PH102: Tutorial Problem set

Tutorial 5

2020-02-18

5.01. (a) Two spherical conducting shells of radii r_a and r_b are arranged concentrically and are charged to the potentials V_a and V_b, respectively. If $r_b > r_b$, find the potential at points between the shell, and at points $r > r_b$.

(b) Two long cylindrical shells of radius r_a and r_b arranged coaxially and are charged to the potentials V_a and V_b, respectively. Find the potential at points between the cylindrical shells.

5.02. Consider a grounded conducting sphere of radius R. A charge q is placed at a distance $a > R$ on the z-axis. The image charge $q' = -Rq/a$ is kept at distance $b = R^2/a$ on the z-axis.

(a) Using the law of cosines, show that $V(r) = \frac{1}{4\pi\epsilon_0}\left(\frac{q}{r'} + \frac{q'}{r''}\right)$ (where r' and r'' are the distances from q and q', respectively) can be written as follows:

$$V(r,\theta) = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{\sqrt{r^2 + a^2 - 2ar\cos\theta}} - \frac{1}{\sqrt{R^2 + (ar/R)^2 - 2ar\cos\theta}}\right]$$

where r and θ are the usual spherical polar coordinates. In this form it is obvious that $V = 0$ on the sphere $r = R$.

(b) Find the induced charge on the sphere, as a function of θ. Integrate this to get the total induced charge.

(c) Calculate the energy of this configuration.

5.03. Consider a point charge q situated at a distance a from the center of a grounded conducting sphere of radius R ($a > R$). The same basic model will handle the case of a sphere at any potential V_0 (relative to infinity) with the addition of a second image charge. What charge should you use, and where should you put it? Find the force of attraction between a point charge q and a neutral conducting sphere.

5.04. Two long, straight copper pipes, each of radius R, are held with their axes at a distance $2d$ apart. One is at potential V_0, the other at $-V_0$. Find the potential everywhere.

5.05. A rectangular pipe, running parallel to the z-axis (from $-\infty$ to ∞), has two grounded metal sides, at $y = 0$ and at $y = a$. At $x = 0$ side, the normal component of the electric field is zero, that is $\partial V/\partial x = 0$, where V is the potential function. The fourth side at $x = b$ is maintained at a constant potential V_0.

(a) Use the method of variable separation and write down the product solutions which satisfy boundary conditions at $y = 0, y = a$ and $x = 0$.

(b) Find the potential everywhere inside the pipe. Leave your answer in series form.

(c) What is the induced charge density on the $y = a$ surface? Again leave your answer in series form.

Take home problems

H5.01. Find the average potential over a spherical surface of radius R due to a point charge q located inside. Show that in general, $V_{ave} = V_{center} + \frac{Q_{enc}}{4\pi\epsilon_0 R}$, where V_{center} is the potential at the center due to all the external charges, and Q_{enc} is the total enclosed charge.

H5.02. A cubical box (sides of length a) consists of five metal plates, which are welded together and grounded. The top is made of a separate sheet of metal, insulated from the others and held at a constant potential V_0. Find the potential inside the box.