral conducting sphere of radius R (see Fig. 3). At the center of each cavity a point charge is placed, call these charges q_a and q_b respectively.

- (a) Find the surface charges σ_a , σ_b and σ_R .
- (b) What is the field outside the conductor?
- (c) What is the field within each cavity?
- (d) What is the force on q_a and q_b ?
- (e) Which of these answers would change if a third charge, say q_c , were brought near the conductor?

Take home problems

H4.01. Which of these is an electrostatic field?

- (a) $\vec{E}_1 = k(xy\hat{x} + 2yz\hat{y} + 3xz\hat{z})$;
- (b) $\vec{E}_2 = k(y^2\hat{x} + (2xy + z^2)\hat{y} + 2yz\hat{z})$.
- (c) $\vec{E}_3 = 2r \sin \theta \cos \phi \hat{r} + r \cos \theta \cos \phi \hat{\theta} - r \sin \phi \hat{\phi}$.

Here k is a constant. For the possible field, find the electrostatic potential V at (x_0, y_0, z_0) using the origin as your reference point. Check your answer by computing $\vec{\nabla}V$.

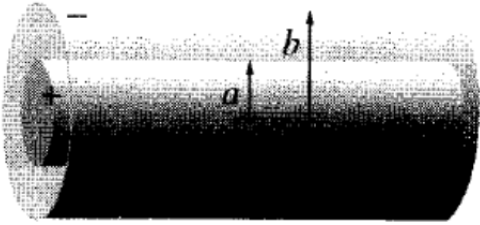


Figure 1: Problem 4.02

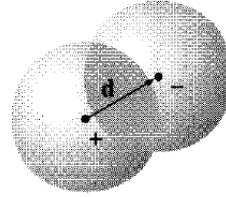


Figure 2: Problem 4.03

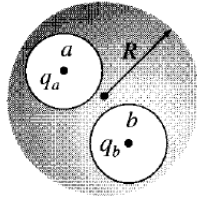


Figure 3: Problem 4.05

H4.02. Suppose the electric field in some region is found to be $\vec{E} = 2r \sin \theta \cos \phi \hat{r} + r \cos \theta \cos \phi \hat{\theta} - r \sin \theta \hat{\phi}$ in spherical polar coordinates (k is some constant).

(a) Find the charge density ρ .

(b) Find the total charge contained in the sphere of radius R , centered at the origin. (Do it two different ways.)

H4.03. If the potential $V(s, \phi, z) = s^2 z \sin \phi$ in cylindrical coordinates, calculate the energy within the region defined by $1 < s < 4$, $-2 < z < 2$ and $0 < \phi < \pi/3$.

H4.04. A spherical surface of radius R and center at the origin carries a surface charge $\sigma(\theta, \phi) = \sigma_0 \cos \theta$. Find the electric field at z on z -axis. Treat the case $z < R$ (inside) as well as $z > R$ (outside).

[Hint: Be sure to take the positive square root : $\sqrt{R^2 + z^2 - 2Rz} = (R - z)$ if $R > z$; but it is $(z - R)$ if $R < z$.]