- given an undirected simple graph $G(V, E)$, compute a vertex cover $V' \subseteq V$ of minimum cardinality
  — an NP-complete problem

- greedy algorithm: find a maximal matching M in G, and output the set S of vertices incident to edges of M

- takes polynomial time

- outputs a feasible solution since for any edge e not covered by vertices in S, e can be included to further extend M

- $OPT \leq |S| = 2|M| \leq 2 \cdot OPT$

- noting that S is a feasible solution, the first inequality is due to the size of an optimal vertex cover is a lower bound on the size of any vertex cover

- the last inequality, since any vertex cover has to pick at least one vertex of each edge in M

- since there is a constant factor approximation algorithm, this problem belongs to APX class

- for tight examples, consider $K_{n,n}$, for any n

- known lower bound: if there is an (< 1.36)-approximation algorithm for this problem, then $P = NP$
  — not proved in class

- homework: reduce any vertex cover problem instance to a weighted set cover problem instance