I.   Setup the Hamiltonian for the following systems:

1.   A particle of mass $m$ in a gravitational field of a large stationary mass $M$. Assume that the motion is on the $xy$-plane.

2.   A pendulum has a bob of mass, $m$, attached to the ceiling trough a light string whose length, $l(t)$, is varied externally (as a known function of time). Assume that the motion is in the $xy$-plane under gravity.

II.   Obtain the path followed by light in a medium whose refractive index vary radially as, $n(r,\theta) = \frac{a}{r}$, where $a$ is +ve constant and $0 < r \leq a$. Consider rays originating from the point, $A(R,0)$ on the $x-$axis and reaching different points on the $y-$axis; say, $B\,(0,R), C\,(0,R/2)$ and $D(0,2R)$.

III.   Find an expression for the geodesic on the surface of a cone of half angle, $\alpha$.

IV.   Find the **general** shape of a rope hanging between two pegs under gravity. The shape of the rope is such that it minimizes its total potential energy.