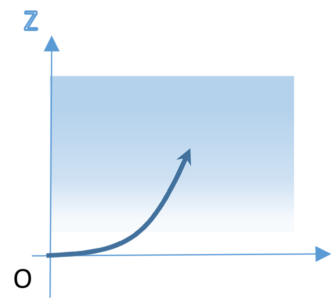
1. The cost of flying an aircraft at an altitude z is e^{-kz} per unit distance of flight path, where k is a positive constant. Consider an airplane that takes off from $(x = -a, z = 0)$ to land at $(x = a, z = 0)$. Assume that the airplane flies in the xz-plane and Earth's surface (xy-plane) is flat. Show that the trajectory that minimizes the travel cost has the form, $z = \frac{1}{k}\ln\left(\frac{\cos kx}{\cos ka}\right)$.

2. Using Fermat's principle find the path of a ray of light travelling on the xz-plane in a medium whore refractive index varies as, $n(z) = n_0\sqrt{1 + {}^{z}/_{a}}$, where n_0 and a is are positive constants and z is the depth of the medium. Assume that the light ray enters the medium at the origin, directed along the positive x-axis, and proceeds to the 1st quadrant .



3. *A soap film is formed between two co-axial rings, each of radius R, located at, $x = \pm a$. Find the function, $y(x)$, that describes the surface of the film (see figure on right). Note that the area of the surface (-which is the "surface of revolution" of the function $y = f(x)$ in question, will be minimum in order to reduce its surface energy due to surface tension.* Ignore gravity).