PH101: Physics 1

Module 2: Special Theory of Relativity - Basics

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Evaluations

Quiz-I of 10% marks on 27th August 2018 (*tentatively*)

Mid-Semester Exam of 40% (as per institute time table)

Quiz-II of 10 marks (22 October 2018)

End-Semester exam of 40% (as per institute time table)

PH101: Physics 1

Module 2: Special Theory of Relativity - Basics

Introduction (Galilean Relativity/Necessity of STR)

Some details (Lorentz transformation / Length Contraction / Time Dilation)

Addition of Velocities

Energy Momentum Relation and Kinematics

Galilean Relativity

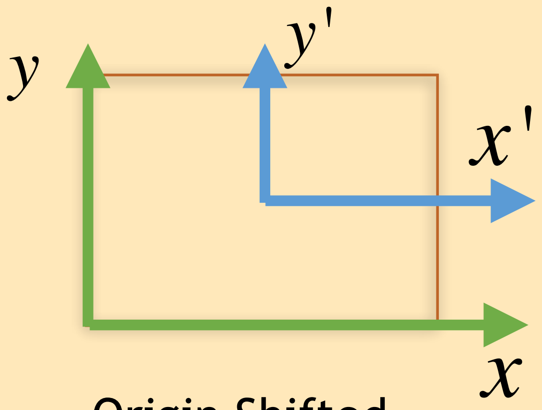
Relativity broadly refers to the idea that values of physical quantities such as **position, time, velocity, acceleration**, etc depend on the reference frame in which it is measured.

Inertial Frame: A frame in which a particle not acted upon by a force remains at rest or uniform motion.

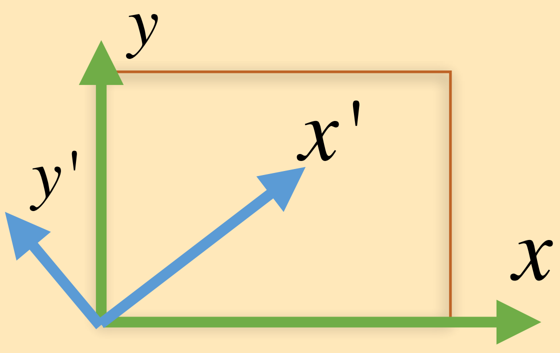
Event: A happening at a given point in space at a certain instant.
e.g. lighting of a birthday candle, flashing a search light

Choice of Frames

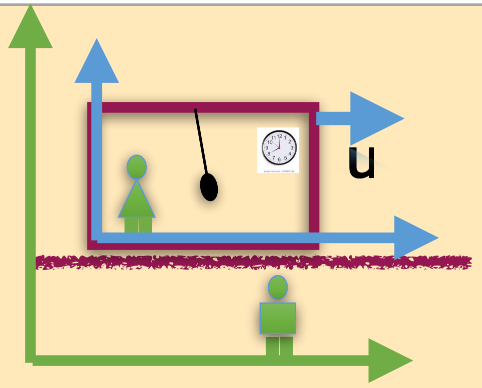
Two frames:



Origin Shifted



Axes rotated



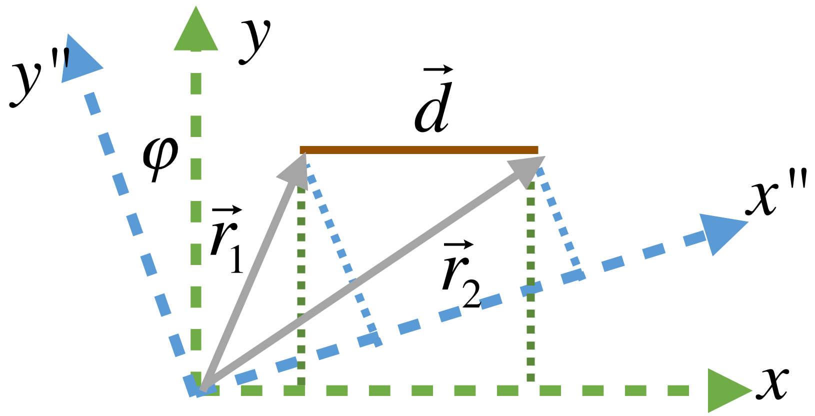
In relative motion

Galilean Relativity: Relating the coordinates in the two frames

Time: $t' = t + t_0$ where t_0 is a constant (with dimensions of time).

This ensures that time interval is the same in both the frames

Space coordinates: Adding a constant vector to the position vector, or making a rotation about any arbitrary axis leaves the distance between two points the same. (we are familiar with this).



$$\vec{r}\,'' = A\vec{r}$$

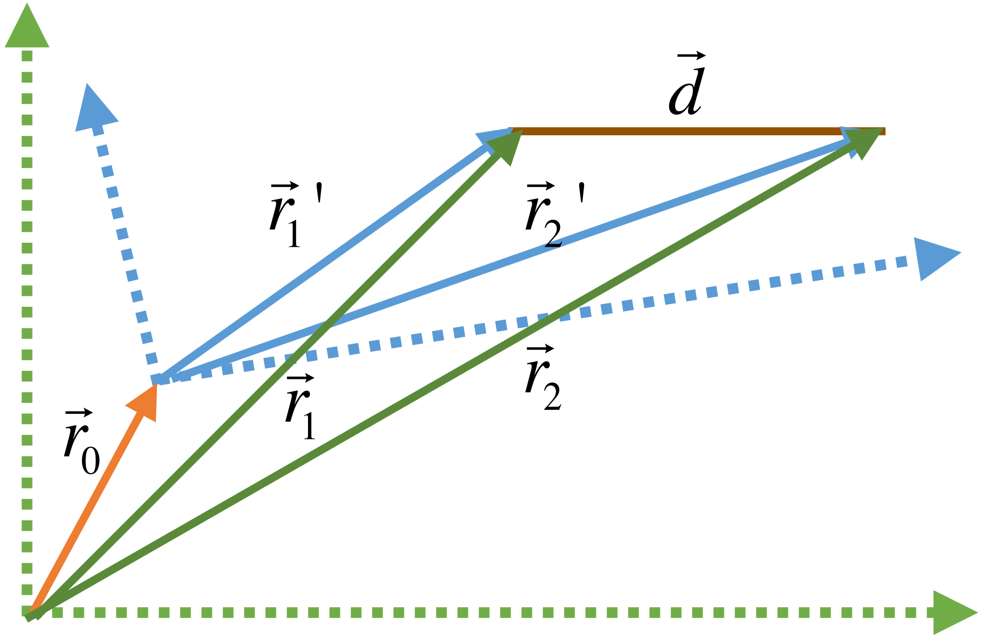
$$\begin{pmatrix} x'' \\ y'' \end{pmatrix} = \begin{pmatrix} \cos\varphi & \sin\varphi \\ -\sin\varphi & \cos\varphi \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

$$\vec{r}_1\,'' = A\vec{r}_1 \qquad \vec{r}_2\,'' = A\vec{r}_2$$

$$\vec{d} = \vec{r}_2 - \vec{r}_1$$

$$d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2 = (x''_2 - x''_1)^2 + (y''_2 - y''_1)^2$$

Galilean Relativity: Relating the coordinates in the two frames



Adding a constant vector to the position vector.

$$\vec{r}\,' = \vec{r}_0 + \vec{r}\,'' = \vec{r}_0 + A\vec{r}$$

$$\vec{d} = \vec{r}_2\,' - \vec{r}_1\,' = \vec{r}_2\,'' - \vec{r}_2\,''$$

This leads to

$$d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2 = (x'_2 - x'_1)^2 + (y'_2 - y'_1)^2$$

In addition, one frame could be moving in relative to the other (with constant velocity).

$$\vec{r}\,' = \vec{r}_0 + A(\vec{r} - \vec{v}t)$$

This summarises the most general relation between coordinates in two space-time frames in Galilean Relativity.

Galilean Relativity

An event seen from two frames, S and S' recorded as (t, x, y, z) and (t', x', y', z')

Consider two such events: (t_1, x_1, y_1, z_1); (t_1, x_2, y_2, z_2) **in S**

and (t'_1, x'_1, y'_1, z'_1); (t'_2, x'_2, y'_2, z'_2) **in S'**

Galilean Relativity: If S and S' are two inertial frames, then distance between two points (where two events take place) do not dependent on who measures them.

Note that, to measure the distance between two points we need to find their positions at the same time.

That means: $t_2 - t_1 = 0, \quad \Rightarrow t'_2 - t'_1 = 0$ Two events simultaneous in S is simultaneous in S' as well.

In general, $t_2 - t_1 = t'_2 - t'_1$ Clocks tick at the same rate.

$$(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2 = (x'_2 - x'_1)^2 + (y'_2 - y'_1)^2 + (z'_2 - z'_1)^2$$

Velocity, as seen in S and S'

Consider a particle moving with velocity

$$\vec{u} = \frac{d\vec{r}}{dt} \qquad \text{in frame S.}$$

Its velocity as measured in frame S' is

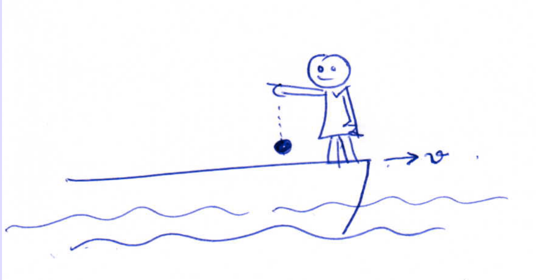
$$\vec{u}\,' = \frac{d\vec{r}\,'}{dt\,'} = \frac{d\vec{r}\,'}{dt} = \frac{d\left[\vec{r}_0 + A\left(\vec{r} - \vec{v}t\right)\right]}{dt}$$

$$= A\left(\frac{d\vec{r}}{dt} - \vec{v}\right) = A\left(\vec{u} - \vec{v}\right)$$

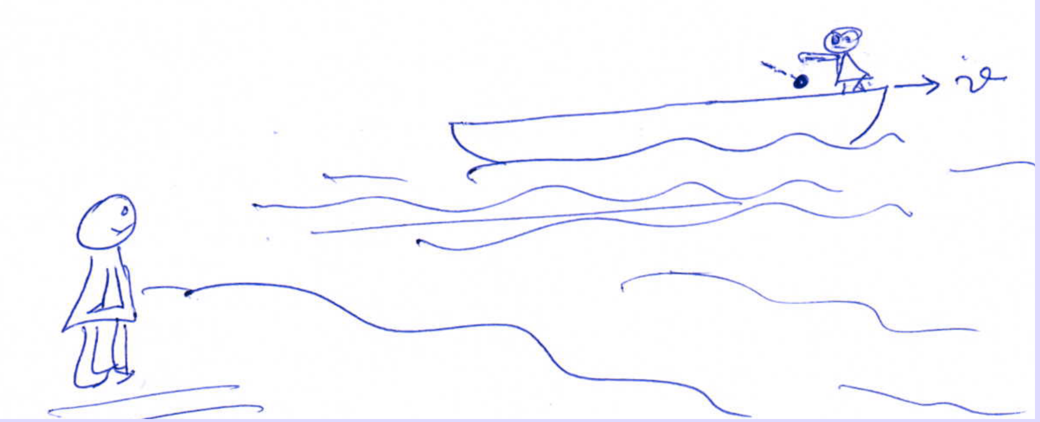
Notice that the matrix **A** is the effect of rotation. In cases where there is no rotation of the axes, but the two frames are in relative motion with respect to each other, A is a unit matrix.

Example

Consider a man in a boat moving with constant speed dropping a ball.



The man will see the ball falling vertically down.



His friend on the shore will see a different trajectory

Galileo's conclusions:

velocities have to be vectorially added

Newtonian Mechanics

Newtonian Mechanics is invariant under Galilean Transformation

Consider a particle with mass m.
In frame S, it is seen as moving with velocity $\vec{u}$

Its momentum: $\vec{p} = m\vec{u}$

Force on it is related to change of momentum as $\vec{F} = \frac{d\vec{p}}{dt}$

Seen from frame S' moving with relative velocity $\vec{v}$ $\Rightarrow$ $\vec{p}\,' = m(\vec{u} + \vec{v})$

$$\vec{F}\,' = \frac{d\vec{p}\,'}{dt\,'} = \frac{d\vec{p}}{dt} = \vec{F}$$

The particle experience the same force in all inertial frames.

Why Special Theory of Relativity

Speed of light and reference frames

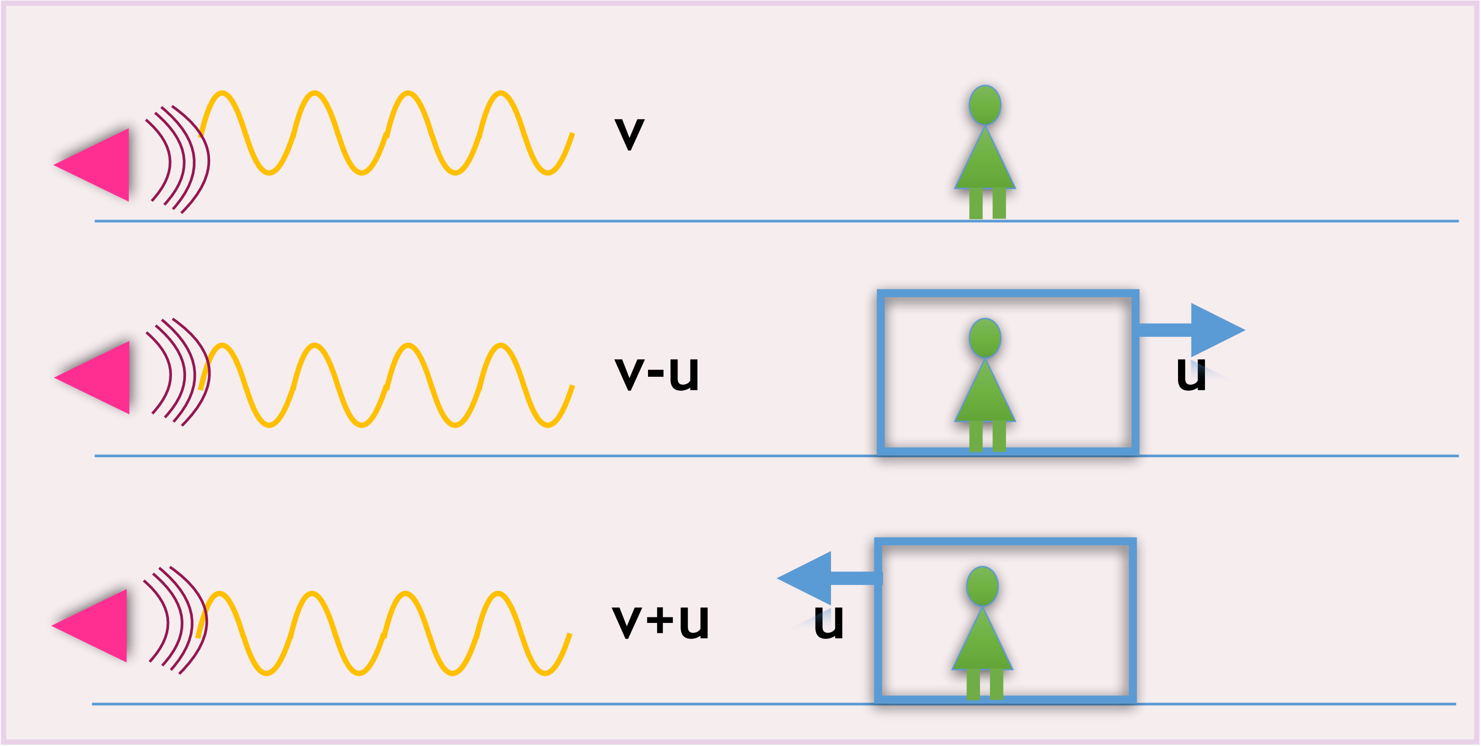
- Speed of sound in air: 340 m/s

  Air is the medium in which it propagates

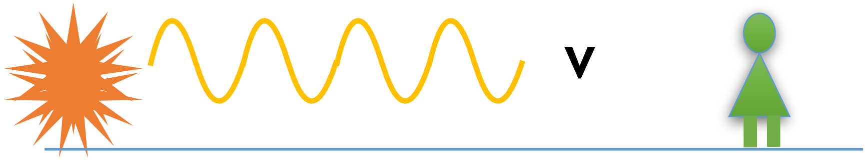
In which reference frame ?

The reference frame in which air is at rest

- For an observer moving w.r.to air



What about light?



It is thought (Newtonian/Galilean Relativity) that it behaves similar to sound

Speed of light and reference frames

Light travels in outer space, where there is no air

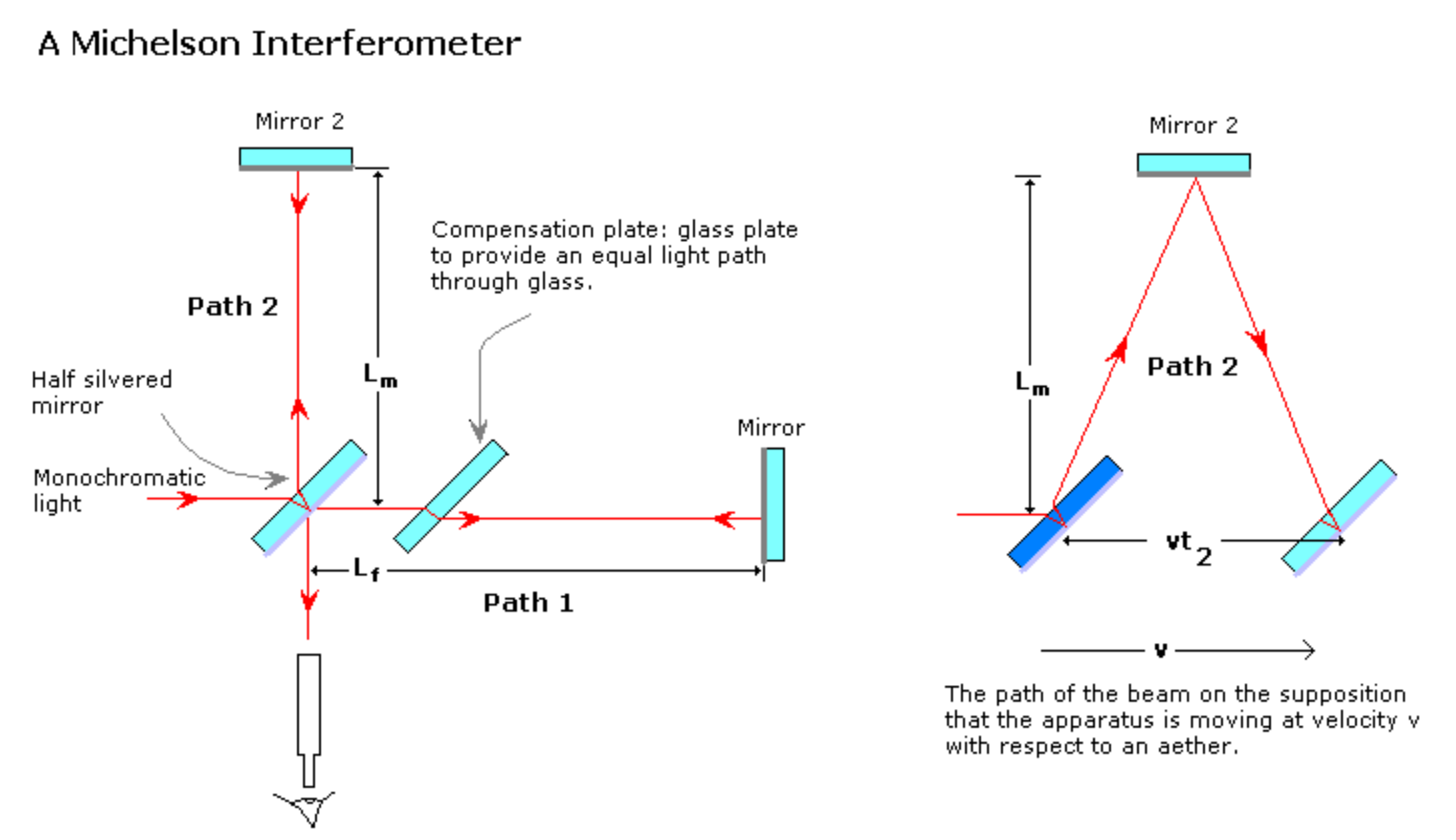
Perhaps there is an all pervading “Ether”

- the medium for light to propagate

The Test: The effect of Earth moving with respect to Ether.

As per James Clerk Maxwell,

light is electromagnetic waves with constant speed $c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$



Michelson-Morley Experiment

Expected:
Shift in the interference pattern

Observation: **NULL Result**

Conclusion: No relative speed

Electromagnetic force

Electromagnetic force under Galilean Transformation

Consider two long line charges of linear charge density λ and $-\lambda$ placed parallel to each other separated by distance d.

In frame S, these two line charges are at rest. Force on it $F_C = -\dfrac{\lambda^2}{2\pi\varepsilon_0 d}$

Seen from frame S' moving with relative velocity $\vec{v}$

These two line charges constitute current, say I in addition to having net charge on each other.

Total force = Coulomb force + Force due to two parallel currents

(one current seeing the magnetic field produced by the other)

$$F' = F_C + F_M$$

Galilean invariance is not applicable to electromagnetic force

Resolving the case: We need Special Theory of Relativity

Einstein: Space-time structure is more dynamic than we thought of.

Speed of light is constant in all frames (as required by Maxwell's EM waves).

(Consequently) Time is not absolute, as in the classical Galilean relativity.

New ideas to compare things in different frames.
(in place of constancy of spatial distance)

We shall continue the discussion in the next class